Chapter 13
Development of Nonlinear Filtering Algorithms of Digital Half-Tone Images

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ABSTRACT

This chapter is devoted to solving the problem of algorithms and structures investigations for Radio Receiver Devices (RRD) with the aim of the nonlinear filtering of Digital Half-Tone Images (DHTI) representing the discrete-time and discrete-value random Markovian process with a number of states greater than two. At that, it is assumed that each value of the DHTI element is represented by the binary g-bit number, whose bits are transmitted via digital communication links in the presence of Additive White Gaussian Noise (AWGN). The authors present the qualitative analysis of the optimal DHTI filtering algorithm. The noise immunity of the optimal radio receiver device for the DHTI filtering with varying quantization and dimension levels is investigated.

INTRODUCTION

The assumption in synthesis of algorithms and devices for the one-dimension (per line) optimal and quasi-optimal digital half-tone image filtering that the filtering process represents the discrete valued Markovian Process (MP) with several states, has no a practical significance. It however permits better understanding of the synthesis approach of more complicated algorithms and structures of static and dynamic digital half-tone image filtering. That notwithstanding, we shall assume that the sample volume of the discrete multi-level MP is limited (for example, by the image line length) and each sample can be represented by the binary g-bit number, whose bits are transmitted through digital communication links in the presence White Gaussian Noise (WGN). The realization of a line (a column) of the digital half-tone image can be an example of such processes.

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First solutions of the continuous prototype of this type of problems for binary signals were obtained by Stratoniovich, Kulman, Tikhonov, Yarlykov, Sosulin, and others (Kulman, 1961; Stratonovich, 1959, 1960; Tihanov, 1970; Yarlykov, 1980). The filtering equations in (Kulman, 1961; Stratonovich 1959, 1960; Tihanov 1970; Yarlykov 1980) represented in the continuous form are rather complicated in realization and are not suitable for the investigation of the characteristics of pulse signal processing devices. Moreover, the absence of qualitative and quantitative features does not permit the evaluation of its effectiveness. Another interpretation of the sequence of multi-level correlated pulse signals is offered in the literature (Trubin et al, 2004; Petrov, Trubin, & Butorin, 2005; Petrov, Trubin, & Chastikov, 2007; Petrov, Trubin, & Tikhonov, 2003). The discrete signal parameter can be approximated in them via the Markov chain (MC) with several states. Equations of nonlinear filtering for the uniform MC with two equiprobable states obtained in (Petrov, Trubin, Butorin; 2005; Petrov, Trubin, & Chastikov; 2007), and devices for binary correlated signal filtering synthesized on its basis had demonstrated a high efficiency, have the simple structure, are suitable in implementation and research. The significant peculiarity of the filtering devices synthesized in (Petrov & Trubin; 2007; Petrov, Trubin, & Butorin; 2005; Petrov, Trubin, & Chastikov; 2007) is the presence the nonlinear function unit, which contains all *a priori* data about statistics of the filtered process. This creates the favorable conditions for investigation of the filtering efficiency, its stability to variation of *a priori* data and to construction of the adaptive filtering algorithms. The structure of filtering devices in (Petrov, Trubin, & Butorin, 2005; Petrov, Trubin, & Chastikov, 2007) is such that it can serve as a basis for construction of filtering devices of multi-dimension DHTI.

EQUATIONS OF ONE-DIMENSION NONLINEAR FILTERING OF THE DISCRETE-VALUE MARKOVIAN PROCESSES

Now we suppose that a discrete parameter of pulse correlated signals, which are adequate to elements of static and dynamic DHTI, represents the uniform Markovian chain with the finite state number and the finite dimension. It is necessary to obtain the filtering equations of such signals and to synthesize the filtering device structures for DHTI recovering, which are distorted by the additive WGN $n(t)$ with zero mean value and the variance σ_n^2.

Let us consider the per line (one-dimension) DHTI filtering. Let the discrete parameter μ_k of the pulse signal $s(\mu_k, t_k)$ of $k-$th image element of the independent line takes in the each time step $k = 1 \ldots m-1$ (m is the number of elements in the image line) of operation one of several values $M_1, \ldots, M_N$ with probabilities $p_1, \ldots, p_N$, accordingly. We assume that the process μ_k is the uniform Markovian chain with the given matrix of transition probability (MTP) from the state M_i in $k-$th sample into the value M_j in $(k+1)-$th sample of the following type:

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \ldots & \pi_{1N} \\ \ldots & & & \\ \pi_{N1} & \pi_{N2} & \ldots & \pi_{NN} \end{pmatrix}. \quad (1)$$

Signals applied to the RRD input increase the knowledge about the process $\mu_1, \mu_2 \ldots \mu_{k+1}$ compared with *a priori* information. Now this knowledge is defined in the $(k+1)-$th time step of system operation by the multi-dimension *a posteriori* probability density, which can be calculated in accordance with the inverse probability formula (Stratonovich, 1959, 1960; Tihanov, 1970; Yarlykov, 1980).

$$p^{as}\left(\mu_1,\dots,\mu_{k+1}\right) = cL\left(\mu_1,\dots,\mu_{k+1}\right) p^{ap}\left(\mu_1,\dots,\mu_{k+1}\right), \tag{2}$$

where $L\left(\mu_1,\dots,\mu_{k+1}\right)$ is a multi-dimension likelihood function; $p^{ap}\left(\mu_1,\dots,\mu_{k+1}\right)$ is the multi-dimension *a posteriori* probability density; c is factor of normalization.

In order to construct the estimation of the filtered MP μ_{k+1} in $\left(k+1\right)-$th time step of system operation on the basis of data entered in RRD input, it is necessary to form the final *a posteriori* probability density

$$p^{as}\left(\mu_{k+1}\right) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} p^{as}\left(\mu_1,\dots,\mu_{k+1}\right) d\mu_1 \dots d\mu_k. \tag{3}$$

We assume that on the basis of *a priori* signal knowledge, which are used for transmission of the digital half-tone image elements, and on the basis of RRD properties we can calculate the likelihood function o the parameter μ_{k+1} in $\left(k+1\right)-$th time step

$$L\left(\mu_{k+1}\right) = \exp\left\{f\left(\mu_{k+1}\right)\right\}, \tag{4}$$

where $f\left(\mu_{k+1}\right)$ is the logarithm of the likelihood function.

Thus, if noises from one time step to another are independent, then

$$L\left(\mu_1,\dots,\mu_{k+1}\right) = \exp\left\{\sum_{m=1}^{k+1} f\left(\mu_m\right)\right\}, \tag{5}$$

and the equation for the final a posteriori probability density for parameter μ in $\left(k+1\right)-$th time step with account of the recurrence property of the Markovian chain has the following form (Kulman, 1961; Petrov, Trubin, & Tikhonov, 2003).

$$p^{as}\left(\mu_{k+1}\right) = \exp\left\{f\left(\mu_{k+1}\right)\right\} \int p^{as}\left(\mu_k\right) w\left(\mu_{k+1} \middle| \mu_k\right) d\mu_k, \tag{6}$$

where the one-step probability density for transition $w\left(\mu_{k+1} \middle| \mu_k\right)$ from value of process μ_k to the adjacent μ_{k+1} can be presented in the form (Kulman, 1961) (see Box 1).

Let us write the final *a posteriori* probability density of parameter μ_k in the $k-$th time step

$$p^{as}\left(\mu_k\right) = \sum_{j=1}^{N} p_{jk} \delta\left(\mu_k - M_j\right) \tag{8}$$

and substituting Equations (7) and (8), into Equation (6) we obtain

Box 1.

$$w\left(\mu_{k+1} \middle| \mu_k\right) = \sum_{j=1}^{N} \pi\left(M_i \middle| M_j\right) \quad \delta\left(\mu_{k+1} - M_i\right); \quad i = \overline{1, N}, \tag{7}$$

where $\delta\left(\cdot\right)$ is delta-function.

$$\sum_{i=1}^{N} p_{i(k+1)}\delta\left(\mu_{k+1} - M_i\right) = c \cdot \exp f\left(\mu_{k+1}\right) \int \sum_{j=1}^{N} p_{jk}$$
$$\times\, \delta\left(\mu_k - M_j\right) \sum^{N} \pi\left(M_i \middle| M_j\right) \delta\left(\mu_{k+1} - M_j\right) d\mu_k. \tag{9}$$

Equating coefficients at similar $\delta -$ functions in the right and the left parts of Equation (9) and using the filtering property of δ-function, we obtain the equation for the final *a posteriori* probability of i-th value of parameter μ_{k+1} (see Box 2).

Equation (10) is the basis for synthesis of devices for pulse correlated signal filtering, which are approximated by the stationary discrete-value Markovian chain with $N = 2^g$ values.

Dividing Equation (10) for $i = 1, \ldots N-1$ by equations for $i = N$, we come to the equation system shown in Box 3.

Having taken the logarithm of Equation (11) and introducing a designation $u_{ik} = \ln \dfrac{p_{ik}}{p_{Nk}}$, we obtain the system of nonlinear recurrent equations (see Box 4).

Equation (12) defines those optimal operations, which can be performed by RRD under the received signal of the discrete MP element with the purpose of its best recovering.

If MP samples $\mu_1, \mu_2, \ldots, \mu_k$ are presented by g-bit binary numbers $y_1, y_2, \ldots, y_k$, the number of discrete values of the process will be equal to $N = 2^g$. For example, for $g = 8$ the number of values $N = 256$. Realization of Equation (12) for such number of discrete values requires considerable computation resources. Another approach to realize Equation (12) using a theorem given in (Petrov, Trubin; 2007) is more productive. In accordance with this theorem, the multi-level discrete-value MP presented by 2^g binary numbers y_k may be changed to g binary MP with MTP for $l -$ th bit as

$$\Pi^{(l)} = \begin{pmatrix} \pi_{11}^{(l)} & \pi_{12}^{(l)} \\ \pi_{21}^{(l)} & \pi_{22}^{(l)} \end{pmatrix}, \; l = \overline{1, g}. \tag{14}$$

Elements of bit MTP satisfy the conditions of normalization and coordination

Box 2.

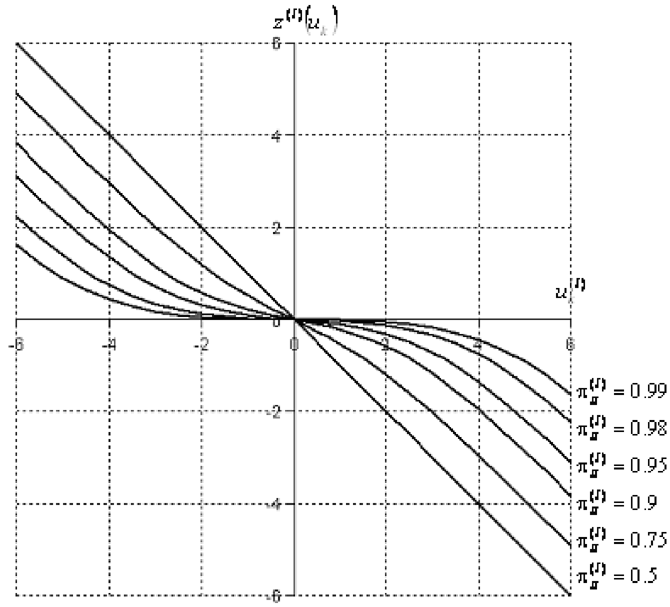
$$p_{i(k+1)} = c \cdot \exp\left\{f_{k+1}\left(M_i\right)\right\} \sum_{j=1}^{g} p_{jk}\pi\left(M_i \middle| M_j\right), \; i, j = \overline{1, g}. \tag{10}$$

Box 3.

$$\frac{p_{i(k+1)}}{p_{N(k+1)}} = \exp\left\{f_{k+1}\left(M_i\right) - f_{k+1}\left(M_N\right)\right\} \frac{p_{ik}}{p_{Nk}} \frac{1 + \sum_{j=1}^{N} \left[\frac{p_{jk}}{p_{Nk}} \Big/ \frac{p_{ik}}{p_{Nk}}\right] \pi_{ji}}{1 + \sum_{j=1}^{N} \frac{p_{jk}}{p_{Nk}} \pi_{jN}}, \tag{11}$$

where $i, j = \overline{1, N}, \;\; i \neq j.$

Figure 2. The family of $z\left(u_{1k}, \pi_{ij}\right)$ *– functions*



ANALYSIS OF NOISE-IMMUNITY OF THE ONE-DIMENSION DHTI FILTERING ALGORITHM

Synthesized devices of nonlinear filtering allow realization of the BBI statistical redundancy owing to the most probable value prediction of each following image element on the basis of previous element and *a priori* knowledge about the correlation degree of the received signals of image elements.

