Dependence of Outage Probability of Cooperative Systems with Single Relay Selection on Channel Correlation

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***Abstract*—The relay selection method is a promising technique for improving the performance of cooperative systems. Most of the existing studies assume that wireless channels are statistically independent. However, in reality, channel correlation is more likely to be non-negligible. In this study, we investigate the dependence of the performance of cooperative systems with single relay selection in equally correlated environments. A tight upper bound of the system outage probability is given as a function of the channel correlation coefficients. We show that even though the system performance is considerably degraded in the high signal-to-noise ratio (SNR) region when the channel correlations are sufficiently large, yet less than one, the system still achieves a diversity order equal to the number of available relays.**

***Index Terms*—Cooperative communications, relay selection, correlated channels.**

I. Introduction

BECAUSE of the broadcast nature of the wireless medium, a transmission from a source node to a destination node can be overheard by the neighboring nodes. Cooperative communication allows these neighbors, called relay nodes, to forward the source information to the destination [1]. The use of all potential relays significantly improves the system performance; it however, entails several drawbacks such as a decrease in the system spectral efficiency, a requirement for strict synchronization, and an increase in complexity at the receiver side [2]. Single relay selection was introduced to overcome these drawbacks while maintaining the system diversity order. The single relay selection method selects the most suitable relay for the source-destination transmission [3].

Most of the existing studies on the relay selection of cooperative systems assume that wireless channels are statistically independent [3]-[5]. However, in reality, these channels are more likely to be correlated, particularly when the relays are close to each other. Since the relays may share the same obstructions, the source-to-relay channels (as well as the relay-to-destination channels) are correlated to each other due to the effect of shadow fading [6].

In this work, we investigate the dependence of the outage probability of cooperative systems with single relay selection under conditions of equally correlated source-to-relay channels and equally correlated relay-to-destination channels. The equally correlated model may be used as a worst-case benchmark or as a rough approximation by replacing every ρ_{ij} $(i \neq j)$ in the correlation matrix with the corresponding average value [7]. We first derive a tight upper bound of the system outage probability. Then, we prove that even when the channel correlations are sufficiently large, yet less than one, the system achieves the same diversity order as the system under conditions of independent channels. Note that the considered correlation effect is path correlation, which is different from other types of correlation forms such as antenna correlation in multiple antennas relay networks, i.e., [8], and correlation between the actual channel and the corresponding estimate in relay networks under time varying channels, i.e., [9].

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II. System Model and Assumptions

We consider a multi-relay system, comprising source S, destination D, and K number of relays. Every node has only one antenna that can be used for both transmission and reception. A direct link between S and D is assumed to be unavailable. Communication in the network is divided into two phases. In the first phase, the source broadcasts its information to the K relays. In the second phase, the selected relay uses the amplify-and-forward (AnF) relaying protocol [1] to forward the source signal toward the destination. The source-to-relay channels are assumed to be correlated. The same assumption is applied for the relay-to-destination channels. For each relay k, its channels (source-to-relay k and relay k-to-destination) are assumed to be independent.

Channel coefficients from S to the relays and from the relays to D are denoted by $\{h_{SR_k}\}_{k=1}^{K}$ and $\{h_{R_kD}\}_{k=1}^{K}$, respectively. $\{h_{SR_k}\}_{k=1}^{K}$ ($\{h_{R_kD}\}_{k=1}^{K}$) are modeled as correlated zero-mean complex Gaussian random variables with variance δ_{SR}^2 (δ_{RD}^2) [7], [10]. The cross-correlation coefficients between any h_{SR_k} and h_{SR_j}, and h_{R_kD} and h_{R_jD} $(k \neq j)$ are ρ_{SR} and ρ_{RD}, respectively. The noise associated with every channel is modeled as a mutually independent AWGN with zero-mean and variance N_0. We do not consider power allocation issues as they are outside the scope of this work. We assume that each transmitter (source or relay) transmits information with a fixed power P. The aforementioned assumptions of the unavailable direct link, complex Gaussian channel gains, AWGN, and the transmission power are commonly used in cooperative system analyses, i.e., [3]-[5].

