1.0 Introduction

Many tunnelling measurements of electron doped cuprates and other HTS samples support the s-wave picture while other STM data supports the d-wave pairing (Ruslan et al., 2000) & (Gordon et al., 2008). The London penetration depth to a large extent has given a precise measure of the quasiparticle population though it has been reported that at low temperatures, there is an exponential suppression of quasiparticle as stated by Fumiaki et al. (1998), (which supports the s-wave pairing). One of the unique properties of the cuprates is its ability to allow magnetic flux to thread them by means of vortices of flux. Each vortex is the normal region in the sample, while the regions in between vortices are the superconducting part of the material. Lately, interest has shifted to how externally applied magnetic fields penetrate the cuprates and how wide a single vortex might be in such cases. Significantly, the penetration depth has been experimentally proven to be proportional to the volume of the magnetic field penetrating the samples from different sides i.e. along and perpendicular to the c-axis. Gordon et al. (2008) in his research shows when the excitation field is applied parallel to the c-axis, only the in-plane currents penetrates and λab is measured directly. When the excitation field is applied perpendicular to the c-axis, currents circulates in both the ab-plane and along the c-axis direction. The Bloch NMR equation has successfully used to study biological and physiological properties of living tissue as discussed by Awojoyogbe (2004) & (2003). Besides medical applications, the Bloch NMR equation can be used to investigate the thermodynamic properties of system. One of the Bloch NMR successes is the Wegner distribution function (Wigner, 1932), (which is to provide a framework for the treatment of quantum-mechanical problems), Kirkwood distribution function (Kirkwood, 1993) and the phase-space correspondence of the Bloch equations (O’Connell, 1985). Through the re-modeled Bloch-NMR equations, another parameter for measuring penetration depth was theoretically discovered.

2.0 Methods

2.1 Theoretical Remodelling Bloch-NMR Equation

In this study, a mathematical algorithm to describe in detail the translational mechanical properties of the Bloch NMR equation was developed. A sample of HTS sample can be analyzed by the x, y, z component (in the rotating frame) of magnetization given by the Bloch equations which may be written as follows:
Where $\Delta \omega = \omega_1 - \omega_0$ the frequency difference between Larmor frequency and frame of reference, 
$\omega_1 = -\gamma B_1$ is the Rabi frequency, $\omega_0 = -\gamma B_0$ is the Larmor frequency, $M_x, M_y$ are the transverse magnetization, $M_z$ is the longitudinal magnetization, $M_o$ is the equilibrium magnetization.

Figure 1: A HTS sample under consideration

Figure 1 shows a HTS sample with thickness $2c$, width $2b$, and length $2a$. It has two different London penetration depths $\lambda_{ab}$ and $\lambda_c$. A magnetic field is applied in the z-direction so demagnetizing corrections are absent. To facilitate comparism with the copper oxides, the y-direction is along the c-axis and x and z directions correspond to the a and b axes. So $\lambda_{xxyy} = \lambda_{ab}$ and $\lambda_{yy} = \lambda_c$. The in-plane supercurrent flowing in the x-direction penetrates from the top and bottom faces a distance $\lambda_{ab}$. Interplane(c-axis) supercurrent flowing in the y-direction penetrates from the left and right edges a distance $\lambda_c$. This assumption transforms the bloch as follows:

\[
\frac{dM_y}{dt} = \frac{\omega}{v_{pe}} - \frac{M_x}{v_{pe}T_2} 
\]
\[
\frac{dM_x}{dt} = -\Delta \omega M_y - \omega_0 M_z - \frac{M_x}{T_2} 
\]
\[
\frac{dM_z}{dt} = -\omega_1 M_y - \frac{M_z}{T_2} 
\]

Where; $v_{pe} = \frac{dM_x}{dt}$

The solution of the above equations can be arranged in matrix form as shown below

\[
\begin{bmatrix}
-\frac{\Delta \omega}{v_{pe}} & \frac{\Delta \omega}{v_{pe}} & 0 \\
-\frac{\omega_0}{v_{pe}T_2} & \frac{\omega_0}{v_{pe}T_2} & -\frac{M_z}{v_{pe}T_2} \\
-\frac{\omega_1}{v_{pe}} & \frac{(\omega_0 - \omega_1)}{v_{pe}} & -\frac{M_z}{v_{pe}}
\end{bmatrix}
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

The matrix multiplication as shown below leads to the steady solutions of the Bloch equations in the rotating frame of reference are shown below.

\[
M_x = \frac{M_{oe} \Delta \omega}{\Delta \omega - \omega_1 - \omega_0} 
\]
\[
M_y = \frac{M_{oe} \Delta \omega}{\Delta \omega + \omega_1 - \omega_0} 
\]
These solutions directly give the frequency response of the magnetization. This idea gives the possibilities of quantitatively calculating the measured signal if a spin system is characterized by relaxation times $T_1$ and $T_2$. The term $\nu_0 T_1 T_2$ is proportional to the radio frequency power $P$. At the state of no saturation i.e. for low power $P$, this term $(\omega^2 T_1 T_2) \ll 1$

\begin{align*}
M_x &= \frac{-\nu_0 T_1 T_2 \nu_{ab}^2 M_{ext}}{1 - \omega^2 \nu_{ab}^2 \nu_{c}^2 T_1 T_2} \\
M_y &= \frac{-\nu_0 T_1 T_2 \nu_{c}^2 M_{ext}}{1 - \omega^2 \nu_{ab}^2 \nu_{c}^2 T_1 T_2}
\end{align*}