The more correlation between binary signals, the more exact the prediction of the received signal, and the higher accuracy of the transmitted information, i.e. the higher the noise immunity of the synthesized filtering device. Investigations (Petrov, Trubin, Butorin; 2005; Petrov, Trubin, Chastikov; 2007) showed that application of the BBI statistical redundancy in *l*-th RRD channel (Figure 1) allows obtaining the essential benefit in signal power $\eta^{(l)}\left(\pi_{ii}^{(l)}, \rho_e^2\right)$ especially at small signal/noise ratio ρ_e^2 at RRD input. The largest benefit in accordance with (17) can be achieved in the BBI transmission channel of high-order bit (l=g) of the binary number, which is the most correlated. The total benefit in all g channels is the impartial estimation of filtering effectiveness of DHTI, represented by the multi-level discrete-value MP.

One of the quantitative estimations of noise immunity of the BBI filtering device is the average error probability $p^{(l)}$ defined by Equation (21). It is necessary to establish connection between the error probability $p^{(l)}$ and MTP ${}^1\Pi^{(l)}$ of one-dimension filtering process.

Let us present the Equation (18) in the standard form of a recursive filter

$$u_{1(k+1)}^{(l)} = a_{k+1}^{(l)}[f_{k+1}^{(l)}(M_1^{(l)}) - f_{k+1}^{(l)}(M_2^{(l)})] + b_{k+1}^{(l)} u_{1k}^{(l)}, \tag{22}$$

where $a_{k+1}^{(l)}$ is a coefficient ensuring the maximal receiving of *a posteriori* data about an image element signal in *l*-th bit, i.e. we can assume that $a_{k+1}^{(l)} = a_0^{(l)} = 1;$

ister of estimation formation of the binary number symbols $\hat{Y}_{k+1} = \left\{ \hat{Y}^{(1)}_{k+1}\nu_1, ..., \hat{Y}^{(g)}_{k+1}\nu_g \right\}$. Nonlinear filters in each channel has the structure similar to $\left(\mathrm{NF}\right)^{(l)}$ of the $l-$th channel.

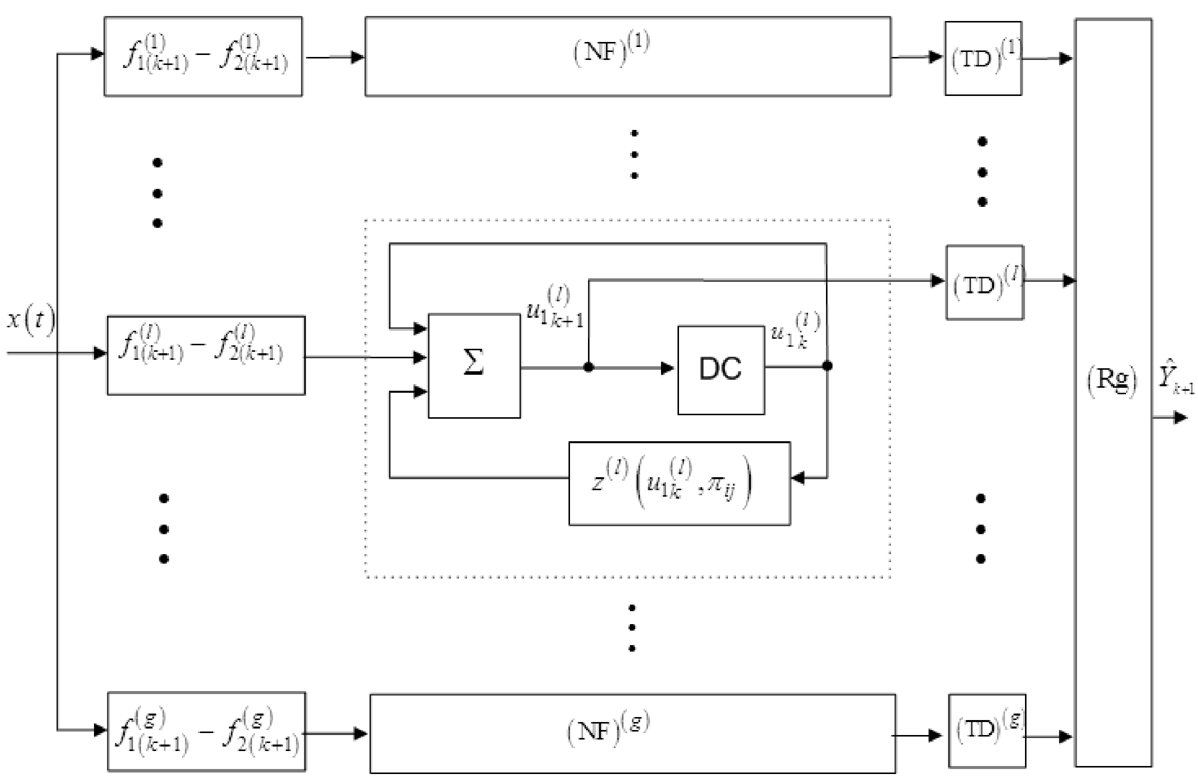
The typical peculiarity of the system of equations of type (18) consists in the fact that all *a priori* information about the extracting symbol $M_i^{(l)}\left(i=1,2\right)$ of the $l-$th bit is included in the last term. The family of curves of conversion $z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)$ for various values of $\pi_{ij}^{(l)}\left(i=j\right)$ is shown in Figure 2.

It follows from the analysis of curve family $z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)$ that for the large signal/noise ratio $\rho_e^2 >> 1$ and for probabilities $\pi_{ii}^{(l)}$, which are not close to 1, the curves of the function $z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)$ can be approximated by the line segments. At that, calculation of function $z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)$ is simplified, which allows the transfer from the optimal RRD to quasi-optimal.

The DHTI representation by the one-dimension Markovian chains is suitable by the possibility of specifying of *a priori* data for the separate line (column) of ê for the line (column) groups of the digital half-tone images, which allows the most complete consideration of the static data non-uniformity in the field of digital half-tone images.

The filtering result of the real DHTI for signal/noise ratio ρ_e^2 is presented in Figure 3. The initial DHTI "The aircraft" is shown in Figure 3a, the noisy DHTI for $\rho_e^2 = -$ 6 dB at RRD input is shown in Figure 3b, filtered DHTI – in Figure 3c.

Figure 1. The structure of the optimal $g-$channel RRD



filtering approximated by the discrete MP with two equiprobable values.

The estimate value of the BBI line element calculated in accordance with Equation (18) can be determined by comparison of $u_{1(k+1)}^{(l)}$ with the threshold chosen in accordance with some criterion of binary signal distinguish. For the considered problem of discrete element filtering of binary correlated signals of BBI element the ideal observer criterion (Stratonovich, 1959, 1960; Tihanov, 1970; Yarlykov 1980) is the most acceptable. In accordance with this criterion the distinguishing of signals with parameter $M_1^{(l)}$ or $M_2^{(l)}$ is performed in RRD on the basis of comparison of $u_{1(k+1)}^{(l)}$ with the threshold $H^{(l)}$, i.e.

$$u_{1(k+1)}^{(l)} \geq H^{(l)}. \tag{20}$$

For pulse signals the condition (20) means that the value $M_1^{(l)}$ is fixed in the cases when $u_{1(k+1)}^{(l)} \geq H^{(l)}$ and the value $M_2^{(l)}$ – when $u_{1(k+1)}^{(l)} < H^{(l)}$. At that, two types of errors may occur (Stratonovich, 1959, 1960; Tihanov, 1970; Yarlykov, 1980): errors of the first kind when the decision is made about the presence of the parameter value M_1 in the received signal, while the signal was transmitted with the parameter value M_2. Errors of the second kind occur at presence of the opposite situation.

Let us designate the probability of the first kind of errors as $p'^{(l)}$ and the second kind as $p''^{(l)}$. In accordance with the ideal observer criterion, the threshold $H^{(l)}$ is set so that to minimize the average error probability $p^{(l)}$ (Stratonovich, 1959, 1960; Tihanov, 1970; Yarlykov, 1980)

$$p^{(l)} = \frac{1}{2}\left[p'^{(l)} + p''^{(l)}\right] = \min. \tag{21}$$

In the specific case of the binary pulse signal reception in the symmetric communication system $H^{(l)} = 0$ for all $l = \overline{1, g}$.

In accordance with the Equation (18), at forming the logarithm of the *a posteriori* probabilities in the $(k+1)$-th time step $u_{1(k+1)}^{(l)}$, the input data defined by the first term are added in the summer with the value of the logarithm of the *a posteriori* probabilities ratio for the previous time step $u_{1k}^{(l)}$ and with calculated value $z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)$, which contains the *a priori* data about the filtering process. At that, our knowledge about the true value of parameter $\mu_{k+1}^{(l)}$ in the received signal increases due to *a priori* data and the average probability of the erroneous decisions $p^{(l)}$ at RDD output decreases. The reduction of the average error probability characterizes this method's benefit at pulse correlated signal processing compared with the uncorrelated case.

ONE-DIMENSION NONLINEAR DHTI FILTERING

To transmit of DHTI represented by $g-$bit binary numbers with the minimal error, it is necessary that binary symbols (0 and 1) of each $l-$th bit $\left(l \in g\right)$ were transmitted through communication link with the minimal error. The structure of optimal $g-$channel RRD performing the reception of the $g-$bit DHTI is shown in Figure 1. Each of g channel of the RRD contains a discriminator calculating the logarithm difference of the likelihood functions of discrete parameter values of binary RDD signals

$$\left[f_{k+1}^{(l)}\left(M_1\right) - f_{k+1}^{(l)}\left(M_2\right)\right],$$

a nonlinear filter $\left(\mathrm{NF}\right)^{(l)}$, a threshold device $\Pi^{(l)}\left(l \in g\right)$, which output is connected to a reg-

Box 4.

$$u_{i(k+1)} = f_{k+1}\left(M_i\right) - f_{k+1}\left(M_N\right) + u_{ik} + z\left(u_{ik}, \pi_{ii}\right),\ \ i = \overline{1,N}, \tag{12}$$

where

$$z\left(u_{ik}, \pi_{ii}\right) = \ln \frac{1 + \sum_{j=1}^{N} \exp\left(u_{jk} - u_{ik}\right) \cdot \pi_{ji}}{1 + \sum_{j=1}^{N} \exp\left(u_{jk}\right) \cdot \pi_{jN}}. \tag{13}$$

$$\sum_{j=1}^{2} \pi_{ij}^{(l)} = 1,\ \ i = 1,2;\ \ l = \overline{1,g} \tag{15}$$

$$p_i^{(l)} = \sum_{j=1}^{2} p_j^{(l)} \pi_{ji}^{(l)},\ \ i = 1,2;\ \ l = \overline{1,g}. \tag{16}$$

For binary MP the following condition holds

$$\pi_{ii}^{(1)} \le \pi_{ii}^{(2)} \le \ldots \le \pi_{ii}^{(g)}. \tag{17}$$

Having considered that the symbol sequence of $l -$ th bit of binary numbers $y_1^{(l)}, \ldots, y_{k+1}^{(l)}$ forms the discrete-value MP with two equiprobable $\left(p_1^{(l)} = p_2^{(l)}\right)$ values M_1 and M_2 and MTP (14) and assuming (although it is not principal) that all bits are transmitted simultaneously through binary communication links, we obtain the system of g nonlinear recurrent equations for binary MP filtering similar to (12) (see Box 5).