Suppose that S communicates with D through the relay R_k, then the received SNR at D is as follows [4]:

$$\gamma_{D_{R_k}} = \frac{\gamma_{SR_k}\gamma_{R_kD}}{\gamma_{SR_k}+\gamma_{R_kD}+1} \le \min\left\{\gamma_{SR_k},\gamma_{R_kD}\right\}, \quad (1)$$

where $\gamma_{ij} = \frac{P}{N_0}|h_{ij}|^2$ denotes the instantaneous SNR over a single hop (for $i \to j$ link). It can be verified that γ_{SR_k} (γ_{R_kD}) follows an exponential distribution with parameter λ_{SR} (λ_{RD}) where $\lambda_{SR} = \frac{P}{N_0}\delta_{SR}^2$ and $\lambda_{RD} = \frac{P}{N_0}\delta_{RD}^2$ are average SNRs over source-to-relay and relay-to-destination links, respectively.

III. System Outage Probability

In this section, we will analyze the system outage probability based on the upper bound of the received SNR at the destination given in (1). Assuming that the Max-Min relay selection (MMRS) method is employed, a suitable relay is chosen as follows [4]:

$$R^* = \arg\max_{k=1,\ldots,K}\left\{\gamma_{D_{R_k}}\right\}. \quad (2)$$

The corresponding end-to-end SNR at the destination is [4]:

$$\gamma_{D_{R^*}} = \max_{k=1,\ldots,K}\left\{\gamma_{D_{R_k}}\right\}. \quad (3)$$

The instantaneous system capacity is defined as [5]:

$$C = \frac{1}{2}\log_2\left(1+\gamma_{D_{R^*}}\right). \quad (4)$$

The outage probability is $P_{out} = Pr[C < R_0]$ where R_0 is the system spectral efficiency. P_{out} can be expressed as:

$$P_{out} = Pr\left[\gamma_{D_{R^*}} < 2^{2R_0}-1\right] = F_{\gamma_{D_{R^*}}}\left(2^{2R_0}-1\right), \quad (5)$$

where $F_{\gamma_{D_{R^*}}}(x)$ is the cumulative distribution function (CDF) of $\gamma_{D_{R^*}}$. Note that, when $\rho_{SR} = 1$, all source-to-relay links experience the same fading, then we only need to select a relay that has the strongest relay-to-destination link. It is similar for $\rho_{RD} = 1$ with the difference of selecting a relay that has the strongest source-to-relay channel. Both cases are known as the nearest neighbor selection (NNS) scheme [3]. The nearest relay is not necessarily the spatially nearest relay to the transmitter or receiver, but the relay with the strongest channel to the transmitter or receiver. When both $\rho_{SR} = 1$ and $\rho_{RD} = 1$, the multi-relay network reduces to a single relay network. Since single relay networks and multi-relay networks with NNS are known to have a maximum diversity order of one [1], [3], those cases are discarded in our analysis.

We now provide an approach how to derive an upper bound of the system outage probability. Let $Y_k = \gamma_{SR_k}$ and $Z_k = \gamma_{R_kD}$ for $k = \{1,2,\ldots,K\}$. Then, the CDF of $\gamma_{D_{R^*}}$ can be expressed as the probability of the intersection (denoted by $\bigcap$) of functions of Y_k and Z_k:

$$\begin{aligned} F_{\gamma_{D_{R^*}}}(x) &= \Pr\left[\gamma_{D_{R^*}} \le x\right] \\ &= \Pr\left[\bigcap_{k=1}^{K}\left\{\min\left(Y_k,Z_k\right) \le x\right\}\right] = \Pr\left[V\right]. \end{aligned} \quad (6)$$