$f_{k+1}^{(l)}\left(M_1^{(l)}\right) - f_{k+1}^{(l)}\left(M_2^{(l)}\right)$ is the logarithm difference of likelihood functions of the binary signal parameter values of *l*-th bit of number y_{k+1};

$u_{ik}^{(l)} = \ln \frac{p_{ik}^{(l)}}{p_{nk}^{(l)}}$ is the logarithm of *a posteriori* probabilities ratio in $k -$ th time step of $l -$ th bit of number y_k.

SYNTHESIS OF A DEVICE FOR ONE-DIMENSION NONLINEAR FILTERING OF BIT BINARY IMAGES

On the basis of the filtering equations obtained in the previous section, we synthesize an optimal device of nonlinear Bit Binary Images (BBI)

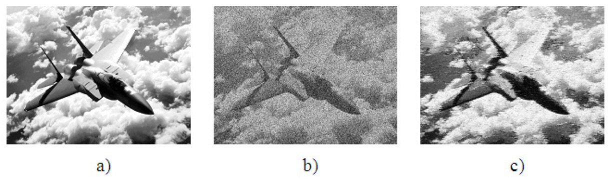
Box 5.

$$u_{1(k+1)}^{(l)} = f_{k+1}^{(l)}\left(M_1^{(l)}\right) - f_{k+1}^{(l)}\left(M_2^{(l)}\right) + u_{1k}^{(l)} + z_1^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right), \tag{18}$$

where

$$z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right) = \ln \frac{\pi_{11}^{(l)} + \pi_{21}^{(l)} \exp\left\{-u_{1k}^{(l)}\right\}}{\pi_{22}^{(l)} + \pi_{12}^{(l)} \exp\left\{u_{1k}^{(l)}\right\}};\ i,j = 1,2;\ \ l = \overline{1,g}; \tag{19}$$

Figure 3. An example of one-dimension nonlinear filtering of the artificial DHTI



$$b_{k+1}^{(l)} = \frac{u_{1k}^{(l)} + z^{(l)}\left(u_{1k}^{(l)}, \pi_{ij}^{(l)}\right)}{u_{1k}^{(l)}} \tag{23}$$

is a coefficient in the feedback circuit of the recursive filter unambiguously dependent on *a priori* statistics of the binary filtering process.

The Equation (22) is similar to the equation of recirculator of the equidistant pulse burst (Lezin, 1969) if we use (as a pulse burst) the average pulse burst equaled to an average length of pulse series containing the discrete values $M_1^{(l)}$ or $M_2^{(l)}$ and the coefficient in the feedback circuit $b_{k+1}^{(l)}$ can be averaged during the action time of such pulse burst.

Since the sequence $\left\{\mu_{k+1}^{(l)}\right\}$ is by the data the Markovian chain, the probability density of events consisting in the fact that series of n time steps contain only values $M_1^{(l)}$ or $M_2^{(l)}$, is equal

$$w^{(l)}\left(n\right) = p_i^{(l)} \cdot \left(w^{(l)}\right)^{n-1}; \; i = 1,2, \tag{24}$$

where $w^{(l)} = \pi_{ii}^{(l)}\delta\left(w^{(l)} - \pi_{ii}^{(l)}\right)$ is a probability density of transition absence from one value $\mu_k^{(l)}$ ti adjacent $\mu_{k+1}^{(l)}$ in the line of $l-$th BBI, $p_i^{(l)}$ is the initial probability of i-th state of the Markovian chain.

The average length of the sequence (the burst) $\chi^{(l)} = \left\langle n^{(l)} \right\rangle$ consisting of values $M_1^{(l)}$ or $M_2^{(l)}$ only, we can obtain from the equation

$$\chi^{(l)} = \frac{p_i^{(l)}}{1 - \pi_{ii}^{(l)}}. \tag{25}$$

Knowing $\chi^{(l)}$ and considering that at bipolar signals the bursts consist of the positive and negative pulses, we can calculate from (25) the value of MTP elements of the Markovian chain of type (14):

$$\pi_{ii}^{(l)} = 1 - \frac{2p_i^{(l)}}{\chi^{(l)}}, \; \pi_{ij}^{(l)} = 1 - \pi_{ii}^{(l)}, \; i = 1,2, \; i \neq j. \tag{26}$$

Having averaged the coefficient $b_{k+1}^{(l)}$ for the action time of the pulse burst wit length $\chi^{(l)}$, we obtain

$$\left\langle b_{k+1}^{(l)} \right\rangle_{\chi^{(l)}} = b^{(l)}. \tag{27}$$

Taking into consideration that burst consist of pulses of the same sign, the effect of accumulation of pulse bursts with the opposite signs in RRD with nonlinear filtering decreases two times compared with the recirculator accumulating the burst of single-sign pulses. Therefore, the signal/noise ratio in signal power at the output of the l-th channel of RRD (Figure 1) $\rho_{e\text{ out}}^2$ can be determined similar to (Lezin, 1969).

$$\left(\rho_{e\text{ out}}^2\right)^{(l)} = \rho_e^2 \frac{1 + b^{(l)}}{2\left(1 - b^l\right)} \left(1 - \left(b^{(l)}\right)^{\chi^{(l)}}\right)^2, \tag{28}$$

where ρ_e^2 is signal/noise ratio at RRD input.

From Equation (28) we can find the benefits in signal power at output of the $l-$th channel of RRD with

For binary phase-modulated (PM) for $\left(\pi_{ii}^{(l)} = 0,95\right)$ the average pulse burst length $\chi^{(l)}$ is equal

$$\chi^{(l)} = 2 \cdot 0,5 \Big/ \left(1 - 0,95\right) = 20. \tag{0.1}$$

An estimation of average value of the coefficient $\hat{b}^{(l)}$ in the feedback circuit of nonlinear filter of the $l-$th RRD channel obtained by calculation way on PC for signal/noise ratio at RRD input $\rho_e^2 = -3$ dB is equal to $\hat{b}^{(l)} = \left\langle b_{k+1}^{(l)} \right\rangle_{\chi^{(l)}=20} = 0,62$. Substituting $\hat{b}^{(l)}$ in Equation (0.1), we find benefits in signal power at RRD output

$$\eta_e^{(l)} = \frac{1 + 0,62}{2\left(1 - 0,62\right)} \left(1 - 0,62^{20}\right)^2 \cong \frac{1,62}{0,76} = 3,3 \text{ dB} \tag{0.2}$$

Simulation of the optimal RRD (Figure 1) on PC gives benefits in power $\eta_e^{(l)} = 3,3$ dB confirming correctness of this approach to analysis of RRD noise immunity for binary correlated signals of the DDI elements representing the uniform Markovian chain with two equiprobable values.

Using the formula for error probability at distinguishing of two determined signals (Stratonovich 1959, 1960; Tihanov, 1970; Yarlykov, 1980) we find the error probability $p^{(l)}$

$$p^{(l)} = 1 - \Phi\left[\sqrt{\eta_e^{(l)} \left(\rho_e^{(l)}\right)^2 \left(1 - r_s\right)}\right], \tag{0.3}$$

where r_s is the normalized auto-correlation function of the binary signal; $\Phi\left(\cdot\right)$ is the integral of error probability (Stratonovich 1959, 1960; Tihanov, 1970; Yarlykov, 1980).

STABILITY OF ALGORITHMS OF THE ONE-DIMENSION NONLINEAR DHTI FILTERING TO INACCURACY OF A PRIORI DATA

After development of the device for nonlinear DHTI filtering it is necessary to investigate its stability to inaccuracy of specifying or calculation of *a priori* statistical data about the filtering process. The importance of these investigations is caused by the fact that we understand *a priori* data as the averaged statistical characteristics obtained by averaging over the large image ensemble, however, in the real life these data may not coincide (with different degree) with the specific realization data – the specific real image or even its separate fragments. Thus, it is necessary to investigate at which degree this lack of coincidence may influence on the RRD noise immunity.

For $l-$th channel of the optimal RRD (Figure 1) we conducted the check on stability of its effectiveness for $\pi_{ii}'^{(l)} \neq \pi_{ii}^{(l)}$, $(i = \overline{1,2})$, which is well known in RRD theory. The check was fulfilled for those values of $\pi_{ii}^{(l)}$, which deviation from the true values of transition probability can lead to essential error increase at RRD output.

Investigation was conducted using the artificial images obtained on the model described in the Chapter 10 of this book (sections 1.2 and 1.4). In the nonlinear filtering device $\left(\text{NF}\right)^{(l)}$ (Figure 1) we specified the statistical data different from values used at image generation.

The obtained results showed that the lower the signal/noise ratio ρ_e^2 at RRD input the higher the sensitivity to mismatching. At fixed ρ_e^2, the higher the true value of the transition probability $\pi_{ii}^{(l)}$ in $l-$th BBI of half-tine image, the higher the sensitivity of $\left(\mathrm{NF}\right)^{(l)}$ to mismatching.

So, if for ${}^{1}\pi_{ii}^{(8)} = 0,90$ the mismatching of ±3% does not influence practically on the filtering effectiveness, then for ${}^{1}\pi_{ii}^{(8)} = 0,95$ the similar error may lead to the loss more than 1 dB in the case of overestimation of the transition probability. Taking this into account, the requirements to the presentation accuracy of *a priori* data in the high-order bit (the most correlated) should be higher.

Since the bit binary images of the digital half-tone images are processed independently, stability of the nonlinear filtering algorithm of the digital half-tone images directly depends on stability of the $(NL)^{(l)}$ $l-$th bit binary images.

EQUATIONS OF TWO-DIMENSION NONLINEAR DHTI FILTERING

Let in $t_{k+1}-$th time step of system operation in the interval $T = t_{k+1} - t_k$ one BBI element is transmitted with the help of the signal $s^{(l)}\left(\mu_{i,j}^{(l)}\right)$, $\left(i=\overline{1,m};\ j=\overline{1,n};\ l=\overline{1,g}\right)$, which discrete parameter $\mu_{i,j}^{(l)}$ (a frequency, a phase etc.) takes one of the two possible values $M_1^{(l)}$, $M_2^{(l)}$.

We chose the one-way Markovian random field (Chapter 10) as the DHTI model. In accordance with the mathematical model (MM) of the two-dimension DHTI we assume that the element sequence $\left\{\mu_{i,j}^{(l)}\right\}$ of $l-$th BBI forms the two-dimension stationary Markovian chain on the non-symmetrical half-plane (NSHP) (Chapter 10, Figure 1.1) with equiprobable values $p_1^{(l)} = p_2^{(l)}$ and with the conditional transition probabilities (see Box 6).

For complicated Markovian chain *a priori* distribution $p_{ap}^{(l)}$ of parameter values in $l-$th bit can be presented in the form (Petrov, Trubin, 2007; Petrov, Kharina; 2006) with account of the given MM of two-dimension DHTI (see Box 7).

Since by the data the distorting noise is white, the likelihood function for the signal sequence of elements of $l-$th BBI can be written in the following form (Stratonovich 1959, 1960; Tihanov, 1970; Yarlykov, 1980):

Box 6.

$$\pi_{khqr}^{(l)} = w\left(\mu_{i,j}^{(l)} = M_k^{(l)} \middle| \mu_{i-1,j}^{(l)} = M_r^{(l)};\ \mu_{i,j-1}^{(l)} = M_h^{(l)};\ \mu_{i-1,j-1}^{(l)} = M_q^{(l)}\right), \tag{33}$$

where $i=\overline{1,m}$; $j=\overline{1,n}$; $l=\overline{1,g}$; $n \times m$ - is the size of image field; $k,h,q,r=\overline{1,2}.$

Box 7.

$$\begin{aligned}
&p_{ap}^{(l)}\left(\mu_{1,1}^{(l)}, \mu_{1,2}^{(l)}, \ldots, \mu_{i-1,j}^{(l)}, \mu_{i-1,j-1}^{(l)}, \mu_{i,j-1}^{(l)}\right) \\
&= p_{ap}^{(l)}\left(\mu_{1,1}^{(l)}, \mu_{1,2}^{(l)}, \mu_{2,1}^{(l)}\right) w\left(\mu_{2,2}^{(l)} \middle| \mu_{1,1}^{(l)}, \mu_{1,1}^{(l)}, \mu_{2,1}^{(l)}\right) \\
&\times \ldots \times w^{(l)}\left(\mu_{i,j}^{(l)} \middle| \mu_{i-1,j}^{(l)}, \mu_{i-1,j-1}^{(l)}, \mu_{i,j-1}^{(l)}\right).
\end{aligned} \tag{34}$$

$$L^{(l)}\left(\mu_{1,1}^{(l)};\mu_{1,2}^{(l)};\ldots;\mu_{2,1}^{(l)};\mu_{2,2}^{(l)};\ldots;\mu_{i-1,j}^{(l)};\mu_{i,j}^{(l)}\right) = \exp\left[\sum_{q=1}^{i}\sum_{r=1}^{j} f^{(l)}\left(\mu_{q,r}^{(l)}\right)\right], \tag{35}$$

where $f^{(l)}\left(\mu_{q,r}^{(l)}\right)$ is the logarithm of the likelihood function of the parameter of the element of $l-$th BBI with coordinates $\left(q,r\right)$; $q=\overline{1,m}$; $r=\overline{1,n}$.

With account of (33), (34) and (35), *a posteriori* distribution of the discrete parameter of the two-dimension field of l-th BBI will be that shown in Box 8.

Having integrated (36) over all values of elements $\left\{\mu_{q,r}^{(l)}\right\}$ with account of (34), we obtain the equation of the final *a posteriori* probability density of the element $\nu_4^{(l)}$ value (see Box 9).

The direct realization of the Equation (37) is difficult due to multi-dimension character of the *a posteriori* probability density and the transition probability density in the right part. All *a priori* in formation about the value of the element of l-th BBI $\mu_{i,j}^{(l)}$ with coordinates $\left(i,j\right)$ is contained in the integrand (37).

1. Taking into account properties of the complicated Markovian chain and assumptions made at construction of the spatial model of two-dimension Markovian process with discrete arguments in the Chapter 10, we present (with account of Figure 1.2 of Chapter 10) the integrand in (37) in the form:

$$p_{as}^{(l)}\left(\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\middle|\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)}\right) = \frac{p_{as}^{(l)}\left(\nu_1^{(l)}\right)w^{(l)}\left(\nu_1^{(l)}\middle|\nu_4^{(l)}\right)p_{as}^{(l)}\left(\nu_2^{(l)}\right)w^{(l)}\left(\nu_2^{(l)}\middle|\nu_4^{(l)}\right)}{p_{as}^{(l)}\left(\nu_3^{(l)}\right)w^{(l)}\left(\nu_3^{(l)}\middle|\nu_4^{(l)}\right)}. \tag{38}$$

Having changed in (37) the integrand to (38), we obtain an equation for the final *a posteriori* probability density of the filtering element $\nu_4^{(l)}$

Box 8.

$$p_{as}^{(l)}\left(\mu_{1,1}^{(l)},\mu_{1,2}^{(l)},\ldots,\mu_{2,1}^{(l)},\mu_{2,2}^{(l)},\ldots,\mu_{i-1,j}^{(l)},\mu_{i,j}^{(l)}\right) = c\cdot\exp\left\{\sum_{q=1}^{i}\sum_{r=1}^{j} f^{(l)}\left(\mu_{q,r}^{(l)}\right)\right\}p_{ap}^{(l)}\left(\mu_{1,1}^{(l)},\mu_{1,2}^{(l)},\mu_{2,1}^{(l)}\right) \times w^{(l)}\left(\mu_{2,2}^{(l)}\left(\mu_{1,1}^{(l)},\mu_{1,2}^{(l)},\mu_{2,1}^{(l)}\right)\times\ldots\times w^{(l)}\left(\mu_{i,j}^{(l)}\left(\mu_{i-1,j}^{(l)},\mu_{i,j-1}^{(l)},\mu_{i-1,j-1}^{(l)}\right),\right.\right. \tag{36}$$

where c is the normalizing coefficient.

Box 9.

$$p_{as}^{(l)}\left(\nu_4^{(l)}\right) = c\cdot\exp\left\{f^{(l)}\left(\nu_4^{(l)}\right)\right\}\iiint p_{ac}^{(l)}\left(\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)}\right) \times w^{(l)}\left(\nu_4^{(l)}\middle|\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)}\right)d\nu_1^{(l)}d\nu_2^{(l)}d\nu_3^{(l)}, \tag{37}$$

where $\nu_1^{(l)}=\mu_{i-1,j}^{(l)};\nu_2^{(l)}=\mu_{i,j-1}^{(l)};\nu_3^{(l)}=\mu_{i-1,j-1}^{(l)};\nu_4^{(l)}=\mu_{i,j}^{(l)}$.

and includes the summer $\left(\Sigma^{(l)}\right)$, the memory unit $\left(\mathrm{SD}_1^{(l)}\right)$ for storing m DHTI elements equaled to the element number on the single line (*i* elements of the current line and *m-i* elements of the previous line), and three loops of feedback each containing the unit realizing the nonlinear function $z_r^{(l)}\left(\cdot\right),\ \left(r=\overline{1,3}\right)$.

As it is seen from Figure 4, in formation of $u^{(l)}\left(\nu_4^{(l)}\right)$ the following data participate: input data, data about values of adjacent (to $\nu_4^{(l)}$) elements of $l-$th BBI belonging to the vicinity $\Lambda_{i,j}^{(l)}=\left\{\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)}\right\}$ and calculated values of nonlinear functions $z_r^{(l)}\left(\cdot\right),\ \left(r=\overline{1,3}\right)$, in which there is *a priori* information about the filtering process.

Because of the fact that nonlinear functions $z_r^{(l)}\left(\cdot\right),\ \left(r=\overline{1,3}\right)$ depend on *a priori* statistics, they have the special significance for formation of $u^{(l)}\left(\nu_4^{(l)}\right)$.

In the case of statistical independence of field elements $\left({}^{r}\pi_{ii}^{(l)}=0,5\right)$ functions $z_r^{(l)}\left(\cdot\right),\ \left(r=\overline{1,3}\right)$ take the values

$$z_1^{(l)}\left(u^{(l)}\left(\nu_1^{(l)}\right),\ {}^{1}\pi_{\alpha\beta}^{(l)}\right)=-u^{(l)}\left(\nu_1^{(l)}\right);$$
$$z_2^{(l)}\left(u^{(l)}\left(\nu_2^{(l)}\right),\ {}^{2}\pi_{\alpha\beta}^{(l)}\right)=-u^{(l)}\left(\nu_2^{(l)}\right);$$
$$z_3^{(l)}\left(u^{(l)}\left(\nu_3^{(l)}\right),\ {}^{3}\pi_{\alpha\beta}^{(l)}\right)=-u^{(l)}\left(\nu_3^{(l)}\right),$$

and $u^{(l)}\left(\nu_4^{(l)}\right)$ is forming only on the basis of input data, i.e. on the basis of logarithm difference of the likelihood functions

$$u^{(l)}\left(\nu_4^{(l)}\right)=\left[f\left(M_1^{(l)}\left(\nu_4^{(l)}\right)\right)-f\left(M_2^{(l)}\left(\nu_4^{(l)}\right)\right)\right]. \quad (47)$$

At that, loops of feedback (Figure 4) are breaking and a decision about the presence in the received signal of the image element with the value $M_k^{(l)},\left(k=1,2\right)$ is made on the basis of a single sample.

After filtering, the BBI elements are collected in the $g-$bit register $\left(\mathrm{Rg}\right)$ for estimation of the binary number with the appropriate weighting coefficients:

$$\overline{y}_{i,j}=\sum_{l=1}^{g}y_{i,j}^{(l)}\cdot 2^{l-1} \quad (48)$$

Factual summing is absent because each BBI corresponds to own bit position in the register with the appropriate weight.

THE NOISE IMMUNITY ANALYSIS OF ALGORITHMS FOR TWO-DIMENSION NONLINEAR DHTI FILTERING

2. Radio receiving device (Figure 4) was modeled on PC. The artificial binary and half-tone images with the number of elements 1024x1024 constructed on the basis of the model described in Chapter 10 and the real images represented in the digital view, which have 2^8=256 brightness levels and is enough for description of the half-tone images, were taken as the initial images. At that, we assumed that the image elements are transmitted in bit-by-bit manner at different signal/noise ratio in power ρ_e^2 at RRD input. The following ratio (Petrov, Trubin; 2007) was taken as the quantitative estimation of recovering of $l-$th BBI:

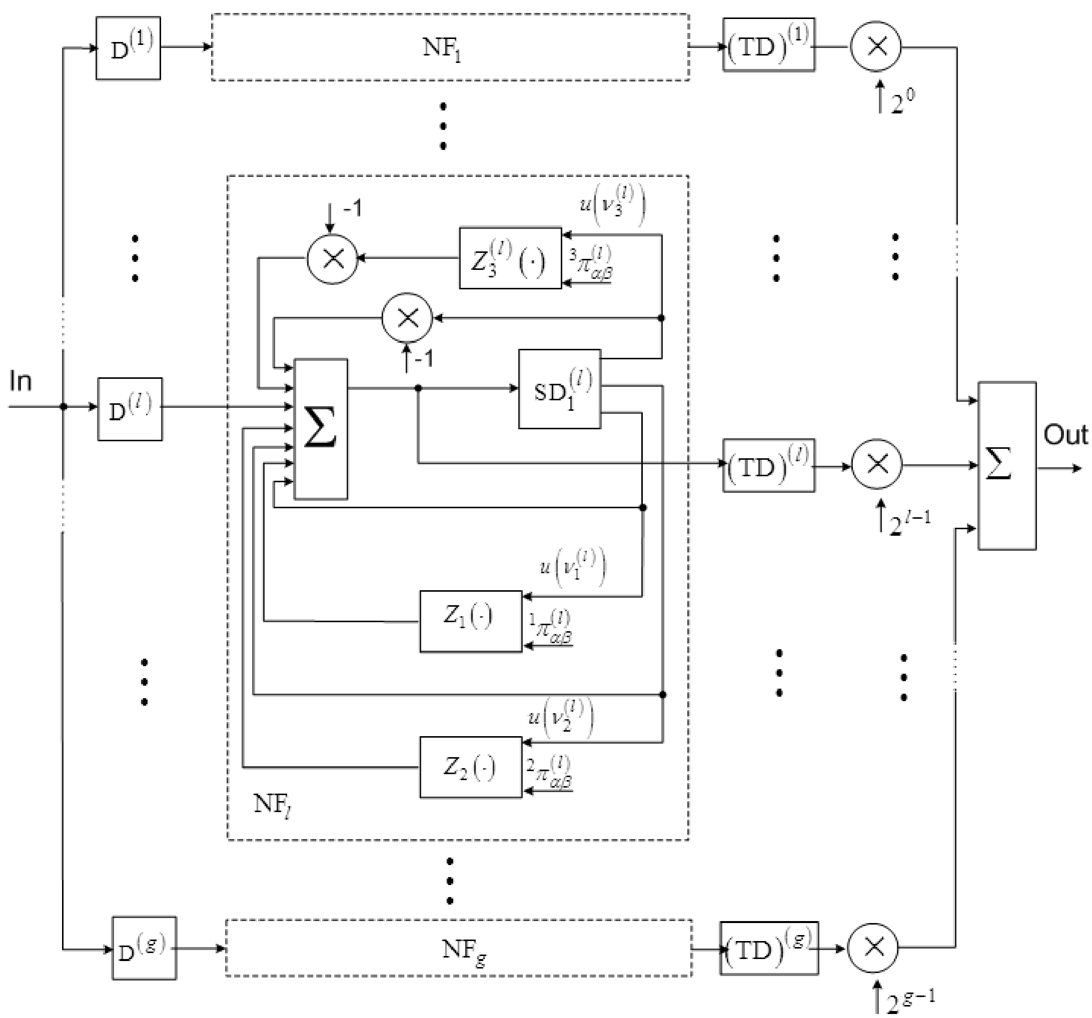
$$u^{(l)}\left(\nu_4^{(l)}\right) \geq H^{(l)}. \qquad (46)$$

The RRD structure for two-dimension filtering of $g-$bit DHTI, which simulates the system of g Equation (43) and the decision rule (46) is presented in Figure 4.