Recognizing the axioms of probability that for any event V, we have $\Pr[V] = 1 - \Pr\left[\bar{V}\right]$, where $^-$ denotes the complement operation. $\bar{V}$ can be express as:

$$\bar{V} = \left[\bigcup_{k=1}^{K}\left\{\min\left(Y_k,Z_k\right) > x\right\}\right], \quad (7)$$

where $\bigcup$ denotes the union operation. Then, using the principle of inclusion and exclusion for probability [11] on $\bar{V}$ we can decompose $\Pr[V]$ into several simpler probabilities as follows:

$$F_{\gamma_{D_{R^*}}}(x) = 1 - \sum_{m=1}^{K}(-1)^{m+1}\sum_{\substack{n_1,\cdots,n_m:\\ 1\le n_1<\cdots<n_m\le K}}\Pr\left[\bigcap_{i=1}^{m}\Xi_{n_i}\right] \quad (8)$$

where Ξ_{n_i} denotes the event $\{\min\left(Y_{n_i},Z_{n_i}\right) > x\}$, which is equal to $\{Y_{n_i} > x \cap Z_{n_i} > x\}$. Since Y_{n_i} and Z_{n_i} are independent, $\Pr\left[\bigcap_{i=1}^{m}\Xi_{n_i}\right]$ can be decomposed as follows:

$$\Pr\left[\bigcap_{i=1}^{m}\Xi_{n_i}\right] = \Pr\left[\bigcap_{i=1}^{m}\left\{Y_{n_i} > x\right\}\right]\Pr\left[\bigcap_{i=1}^{m}\left\{Z_{n_i} > x\right\}\right]. \quad (9)$$

The two probabilities in the right hand-side (RHS) of Eq. (9) can be obtained by using again the axioms of probability and the principle of inclusion and exclusion for probability, i.e.,

$$\begin{aligned} \Pr\left[\bigcap_{i=1}^{m}\left\{Y_{n_i} > x\right\}\right] &= 1 - \sum_{l=1}^{m}(-1)^{l+1} \\ &\cdot \sum\nolimits_{\substack{q_1,\cdots,q_l:\\ 1\le q_1<\cdots<q_l\le m}}\Pr\left[\bigcap_{i=1}^{l}\left\{Y_{q_i} \le x\right\}\right], \end{aligned} \quad (10)$$

and the probability $\Pr\left[\bigcap_{i=1}^{l}\left\{Y_{q_i} \le x\right\}\right]$ is [7]:

$$\begin{aligned} &\Pr\left[\bigcap_{i=1}^{l}\left\{Y_{n_i} \le x\right\}\right] = \\ &\int_0^\infty\left[1 - Q\left(\sqrt{\tfrac{2\rho_{SR}u}{1-\rho_{SR}}},\sqrt{\tfrac{2x}{\lambda_{SR}(1-\rho_{SR})}}\right)\right]^l e^{-u}du \end{aligned}, \quad (11)$$

where $Q(\alpha,\beta)$ is the first order Marcum Q-function. Substituting Y, ρ_{SR}, and λ_{SR} in equations (10) and (11) by Z, ρ_{RD}, and λ_{RD} we will obtain $\Pr\left[\bigcap_{i=1}^{m}\left\{Z_{n_i} > x\right\}\right]$. Finally, substituting (11) and (10) into (9), (8), and (5) yields an upper bound of the outage probability of a multi-relay system.

In summary, as a result of the outage probability of two-relay systems ($K = 2$), we provide a closed form upper bound to give some insight on the effect of channel correlations on the system performance.

Theorem 1: The system outage probability of two-relay cooperative systems with single relay selection in correlated environments can be upper bounded as:

$$\begin{aligned} P_{out} \le P_{up} = f\left(\lambda_{SR},\rho_{SR}\right)f\left(\lambda_{RD},\rho_{RD}\right) + 1 \\ -2e^{-A\left(\frac{1}{\lambda_{SR}}+\frac{1}{\lambda_{RD}}\right)}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} &f\left(\lambda_i,\rho_i\right) = 2e^{\frac{-A}{\lambda_i}} - 1 + \frac{A}{\lambda_i} \\ &\cdot\left[1 - e^{\frac{-2A}{B(\lambda_i,\rho_i)}}\left\{I_0\left(\frac{2A}{B(\lambda_i,\rho_i)}\right) + I_1\left(\frac{2A}{B(\lambda_i,\rho_i)}\right)\right\}\right]; \end{aligned} \quad (13)$$

with $i = \{SR, RD\}$; $A = 2^{2R_0}-1$; $B\left(\lambda_i,\rho_i\right) = \lambda_i\left(1-\rho_i^2\right)$; $I_n(\cdot)$ is the modified Bessel function of the first kind of order n^{th}.