This structure contains discriminators of binary signals $\left(\mathrm{D}^{(1)},\ldots,\mathrm{D}^{(g)}\right)$ realizing operations of calculation the logarithm difference of the likelihood functions of l-th channel $\left[f\left(M_1^{(l)}\left(\nu_4\right)\right)-f\left(M_2^{(l)}\left(\nu_4\right)\right)\right]$, $\left(l=\overline{1,g}\right)$; nonlinear bit filters $\left(\mathrm{NF}^{(1)},\ldots,\mathrm{NF}^{(g)}\right)$, memory units $\left(\mathrm{SD}_1^{(l)}\right)$ for delay of image elements in horizontal, vertical and diagonal lines and $\left(\mathrm{SD}_2^{(l)}\right)$ for storage of MTP element values $\left({}^1\boldsymbol{\Pi}^{(l)}, {}^2\boldsymbol{\Pi}^{(l)}, {}^3\boldsymbol{\Pi}^{(l)},\ l=\overline{1,g}\right)$, summers, computational units for nonlinear function (44) and the same threshold devices $\left[\left(\mathrm{TD}\right)^{(1)},\ldots,\left(\mathrm{TD}\right)^{(g)}\right]$.

A nonlinear filter smoothes the single samples arriving from a detector in discrete time moments

Figure 4. The device of two-dimension nonlinear DHTI filtering



Box 13.

$$u^{(l)}\left(\nu_4^{(l)}\right)=f\left(M_1^{(l)}\left(\nu_4^{(l)}\right)\right)-f\left(M_2^{(l)}\left(\nu_4^{(l)}\right)\right)$$
$$+u^{(l)}\left(\nu_1^{(l)}\right)+z_1^{(l)}\left(u^{(l)}\left(\nu_1^{(l)}\right),{}^{1}\pi_{\alpha\beta}^{(l)}\right)$$
$$+u^{(l)}\left(\nu_2^{(l)}\right)+z_2^{(l)}\left(u^{(l)}\left(\nu_2^{(l)}\right),{}^{2}\pi_{\alpha\beta}^{(l)}\right)$$
$$-u^{(l)}\left(\nu_3^{(l)}\right)-z_3^{(l)}\left(u^{(l)}\left(\nu_3^{(l)}\right),{}^{3}\pi_{\alpha\beta}^{(l)}\right),\tag{43}$$

where

$$z_r^{(l)}\left(u^{(l)}\left(\nu_r^{(l)}\right),{}^{r}\pi_{\alpha\beta}^{(l)}\right)=\ln\frac{{}^{r}\pi_{\alpha\alpha}^{(l)}+{}^{r}\pi_{\beta\alpha}^{(l)}\exp\left\{-u^{(l)}\left(\nu_r^{(l)}\right)\right\}}{{}^{r}\pi_{\beta\beta}^{(l)}+{}^{r}\pi_{\alpha\beta}^{(l)}\exp\left\{u^{(l)}\left(\nu_r^{(l)}\right)\right\}};$$
$$\left(l=\overline{1,g};\ r=\overline{1,3};\ \alpha,\beta=\overline{1,2}\right).\tag{44}$$

for the final *a posteriori* probability of the signal discrete parameter in $\left(i,j\right)-$th element of $l-$th BBI of the digital half-tone image (see Box 12).

Dividing Equation (42) at $\beta=1$ by equation at $\beta=2$ and using results obtained in (Kulman, 1961), we come to an equation for nonlinear filtering of elements of $l-$th BBI of the digital half-tone image (see Box 13).

The nonlinear recurrent Equation (43) defines optimal operations, which should be fulfilled under given elements of *l*-th BBI, for the purpose of the best its recovering under influence of the white Gaussian noise. Taking into account that DHTI is presented in the form of g BBI, the DHTI filtering equation we represent as a system from g equations of type (43) (Petrov, Trubin and Butorin, 2003; Petrov, Trubin; 2007).

All *a priori* information about statistical dependence of elements of *l*-th binary image is concentrated in term of type (14), where ${}^{r}\pi_{\alpha\beta}^{(l)}\left(\alpha,\beta=\overline{1,2}\right)$ are elements of the transition probability matrices of elements of *l*-th bit of the digital half-tone image in horizontal ${}^{1}\Pi_{\alpha\beta}^{(l)}$, vertical ${}^{2}\Pi_{\alpha\beta}^{(l)}$ and diagonal ${}^{3}\Pi_{\alpha\beta}^{(l)}$ lines, interconnected by the equation

$${}^{3}\Pi_{\alpha\beta}^{(l)}={}^{1}\Pi_{\alpha\beta}^{(l)}\,{}^{2}\Pi_{\alpha\beta}^{(l)}=\begin{pmatrix}{}^{1}\pi_{11}^{(l)} & {}^{1}\pi_{12}^{(l)}\\ {}^{1}\pi_{21}^{(l)} & {}^{1}\pi_{22}^{(l)}\end{pmatrix}\cdot\begin{pmatrix}{}^{2}\pi_{11}^{(l)} & {}^{2}\pi_{12}^{(l)}\\ {}^{2}\pi_{21}^{(l)} & {}^{2}\pi_{22}^{(l)}\end{pmatrix}.\tag{45}$$

THE STRUCTURE OF THE DEVICE FOR TWO-DIMENSION NONLINEAR DHTI FILTERING

Since *a priori* data about the filtering process is assumed known (similar to algorithms of the one-dimension DHTI filtering), we take as a criterion of distinguishing the BBI element values the criterion of the ideal observer. In accordance with this criterion, the decision about the presence in the received realization of the signal $s^{(l)}\left(\mu_{i,j}^{(l)}\right)$ with BBI element values $M_1^{(l)}$ or $M_2^{(l)}$ is performed on the basis of comparison of the logarithm *a posteriori* probability ratio with some threshold $H^{(l)}$

Box 10.

$$p_{as}^{(l)}\left(\nu_4^{(l)}\right)=c\cdot\exp\left\{f^{(l)}\left(\nu_4^{(l)}\right)\right\}$$
$$\times\iiint\frac{p_{as}^{(l)}\left(\nu_1^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\middle|\nu_1^{(l)}\right)p_{as}^{(l)}\left(\nu_2^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\middle|\nu_2^{(l)}\right)}{p_{as}^{(l)}\left(\nu_3^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\middle|\nu_3^{(l)}\right)}d\nu_1^{(l)}d\nu_2^{(l)}d\nu_3^{(l)}. \qquad (39)$$

Box 11.

$$p_{as}^{(l)}\left(\nu_k^{(l)}\right)=\sum_{\alpha=1}^{2}p_{\alpha}^{(l)}\left(\nu_k^{(l)}\right)\delta\left(\nu_k^{(l)}-M_{\alpha}^{(l)}\right),\ \left(k=\overline{1,4}\right); \qquad (40)$$

$$w^{(l)}\left(\nu_i^{(l)}\middle|\nu_j^{(l)}\right)=\sum_{\beta=1}^{2}{}^{r}\pi_{\alpha\beta}^{(l)}\delta\left(\nu_i^{(l)}-M_{\beta}^{(l)}\right),$$
$$\left(i=3,4;\ j=\overline{1,3};\ r=\overline{1,3};\ \alpha=1,2\right), \qquad (41)$$

where $p_{\alpha}^{(l)}\left(\nu_k^{(l)}\right)$ is *a posteriori* probability of value $M_{\alpha}^{(l)}$ in $\left(\nu_k^{(l)}\right)-$ th element of $l-$ th binary section; ${}^{r}\pi_{\alpha\beta}^{(l)}$ is the transition probability from value $M_{\alpha}^{(l)}$ to $M_{\beta}^{(l)}$ in field elements in horizontal $\left(r=1\right)$, vertical $\left(r=2\right)$ and diagonal $\left(r=3\right)$ lines; $\delta\left(\cdot\right)$ is the delta-function.

Box 12.

$$p_{\beta}^{(l)}\left(\nu_4^{(l)}\right)=c'\exp\left\{f\left(M_{\beta}^{(l)}\left(\nu_4^{(l)}\right)\right)\right\}$$
$$\times\frac{\sum_{\alpha=1}^{2}p_{\alpha}^{(l)}\left(\nu_1^{(l)}\right)\cdot{}^{1}\pi_{\alpha\beta}^{(l)}\sum_{\theta=1}^{2}p_{\theta}^{(l)}\left(\nu_2^{(l)}\right)\cdot{}^{2}\pi_{\theta\beta}^{(l)}}{\sum_{\phi=1}^{2}p_{\phi}^{(l)}\left(\nu_3^{(l)}\right)\cdot{}^{3}\pi_{\phi\beta}^{(l)}};\ \beta=\overline{1,2};\ \mathrm{l}=\overline{1,g}. \qquad (42)$$

expressed through *a posteriori* probability densities of the elements adjacent to $\nu_4^{(l)}$ (Figure 1.2) and one-dimension probability densities of their transitions from one value to another in horizontal or vertical line of $l-$th BBI (see Box 10).

Let us present *a posteriori* probability densities of the field element values and the transition probability densities for the values of image elements in (39) in the form shown in Box 11.

Substituting (40) and (41) into (39) and integrating with delta-functions, we obtain an equation

$$\eta_e^{(l)} = 10\lg\left(\left(\rho_{\text{out}}^2\right)^{(l)} \Big/ \rho_{\text{e}}^2\right) \text{ dB}, \tag{49}$$

where $\left(\rho_{\text{out}}^2\right)^{(l)}$ is the signal/noise ratio in power at the nonlinear filter output of $l-$th BBI ($\rho_e^{2(1)}, \rho_e^{2(2)}, ..., \rho_e^{2(g)} = \rho_e^2$).

The power benefit η is shown in Figure 5 for one-dimension (curve 1) and two-dimension (curve 2) filtering of the same real DHTI. The analysis of curves presented in Figure 5, demonstrates the higher effectiveness of two-dimension filtering of digital half-tone images compared to one-dimension case.

Figure 6 shows the example of two-dimension filtering of the artificial DHTI with ${}^1\pi_{\alpha\alpha} = {}^2\pi_{\alpha\alpha} = 0,9$ distorted by WGN for $\rho_e^2 = -6$ dB and recovered with the help of the optimal algorithm. ${}^1\pi_{\alpha\alpha} = {}^2\pi_{\alpha\alpha} = 0,9$ $\rho_e^2 = -6$ dB. The benefit in signal power $\eta^{(l)}$ is 8.8 dB.

In Figure 7 the example of the real binary image filtering with transition probability matrix elements ${}^1\Pi^{(l)}$ and ${}^2\Pi^{(l)}$ in horizontal and vertical lines accordingly ${}^1\pi_{ii}^{(l)} = 0,946$, ${}^2\pi_{ii}^{(l)} = 0,947$ distorted by WGN $\left(\rho_e^2 = -9 \text{ dB}\right)$ is shown. The benefit $\eta^{(l)}$ is 9.8 dB.

At investigation of filtering effectiveness of artificial digital half-tone images, the MTP was specified in accordance with averaged values obtained on the basis of analysis of large number of real image samples presented in the digital form (Lezin, 1969; Trubin et al, 2004). The analysis of distribution of MTP element values on bits of the digital half-tone images shows that transition probability in horizontal and vertical lines have the similar character: MTP element values in vertical line is usually less than in horizontal line by 3…5%.