Proof: Substituting $K = 2$ into Eq. (8), we obtain:

$$\begin{aligned} F_{\gamma_{D_{R^*}}}(x) = 1 + Pr\left[Y_1 > x, Y_2 > x\right]Pr\left[Z_1 > x, Z_2 > x\right] \\ -Pr\left[Y_1 > x\right]Pr\left[Z_1 > x\right] - Pr\left[Y_2 > x\right]Pr\left[Z_2 > x\right]. \end{aligned} \quad (14)$$

It is known that Y_k (Z_k) follows an exponential distribution with parameter λ_{SR} (λ_{RD}). Consequently,

$$\begin{cases} Pr\left(Y_1 > x\right) = Pr\left(Y_2 > x\right) = e^{\frac{-x}{\lambda_{SR}}} \\ Pr\left(Z_1 > x\right) = Pr\left(Z_2 > x\right) = e^{\frac{-x}{\lambda_{RD}}} \end{cases}. \tag{15}$$

The probabilities of joint events, $\Pr\{Y_1 > x, Y_2 > x\}$ and $\Pr\{Z_1 > x, Z_2 > x\}$, can be obtained by using equations (10) and (11). However, for this special case $K = 2$, instead of using Eq. (11) we prefer to use the approach given in [10]:

$$\begin{aligned} &\Pr\left[Y_1 \le x, Y_2 \le x\right] \\ &= \frac{1}{\lambda_{SR}^2\left(1-\rho_{SR}^2\right)} \int_0^x \int_0^x e^{\frac{-u-v}{B(\lambda_{SR},\rho_{SR})}} I_0\left(\frac{2|\rho_{SR}|\sqrt{uv}}{B(\lambda_{SR},\rho_{SR})}\right), \end{aligned} \tag{16}$$

which gives less difficulty in deriving an asymptotic approximation of the upper bound of the system outage probability. Let $u_1 = {}^{u}/_{B\left(\lambda_{SR},\rho_{SR}\right)}$ and $v_1 = {}^{v}/_{B\left(\lambda_{SR},\rho_{SR}\right)}$. With noting that $I_0\left(2\left|\rho_{SR}\right|\sqrt{u_1 v_1}\right) \le I_0\left(2\sqrt{u_1 v_1}\right)$, the above probability can be upper bounded as [12]:

$$\begin{aligned} &\Pr\left[Y_1 \le x, Y_2 \le x\right] \le \frac{x}{\lambda_{SR}} \\ &\cdot \left[1 - e^{\frac{-2x}{B(\lambda_{SR},\rho_{SR})}} \left\{I_0\left(\frac{2x}{B(\lambda_{SR},\rho_{SR})}\right) + I_1\left(\frac{2x}{B(\lambda_{SR},\rho_{SR})}\right)\right\}\right] \end{aligned} \tag{17}$$

Similarly, an upper bound of $\Pr[Z_1 \le x, Z_2 \le x]$ can be obtained by substituting Y, λ_{SR}, and ρ_{SR} with Z, λ_{RD}, and ρ_{RD} in the inequality (17).

Finally, substituting (17) and (15) into (10) (with $m = 2$ and noting that $\Pr\left[Y_k \le x\right] = 1 - \Pr\left[Y_k > x\right]$, $\Pr\left[Z_k \le x\right] = 1 - \Pr\left[Z_k > x\right]$), (14), and (5) we get (12), which completes the proof. ■

Theorem 2: The two-relay systems achieve a diversity order of two, with a certain amount of tolerance, when $|\rho_{SR}|$ and $|\rho_{RD}|$ are less than one.