Figure 8 shows the function of benefit in signal power η for developed optimal (curve 1) and known (Butorin, 2004) filtering methods of half-tone images such as the method of the two-dimension maximal *a posteriori* estimation (MAP estimation) based on the matrix equation solution for different values of a convergence criterion

Figure 5. Benefit in power η for filtering of the same real DHTI (1- one-dimension-; 2- two-dimension-; 3-three-dimension filtering)

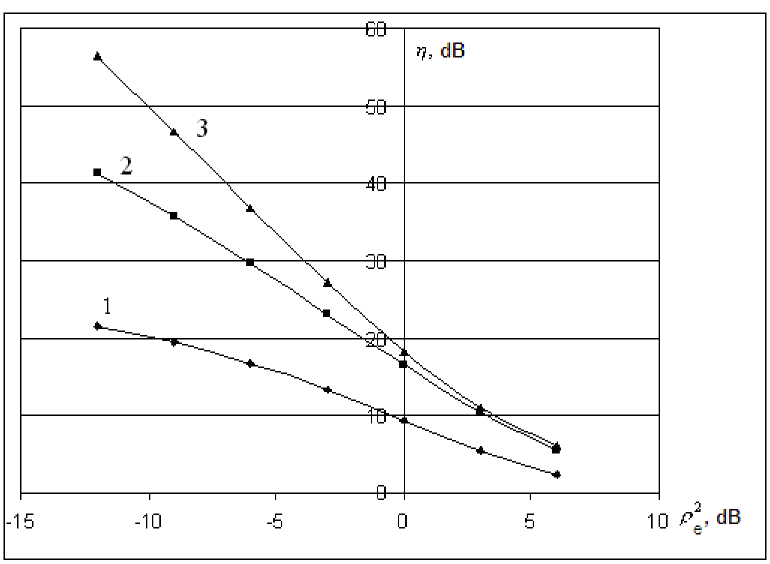
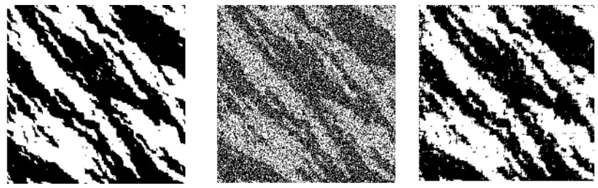


Figure 6. Example of two-dimension filtering of the artificial BBI: a) initial; b) noisy; c) filtered



a) b) c)

(curves 2, 3); the method of consecutive smoothing (curve 4); the method of an one-dimension filter (curve 5) and the two-dimension method of filter loop (curve 6). From analysis of curves of Figure 8 it follows that in the coincident range of signal/noise ratios $\rho_e^2 = 0 \ldots 5$ dB the offered method of two-dimension nonlinear DHTI filtering gives the best results essentially exceeding in computation resource saving.

From analysis of the curves presented in Figures 5 and 8 we see that the offered filtering algorithm is essentially effective in the case when the signal power is equal or less than the noise power. It is evident that more correlated high-order bits make the most contribution to the total filtering benefit. Moreover, distortions in low-order bits are practically indistinguishable by the human eye due to their less contribution to the total brightness level of the image. Therefore, the calculation volume can be essentially reduced if to refuse from low-order bit processing passing to 4-bit representation of digital half-tone images (Kulman, 1961).

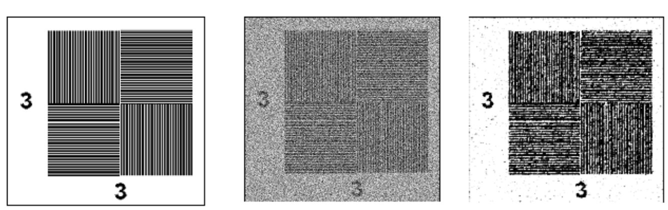
So, if in the simulated image it is possible not to process four low-order bits, the loss is not more than 2 dB compared to the complete processing (~37 dB) for signal/noise ratio at device input up to -12 dB, and at that, the computational complexity will reduce exactly twice. Figure 9 shows an example of image "Church" processing distorted by the white noise at $\rho_e^2 = 0$ dB. The benefit in signal power η is 19 dB.

The analysis of two-dimension filtering results of the static DHTI shows that representation of the digital half-tone image by the set of g BBI permits to obtain filtering algorithms providing the image reception noise immunity, which does not yield the known algorithms, and to keep the high uniformity of RRD structure and essentially reduce the number of computation operations and a memory volume, which size in RRD with nonlinear filtering is equal to the number of image elements of an one line.

NONLINEAR FILTERING OF THE DHTI VIDEO-SEQUENCE

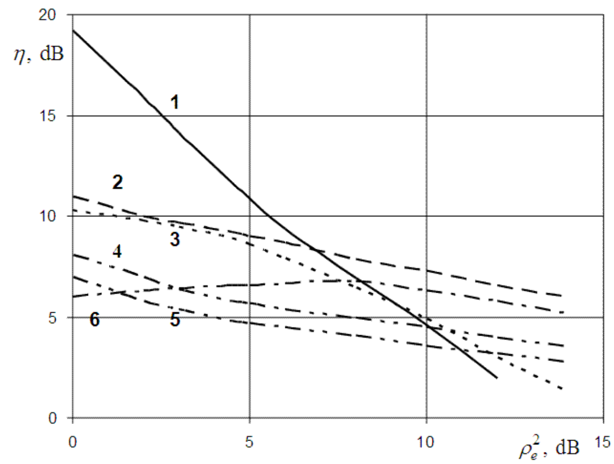
We carry out the synthesis and investigation of the filtering algorithm on the basis of the math-

Figure 7. Example of filtering of the real binary image: a) initial; b) noisy; c) filtered



a) b) c)

Figure 8. Benefit in signal power η for the most known filtering methods for half-tone images



ematical model of three-dimension discrete-value Markovian process considered in Chapter 10.

At digital representation of half-tone images by the set of g BBI (g is a digit capacity of a binary number) the problem of DHTI video-sequence filtering can be reduced to the problem of BBI video-sequence filtering.

Let in $k-$th frame in the interval $T = t_h - t_{h-1}$ $\left(h = \overline{1, m \times n}\right)$ we transmit the one BBI element by the signal $s\left(\mu_{i,j,k}^{(l)}\right)$, $\left(l = \overline{1, g}\right)$, which discrete parameter $\mu_{i,j,k}^{(l)}$ takes one from two possible values $M_1^{(l)}$ or $M_2^{(l)}$.

We assume that the sequence of BBI element values $\left\{\mu_{i,j,k}^{(l)}\right\}$ forms the complicated stationaty Markovian chain with equiprobable $p_1^{(l)} = p_2^{(l)}$, $\left(l = \overline{1, g}\right)$ states $M_1^{(l)}$, $M_2^{(l)}$, and with conditional transition probabilities from one value to another (see Box 14).

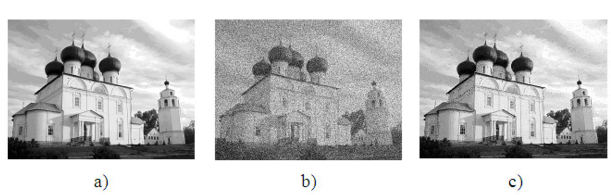
Let $\nu_4^{(l)} = \mu_{i,j,k}^{(l)}$ be the $\left(i, j, k\right)-$th element of l-th BBI in the k-th sequence frame of quantized half-tone images of the Markovian type. Than for the multi-dimension *a priori* probability density we can write the expression of type shown in Box 15.

At presence of the additive WGN distorting the image, the likelihood function for the sequence of element values of $l-$th BBI can be written in the form shown in Box 16.

Taking into consideration (50), (51) and (52), *a posteriori* value distribution of the three-dimension l-th binary Markovian process will have the form shown in Box 17.

The direct realization o Equation (54) is difficult due to the multi-dimension character of probability densities in the right part of and the

Figure 9. Example of processing of "church" image: a) initial DHTI; b) noisy; c) filtered



presence of the statistical correlation between arguments. All *a priori* information concerning the possible element value of $l-$th BBI $\nu_4^{(l)}$ is defined by the multipliers under integral designations in (54).

Let us change an integrand in (54) by some equivalent obtained on the basis calculation of mutual information between elements of $l-$th BBI belonging to the vicinity $\Lambda_{i,j,k}^{(l)}=\left\{\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right\}$ andthe image element $\nu_4^{(l)}$ (Trubin et al, 2004; Petrov, Trubin; 2007) (see Box 18).

Substituting (55) into (54), we obtain the equation for *a posteriori* probability density of the image element value $\nu_4^{(l)}$ expressed through one-dimension *a posteriori* probability densities of elements of $l-$th BBI entering in $\Lambda_{i,j,k}^{(l)}=\left\{\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right\}$ andthe probability density of its possible transitions into values of the signal parameter of the image element $\nu_4^{(l)}$ (see Box 19).

Let us represent *a posteriori* probability densities and the transition probability densities in (56) as shown in Box 20.

Substituting (57), (58) into (56), having integrated with delta-functions and equating coefficients at the same delta-functions, we obtain the equation for the final *a posteriori* probability of the element value of $l-$th BBI $\nu_4^{(l)}$ (see Box 21).

Having divided Equation (59) at $\beta=1$ by the equation at $\beta=2$ and taking the logarithm in left and right parts, we obtain a system from g recurrent equation of nonlinear filtering of $l-$th

Box 14.

$$\pi_{lqrstfgh}^{(l)}=\pi^{(l)}\left\{\mu_{i,j,k}^{(l)}=M_l^{(l)}\middle|\mu_{i,j-1,k}^{(l)}=M_q^{(l)};\mu_{i-1,j,k}^{(l)}=M_r^{(l)};\mu_{i-1,j-1,k}^{(l)}=M_t^{(l)};\right.$$
$$\left.\mu_{i,j,k-1}^{(l)}=M_t^{(l)};\mu_{i,j-1,k-1}^{(l)}=M_f^{(l)};\mu_{i-1,j,k-1}^{(l)}=M_g^{(l)};\mu_{i-1,j-1,k-1}^{(l)}=M_h^{(l)}\right\} \qquad (50)$$

where $l,q,r,s,t,f,g,h=1,2.$

Box 15.

$$P^{(l)}\left\{\mu_{i,j,q}^{(l)};i=\overline{1,m};j=\overline{1,n};q=\overline{1,k}\right\}=\prod_{q=1}^{k}\prod_{i=1}^{m}\prod_{j=1}^{n}P^{(l)}\left\{\mu_{i,j,q}^{(l)}\right\}$$
$$\times\, w^{(l)}\left\{\mu_{i,j,k}^{(l)}\middle|\mu_{i,j-1,k}^{(l)},\mu_{i-1,j-1,k}^{(l)},\mu_{i-1,j,k}^{(l)},\mu_{i,j-1,k-1}^{(l)},\ldots\right. \qquad (51)$$
$$\left.\mu_{i-1,j-1,k-1}^{(l)},\mu_{i-1,j,k-1}^{(l)},\mu_{i,j,k-1}^{(l)}\right\},\quad l=\overline{1,g}.$$

If $i=j=q=1,$ than $P^{(l)}\left\{\mu_{i,j,q}^{(l)}\right\}=0,5;$

If $i=1,j>1,q=1,$ then $P^{(l)}\left\{\mu_{1,j,1}^{(l)}\right\}=P^{(l)}\left\{\mu_{1,1,1}^{(l)}\right\}\prod_{j=2}^{n}w^{(l)}\left\{\mu_{1,j,1}^{(l)}\middle|\mu_{1,j-1,1}^{(l)}\right\};$

If $i>1,j>1,q=1,$ then $P^{(l)}\left\{\mu_{i,j,1}^{(l)}\right\}=P^{(l)}\left\{\mu_{1,1,1}^{(l)}\right\}\prod_{j=2}^{m}\prod_{i=2}^{n}w^{(l)}\left\{\mu_{i,j,1}^{(l)}\middle|\mu_{i-1,j-1,1}^{(l)}\right\}.$

data, the data about values of image elements belonging to the vicinity

$$\Lambda_{i,j,k}^{(l)} = \left\{\nu_1^{(l)}, \nu_2^{(l)}, \nu_3^{(l)}, \nu_1'^{(l)}, \nu_2'^{(l)}, \nu_3'^{(l)}, \nu_4'^{(l)}\right\},$$