Proof: Let $t_i = \frac{2A}{B(\lambda_i,\rho_i)}$, $f\left(\lambda_i,\rho_i\right)$ becomes:

$$f\left(\lambda_i,\rho_i\right) = 2e^{\frac{-A}{\lambda_i}} - 1 + \frac{A}{\lambda_i}\left[1 - e^{-t_i}\left\{I_0\left(t_i\right) + I_1\left(t_i\right)\right\}\right]. \tag{18}$$

Typically, $t_i \sim 0$ and thus $I_0\left(t_i\right) \simeq 1$. Using the following relation [13]:

$$I_1\left(t_i\right) = I_0\left(t_i\right)\frac{t_i}{2}e^{\frac{-t_i^2}{8}}, \tag{19}$$

and noting that $e^{\frac{-t^2}{8}} \simeq 1$, $f\left(\lambda_i,\rho_i\right)$ can be approximated as:

$$f\left(\lambda_i,\rho_i\right) \simeq 1 + \frac{A}{\lambda_i}\left[1 - e^{-t}\left\{1 + \frac{t}{2}\right\}\right] \simeq 1 + \frac{A}{2\lambda_i}t_i \tag{20}$$

Consequently, the upper bound of the system outage probability given in (12) can be approximated as:

$$P_{up} \simeq \frac{1}{\lambda_{SR}^2}\left(\frac{A^2}{1-\rho_{SR}^2} + \frac{A^2}{L^2\left(1-\rho_{RD}^2\right)}\right), \tag{21}$$

where $L = {}^{\lambda_{RD}}/_{\lambda_{SR}}$. For $|\rho_{SR}|$ and $|\rho_{RD}|$ are less than one, P_{up} is proportional to λ_{SR}^{-2}, the system achieves a diversity order of two. ■

The outage probability of two-relay cooperative systems with single relay selection under conditions of independent channels was presented in [14]:

$$P_{out}^I = \left[1 - \exp\left\{-A\frac{\lambda_{SR}+\lambda_{RD}}{\lambda_{SR}\lambda_{RD}}\right\}\right]^2, \tag{22}$$

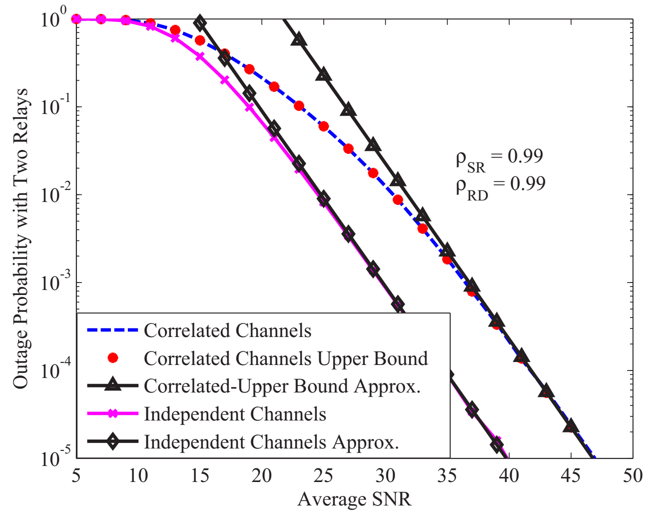

Fig. 1. Outage probability of two-relay systems versus average SNR.

which can be asymptotically approximated as:

$$P_{out}^I \simeq \frac{1}{\lambda_{SR}^2}A^2\left(1 + \frac{1}{L}\right)^2 \tag{23}$$

From equations (21) and (23), it is clear that the channel correlations do not affect the system diversity order. Note that the RHS of Eq. (21) is an approximation of the upper bound of the system outage probability under conditions of correlated channels, whereas the RHS of Eq. (23) is an approximation of the exact system outage probability under conditions of independent channels. In short, when $\rho_{SR} = \rho_{RD} = 0$ in Eq. (21), the approximation of the upper bound is not the same as Eq. (23).

Finally, as another example, we present the upper bound of the system outage probability for the case of $K = 3$ in Eq. (24) at the top of next page.

IV. Simulation Results

In this section, we provide numerical and simulation results to validate our analysis. The simulation setting follows the system model in Section II with $R_0 = 2$ bit/s/Hz and $L = 1$.

Fig.1 illustrates the relation between the outage probability of two-relay systems and the channel correlations. It first shows that the derived result given in (12) is a very tight upper bound of the system outage probability. Secondly, although the system outage probability considerably degrades when the channel correlations approach one, the system still maintains a diversity order of two.

In Fig. 2, we simulate the outage probability of two-relay systems versus the channel correlations for several average SNR values. In the simulation setting, we set $\rho_{SR} = \rho_{RD}$. As expected, the system outage probability decreases when SNR increases. For small values of ρ_{SR} and ρ_{RD}, the effect of the channel correlations on the system outage probability is negligible. However, when the channel correlations approach one, the system outage probability degrades steeply.