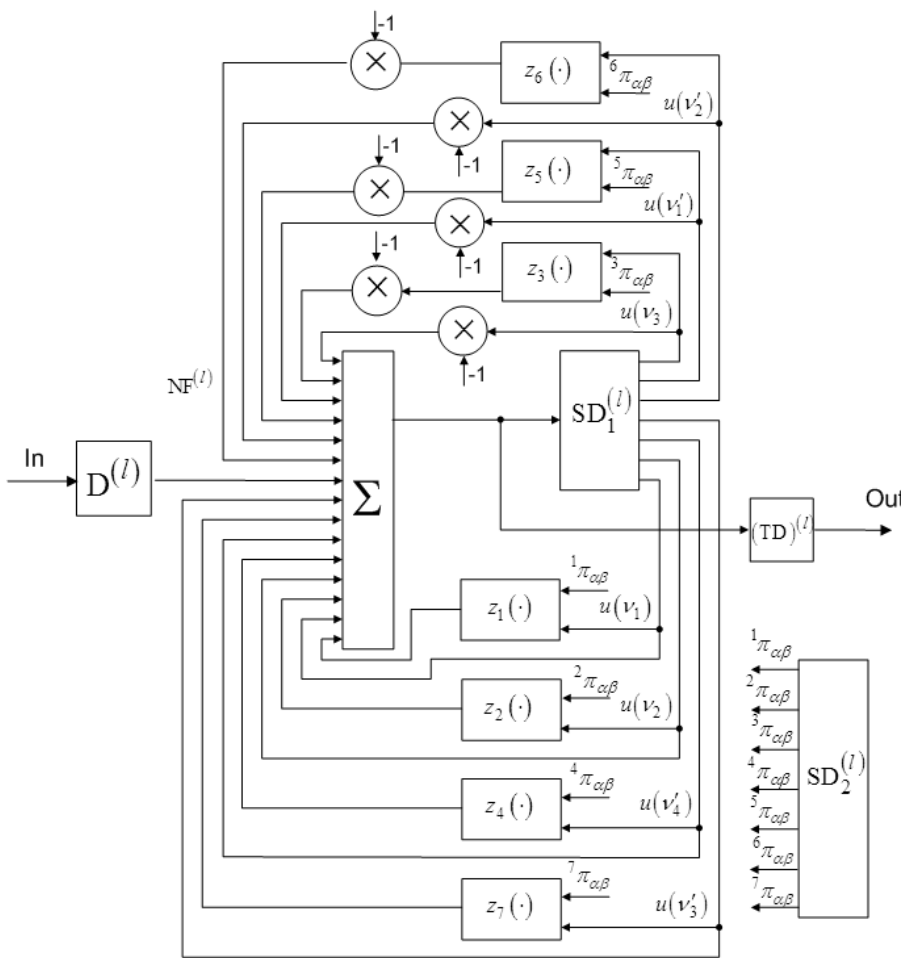
and calculated values of nonlinear functions $z_i^{(l)}\left(\cdot\right)$, $i = \overline{1,7}$, in which *a priori* information about the filtering process is contained.

The optimal RRD for filtering the video-sequence of binary images, which realizes Equations (60) – (62) and the decision rule (63) is presented in Figure 10. It contains of the discriminator $\left(\mathrm{D}^{(l)}\right)$ calculating the difference of logarithms of likelihood functions of the discrete parameter values of binary signals $\left[f_{k+1}^{(l)}\left(M_1\right) - f_{k+1}^{(l)}\left(M_2\right)\right]$; the nonlinear filter including the summer $\left(\Sigma^{(l)}\right)$, the memory cells to store values of the image elements in the vicinity

$$\Lambda_{i,j,k}^{(l)} = \left\{\nu_1^{(l)}, \nu_2^{(l)}, \nu_3^{(l)}, \nu_1'^{(l)}, \nu_2'^{(l)}, \nu_3'^{(l)}, \nu_4'^{(l)}\right\} -$$

$\left(\left(\mathrm{SD}_1^{(l)}\right)\right)$ and values of elements of the matrices of transition probabilities ${}^r\pi_{\alpha\beta}^{(l)}$, $r = \overline{1,7}$ – $\left(\left(\mathrm{SD}_2^{(l)}\right)\right)$, the seven loops of feedback, each of

Figure 10. Optimal RRD for filtering the video-sequence of binary images



Box 22.

$$
\begin{aligned}
u^{(l)}\left(v_4\right) = & \left[f\left(M_1^{(l)}\left(\nu_4^{(l)}\right)\right) - f\left(M_2^{(l)}\left(\nu_4^{(l)}\right)\right)\right] \\
& + u^{(l)}\left(v_1^{(l)}\right) + z_1^{(l)}\left[u^{(l)}\left(v_1^{(l)}\right), {}^1\pi_{ij}^{(l)}\right] + u^{(l)}\left(v_2^{(l)}\right) + z_2^{(l)}\left[u^{(l)}\left(v_2^{(l)}\right), {}^2\pi_{ij}^{(l)}\right] \\
& + u^{(l)}\left(v_4'^{(l)}\right) + z_4^{(l)}\left[u^{(l)}\left(v_4'^{(l)}\right), {}^4\pi_{ij}^{(l)}\right] + u^{(l)}\left(v_3'^{(l)}\right) + z_7^{(l)}\left[u^{(l)}\left(v_3'^{(l)}\right), {}^7\pi_{ij}^{(l)}\right] \\
& - u^{(l)}\left(v_3^{(l)}\right) - z_3^{(l)}\left[u^{(l)}\left(v_3^{(l)}\right), {}^3\pi_{ij}^{(l)}\right] - u^{(l)}\left(v_1'^{(l)}\right) - z_5^{(l)}\left[u^{(l)}\left(v_1'^{(l)}\right), {}^5\pi_{ij}^{(l)}\right] \\
& - u^{(l)}\left(v_2'^{(l)}\right) - z_6^{(l)}\left[u^{(l)}\left(v_2'^{(l)}\right), {}^6\pi_{ij}^{(l)}\right],
\end{aligned}
\tag{60}
$$

where $u^{(l)}\left(\nu_4^{(l)}\right) = \ln \dfrac{p_1^{(l)}\left(\nu_4^{(l)}\right)}{p_2^{(l)}\left(\nu_4^{(l)}\right)}, \quad l = \overline{1,g};$

$$
z_p^{(l)}\left(\cdot\right) = \ln \frac{{}^p\pi_{\alpha\alpha} + {}^p\pi_{\beta\alpha}\exp\left\{-u^{(l)}\left(\nu_q^{(l)}\right)\right\}}{{}^p\pi_{\beta\beta} + {}^p\pi_{\alpha\beta}\exp\left\{u^{(l)}\left(\nu_q^{(l)}\right)\right\}}; p = \overline{1,3};\ \ j = \overline{1,3}; \tag{61}
$$

$$
z_p^{(l)}\left(\cdot\right) = \ln \frac{{}^p\pi_{\alpha\alpha} + {}^p\pi_{\beta\alpha}\exp\left\{-u^{(l)}\left(\nu_q'^{(l)}\right)\right\}}{{}^p\pi_{\beta\beta} + {}^p\pi_{\alpha\beta}\exp\left\{u^{(l)}\left(\nu_q'^{(l)}\right)\right\}}; q = \overline{1,4};\ \ p = \overline{4,7}. \tag{62}
$$

BBI in k-th frame (Petrov, Trubin and Butorin, 2003; Petrov, Trubin; 2007) (see Box 22).

Equations (60) are the basis for synthesis of the device for nonlinear spatial-time filtering of the binary Markovian image belonging to l-th section in k-th frame of the DHTI video-sequence.

SYNTHESIS OF THE DEVICE FOR NONLINEAR FILTERING OF THE DHTI VIDEO-SEQUENCE

The structure of Equation (60) has the high uniformity and it permits to synthesize filtering algorithms being rather simple in implementation.

We chose (as at synthesis of two-dimension filtering algorithms) as a criterion of distinguishing of discrete values $M_1^{(l)},\ M_2^{(l)}$ of l-th BBI the ideal observer criterion (Stratonovich 1959, 1960; Tihanov, 1970; Yarlykov, 1980), in accordance with which the decision about the presence in the received signal realization $s\left(\mu_{i,j,k}^{(l)}\right)$ of the image element $\mu_{i,j,k}^{(l)}$, having the value $M_1^{(l)}\ or\ M_2^{(l)}$ is made on the basis of comparison of logarithm of *a posteriori* probability ratio with some threshold H:

$$
u_{i,j,k}^{(l)}\left(\nu_4^{(l)}\right) \geq H \tag{63}
$$

In the symmetrical system, when $p_1^{(l)} = p_2^{(l)}$, the threshold H=0.

From Equation (60) we see that the following data contribute in forming of $u_{i,j,k}^{(l)}$: the input

Box 19.

$$p_{as}^{(l)}\left(\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right)$$
$$\times w^{(l)}\left(\nu_4^{(l)}\left|\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right.\right)$$
$$=\frac{p^{(l)}\left(\nu_1^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_1^{(l)}\right.\right)p^{(l)}\left(\nu_2^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_2^{(l)}\right.\right)}{p^{(l)}\left(\nu_3^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_3^{(l)}\right.\right)} \tag{56}$$
$$\times\frac{p^{(l)}\left(\nu_4'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_4'^{(l)}\right.\right)p^{(l)}\left(\nu_3'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_3'^{(l)}\right.\right)}{p^{(l)}\left(\nu_1'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_1'^{(l)}\right.\right)p^{(l)}\left(\nu_2'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_2'^{(l)}\right.\right)}.$$

Box 20.

$$p_{as}^{(l)}\left(\nu_q^{(l)}\right)=\sum_{\alpha=1}^{2}p_\alpha^{(l)}\left(\nu_q^{(l)}\right)\delta\left(\nu_q^{(l)}-M_\alpha^{(l)}\right)\ ;$$
$$p_{as}^{(l)}\left(\nu_q'^{(l)}\right)=\sum_{\gamma=1}^{2}p_\gamma^{(l)}\left(\nu_q'^{(l)}\right)\delta\left(\nu_q'^{(l)}-M_\gamma^{(l)}\right);\quad q=\overline{1,4};\ l=\overline{1,g}, \tag{57}$$
$$w^{(l)}\left(\nu_4^{(l)}\left|\nu_j^{(l)}\right.\right)=\sum_{\beta=1}^{2}{}^{r}\pi_{\alpha\beta}^{(l)}\delta\left(\nu_4^{(l)}-M_\beta^{(l)}\right);j=\overline{1,3};r=\overline{1,3};\alpha,\beta=1,2;\ \ l=\overline{1,g};$$
$$w^{(l)}\left(\nu_4^{(l)}\left|\nu_h'^{(l)}\right.\right)=\sum_{\vartheta=1}^{2}{}^{r}\pi_{\alpha\vartheta}^{(l)}\delta\left(\nu_4^{(l)}-M_\vartheta^{(l)}\right);h=\overline{1,4};r=\overline{4,7};\alpha,\vartheta=1,2;\ \ l=\overline{1,g}, \tag{58}$$

where $p_\alpha^{(l)}\left(\nu_q^{(l)}\right)$ is *a posteriori* probability of value $M_\alpha^{(l)}$ in the element $\nu_q^{(l)}$ of $l-$ th BBI in k-th frame; $p_\alpha^{(l)}\left(\nu_q'^{(l)}\right)$ is *a posteriori* probability of value $M_\alpha^{(l)}$ in the element $\nu_q'^{(l)}$ of $l-$ th BBI in $\left(k-1\right)-$ th frame; ${}^{r}\pi_{\alpha\beta}^{(l)},\ \ r=\overline{1,7}$ are elements of $r-$ th MTP ${}^{1}\Pi^{(l)},{}^{2}\Pi^{(l)},{}^{3}\Pi^{(l)},{}^{4}\Pi^{(l)},{}^{5}\Pi^{(l)},{}^{6}\Pi^{(l)},{}^{7}\Pi^{(l)}$ of type (1.32, 1.33, 1.42, 1.48, 1.59); $\delta\left(\cdot\right)$ is the delta-function.