Fig. 3 gives a comparison between the simulation and the analysis upper bound of the outage probability for three-relay

$$P_{out}^{K=3} \leq 1 - 3e^{-A\left(\frac{1}{\lambda_{SR}}+\frac{1}{\lambda_{RD}}\right)} + 3f\left(\lambda_{SR},\rho_{SR}\right)f\left(\lambda_{RD},\rho_{RD}\right) - g\left(\lambda_{SR},\rho_{SR}\right)g\left(\lambda_{RD},\rho_{RD}\right), \quad (24)$$

where

$$g\left(\lambda_i,\rho_i\right) = 3e^{\frac{-A}{\lambda_i}} + 3\left(f\left(\lambda_i,\rho_i\right) - 2e^{\frac{-A}{\lambda_i}} + 1\right) - 2 - \int_0^{\infty}\left[1 - Q\left(\sqrt{\frac{2\rho_i u}{1-\rho_i}}, \sqrt{\frac{2A}{\lambda_i\left(1-\rho_i\right)}}\right)\right]^3 e^{-u}du. \quad (25)$$

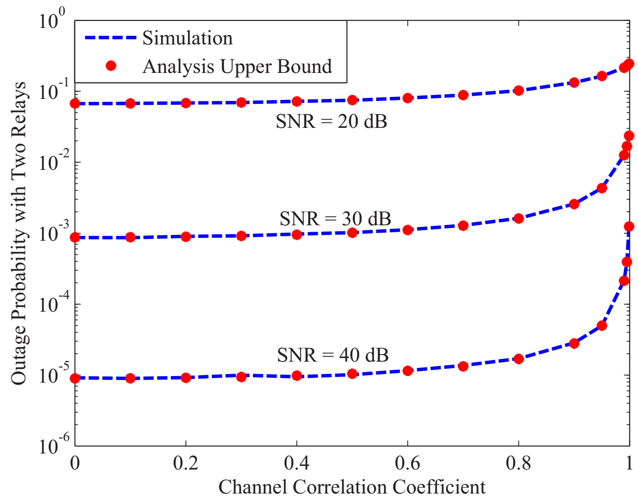

Fig. 2. Outage probability of two-relay systems versus channel correlation.

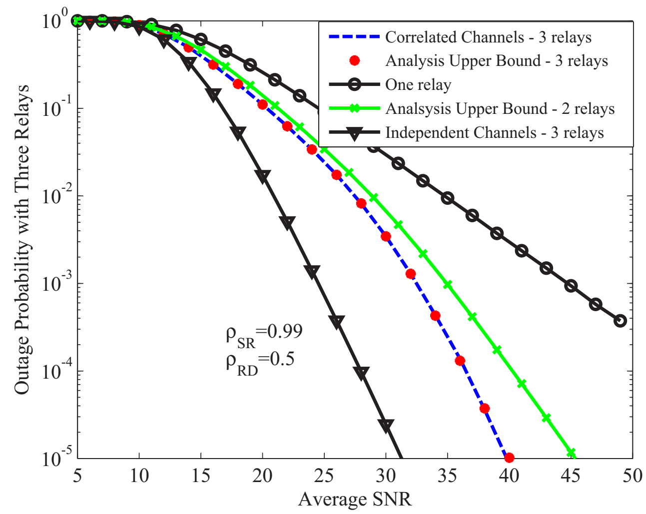

Fig. 3. System outage probability versus average SNR.

systems. It first confirms that the systems still achieve a diversity order equal to the number of available relays. Secondly, it shows that the system performance gain of moving from two relays to three relays is much less than that of moving from one relay to two relays, i.e., 2 (dB) in comparison with 10 (dB) at 10^-3 of the system outage probability.

V. Conclusion

We studied the dependence of the outage probability of cooperative systems with single relay selection on channel correlation. A tight upper bound of the system outage probability was given as a function of the channel correlation coefficients. We showed that the system outage probability degraded noticeably when the channel correlations approach one. However, the system still achieved the same diversity order as the system under conditions of independent channels.

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