Box 21.

$$p_\beta^{(l)}\left(\nu_4^{(l)}\right)=c'\exp\left\{f\left(M_{\beta,\nu_4^{(l)}}^{(l)}\right)\right\}\frac{p_\alpha^{(l)}\left(\nu_1^{(l)}\right)\cdot\left({}^{1}\pi_{\alpha\beta}^{(l)}\right)\cdot p_\theta^{(l)}\left(\nu_2^{(l)}\right)\cdot\left({}^{2}\pi_{\theta\beta}^{(l)}\right)}{p_\phi^{(l)}\left(\nu_3^{(l)}\right)\cdot\left({}^{3}\pi_{\phi\beta}^{(l)}\right)}$$
$$\times\frac{p_\gamma^{(l)}\left(\nu_4'^{(l)}\right)\cdot\left({}^{4}\pi_{\gamma\beta}^{(l)}\right)\cdot p_\omega^{(l)}\left(\nu_3'^{(l)}\right)\cdot\left({}^{7}\pi_{\omega\beta}^{(l)}\right)}{p_\psi^{(l)}\left(\nu_1'^{(l)}\right)\cdot\left({}^{5}\pi_{\psi\beta}^{(l)}\right)\cdot p_\nu^{(l)}\left(\nu_2'^{(l)}\right)\cdot\left({}^{6}\pi_{\nu\beta}^{(l)}\right)};\quad\alpha,\beta,\theta,\phi,\gamma,\omega,\psi,\nu=1,2;\ \ l=\overline{1,g}. \tag{59}$$

Box 16.

$$F\left\{\mu_{i,j,q}^{(l)};i=\overline{1,m};j=\overline{1,n};q=\overline{1,k}\right\}=\exp\left\{\sum_{q=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k}f\left(\mu_{i,j,q}^{(l)}\right)\right\},\tag{52}$$

where $f\left(\mu_{i,j,q}^{(l)}\right)$ is a logarithm of the likelihood function of the element value $\left(i,j,q\right)$ in $q-$ th frame and $l-$ th BBI.

Box 17.

$$\begin{aligned}&p_{as}^{(l)}\left\{\mu_{i,j,q}^{(l)};i=\overline{1,m};j=\overline{1,n};q=\overline{1,k}\right\}\\&=c\exp\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k}f\left[\mu_{i,j,q}^{(l)}\right]\right\}\prod_{q=1}^{k}\prod_{i=1}^{m}\prod_{j=1}^{n}P\left\{\mu_{1,1,1}^{(l)}\right\}\\&\times w^{(l)}\left\{\mu_{i,j,q}^{(l)}\left|\mu_{i,j-1,q}^{(l)},\mu_{i-1,j-1,q}^{(l)},\mu_{i-1,j,q}^{(l)},\mu_{i,j-1,q-1}^{(l)},\mu_{i-1,j-1,q-1}^{(l)},\mu_{i-1,j,q}^{(l)},\mu_{i,j,q-1}^{(l)}\right.\right\},\end{aligned}\tag{53}$$

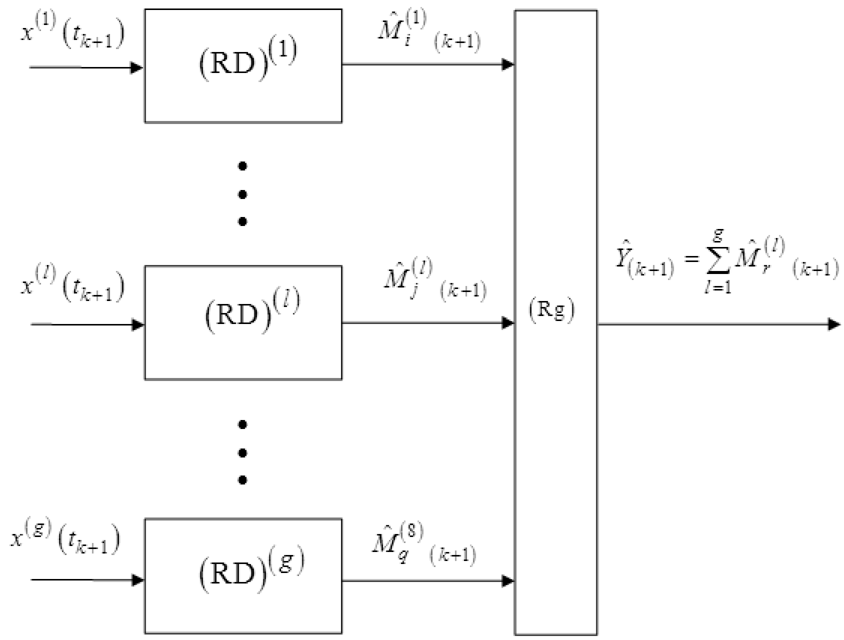
where *c* is the normalizing coefficient, $\alpha=1,2;\ \ l=\overline{1,g}.$ Having integrated (53) over all values of $\left\{\mu_{i,j,k}^{(l)}\right\}$, we obtain the equation of final *a posteriori* probability of the value $\left\{\mu_{i,j,k}^{(l)}\right\}$ in element $\nu_4^{(l)}$ of *l*-th BBI with account of designations taken in Chapter 10 (Figure 1.2):

$$\begin{aligned}&p_{as}^{(l)}\left(\nu_4^{(l)}\right)=c\exp\left\{f\left(\nu_4^{(l)}\right)\right\}\\&\times\int\cdots\int p_{as}^{(l)}\left(\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right)\\&\times w^{(l)}\left(\nu_4^{(l)}\left|\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right.\right)d\nu_1^{(l)}d\nu_2^{(l)}d\nu_3^{(l)}d\nu_1'^{(l)}d\nu_2'^{(l)}d\nu_3'^{(l)}d\nu_4'^{(l)}.\end{aligned}\tag{54}$$

Box 18.

$$\begin{aligned}&p_{as}^{(l)}\left(\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right)\times\\&\times w^{(l)}\left(\nu_4^{(l)}\left|\nu_1^{(l)},\nu_2^{(l)},\nu_3^{(l)},\nu_1'^{(l)},\nu_2'^{(l)},\nu_3'^{(l)},\nu_4'^{(l)}\right.\right)=\\&=\frac{p^{(l)}\left(\nu_1^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_1^{(l)}\right.\right)p^{(l)}\left(\nu_2^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_2^{(l)}\right.\right)}{p^{(l)}\left(\nu_3^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_3^{(l)}\right.\right)}\times\\&\times\frac{p^{(l)}\left(\nu_4'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_4'^{(l)}\right.\right)p^{(l)}\left(\nu_3'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_3'^{(l)}\right.\right)}{p^{(l)}\left(\nu_1'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_1'^{(l)}\right.\right)p^{(l)}\left(\nu_2'^{(l)}\right)w^{(l)}\left(\nu_4^{(l)}\left|\nu_2'^{(l)}\right.\right)}.\end{aligned}\tag{55}$$

Figure 11. The g-channel device for filtering of the DHTI video-sequence



which includes the unit realizing the nonlinear function $z_r(\cdot)$, $\left(r = \overline{1,7}\right)$; the threshold device.

With consideration of assumptions about the transmission method, the device for filtering of DHTI video-sequence presented by 2^g binary numbers should be the g-channel device, each channel of which represents the device identical to the device of nonlinear filtering of the video-sequence $\left(\mathrm{RD}\right)^{(l)}$ (Figure 11).

In accordance with Equation (60) and the optimal criterion (63), the digital model of RRD was constructed (Figure 11) for filtering of the DHTI video-sequence represented by $q = 2^8$ $\left(g = 8\right)$ binary numbers. During simulation on PC, artificial and real quantized images were subject of filtering.

Investigation of effectiveness of the three-dimension nonlinear filtering of the dynamic DHTI was conducted (Petrov, Trubin; 2007) on artificial DHTI video-sequences having 512 frames with the element number in the frame 512×512, formed on the basis of MM (Chapter 10), developed in (Trubin et al, 2004; Petrov, Trubin, Chastikov; 2007), for the BBI number $g = 8$.

The matrices of transition probabilities in lines and columns were specified as equal: ${}^{1}\Pi^{(l)} = {}^{2}\Pi^{(l)}$ with elements:

$$\begin{aligned}
{}^{1}\pi_{ii}^{(1)} &= {}^{2}\pi_{ii}^{(1)} = 0.5; {}^{1}\pi_{ii}^{(2)} = {}^{2}\pi_{ii}^{(2)} = 0.53;\\
{}^{1}\pi_{ii}^{(3)} &= {}^{2}\pi_{ii}^{(3)} = 0.58; {}^{1}\pi_{ii}^{(4)} = {}^{2}\pi_{ii}^{(4)} = 0.65;\\
{}^{1}\pi_{ii}^{(5)} &= {}^{2}\pi_{ii}^{(5)} = 0.78; {}^{1}\pi_{ii}^{(6)} = {}^{2}\pi_{ii}^{(6)} = 0.88;\\
{}^{1}\pi_{ii}^{(7)} &= {}^{2}\pi_{ii}^{(7)} = 0.95; {}^{1}\pi_{ii}^{(8)} = {}^{2}\pi_{ii}^{(8)} = 0.98.
\end{aligned} \quad (64)$$

Matrices ${}^{4}\Pi^{(l)}$ for all l $\left(l = \overline{1,g}\right)$ were taken with the same elements ${}^{4}\pi_{ii}^{(l)} = 0.9$.

Figure 5 (curve 3) shows the benefit in signal power η at the output of the filtering device for the video-sequence from 512 frames of artificial 8-bit DHTI with size 512×512 for different signal/noise ratios in power in the single pulse at the device input ρ_e^2.

Obtained results confirm the high effectiveness of the synthesized algorithm the dynamic DHTI filtering especially for slowly-varied scenes of video-sequences of the large-structured DHTI.

From analysis of the curves presented in Figure 5 we may conclude that at small signal/noise ratios $\rho_e^2 < 0$ dB the three-dimension filtering ensures the considerable benefit compared to one-dimension and two-dimension ones.

CONCLUSION

The main conclusions are the following:

1. The area of application of theory of Markov process is widened vis-à-vis the solution of problems of synthesis of mathematical models and the nonlinear filtering algorithm synthesis for digital half-tone images and video-sequences representing the multi-dimension, multi-valued random processes.
2. On the basis of the mathematical models developed and the filtering theory of conditional Markov processes, the algorithms and structures of the optimal radio receivers for nonlinear filtering of static and dynamic digital half-tone images are derived. The developed radio receiver device has a rather simple structure and high uniformity, which allows for ease of combination of radio receiver devices for increased dimension and number of discrete values of filtering process.
3. Qualitative and quantitative analysis of developed algorithms for nonlinear filtering of static and dynamic DHTI shows that filtering effectiveness increases with reduction of the signal/noise ratio and with growth of dimension of filtering process. At varying the signal/noise ratio at radio receiver device input ρ_e^2 from 0 dB to –9 dB, the benefit in signal power η increases from 9 to 19 dB for one-dimension filtering, from 17 to 38 dB for two-dimension one and from 18 dB to 46 dB for three-dimension filtering of the video-sequence of 8-bit DHTI.

DIRECTIONS OF FURTHER RESEARCHES AND DEVELOPMENTS

Starting from the obtained results of qualitative and quantitative investigations of the synthesized MM and filtering algorithms of static and dynamic DHTI, we can predict the behavior of mathematical models and filtering algorithms for higher order. For this, it is necessary in each specific case to investigate the problems of MM construction and development of nonlinear filtering algorithms differing by more complicated statistical peculiarities with the help of methods, which utility has been approved and generalized for several types of real processes. The solution of such problems may probably lead to obtaining new unknown results.

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