$M_x$ is in phase with the exciting alternating field $B_1$ and $M_y$ is $90^\circ$ out of phase with the exciting alternating field $B_1$.

\begin{align*}
M_x &\propto (\omega_1 M_p) \frac{\nu_{ab}^2 \nu_{c}^2 T_1 T_2}{1 - \omega^2 \nu_{ab}^2 \nu_{c}^2 T_1 T_2} = \nu_1 M_p F(\Delta \omega) \\
M_y &\propto (\omega_1 M_p) \frac{\nu_{c}^2 \nu_{ab}^2 T_1 T_2}{1 - \omega^2 \nu_{ab}^2 \nu_{c}^2 T_1 T_2} = \nu_1 M_p G(\Delta \omega)
\end{align*}

$\nu_{ab}$ and $\nu_c$ are the penetration velocity along the ‘ab’ and ‘c’ axes respectively. $F(\Delta \omega)$ and $G(\Delta \omega)$ are the dispersion and the absorption for a particular kind of line known as the Lorentzian line. Before the superconducting state is attained, the magnetic flux is written as

$$M_{tot} = M_{ext} - M_{HTS}$$ \hspace{1cm} (18)

When the HTS sample starts superconducting, the magnetic flux expels-leaving the net magnetic flux as

$$M_{tot} = - M_{HTS}$$ \hspace{1cm} (19)

Experimentally, $M_{HTS} = \frac{\lambda}{\pi(\xi)} ln \left( \frac{\nu_{ab}}{\xi} \right)$

$\lambda$ and $\xi$ are the penetration depth and the vortex cross section in the HTS material. Here the total magnet flux is the transverse magnetic flux along the $y$-axis i.e. $M_y = M_{tot}$

Therefore, $M_y = - \frac{\lambda}{\pi(\xi)} ln \left( \frac{\nu_{ab}}{\xi} \right)$

\begin{align*}
4\pi \left( \frac{\lambda}{\nu_{ab}} \right)^2 T_2 \omega_1 \nu_{ab} \nu_{c} M_0 &= ln \left( \frac{\nu_{ab}}{\xi} \right) (\nu_{c}^2 + \Delta \omega^2 \nu_{ab}^2 \nu_{c}^2 T_1^2)
\end{align*}

Equation (21) can be written into two forms according to the terms $\nu_{ab}$ and $\nu_{c}$

\begin{align*}
4\pi \left( \frac{\lambda}{\nu_{ab}} \right)^2 \omega_1 &= F \left( \frac{\lambda}{\nu_{ab}} \right) \Delta \omega^2 T_1^n \\
\text{Where } \frac{\lambda}{\nu_{ab}} &= \frac{\lambda}{\nu_{ab}(\nu_c)} \sqrt{\frac{1}{\xi} - \frac{\nu_{ab}^2}{\nu_c^2}} \\
\frac{\nu_{ab}}{\nu_c} &= \frac{\nu_{ab}}{\nu_c(\xi)} \sqrt{\frac{1}{\xi} - \frac{\nu_{ab}^2}{\nu_c^2}} 
\end{align*}

\( T \) is the reduced temperature
This shows directly proportional relationship between penetration depth and coherent depth with a constant $\xi_{ab}$. Figure 2 (a) is an illustrational graph of $\lambda_{ab}$ and $\frac{\xi_{ab}}{\eta_{s}}$ when $\xi_{ab} = 80\text{nm}$.

Figure 3: The relationship between ground state penetration depth with the reduced temperature and the magnetic penetration depth. Five lines showing the increasing external magnetic field.
The second form of equation (22) is given below

\[ 4\pi \left( \frac{d^2}{d^2} \right) T_2 \omega_1 v_z = \ln \left( \frac{d^2}{d^2} \right) (v_z^2 - \Delta \omega^2 v_z^2) \]  

(24)

Considering motion of the transverse magnetization when \( \omega_1 = \gamma B_1(t) = \cos \omega t \), equation transforms to

\[ \frac{\partial^2}{\partial t^2} \lambda_{ab} - \frac{\partial}{\partial t} = -c \cos \omega t \]  

(25)

Where \( a = \ln \left( \frac{d^2}{d^2} \right) \), \( b = \ln \left( \frac{d^2}{d^2} \right) \Delta \omega^2 T_2 \), \( c = 4\pi \left( \frac{d^2}{d^2} \right) T_2 \)

\( \lambda_{ab} \) is the effective penetration depth. From Figure 2 (b) the solution of the two penetration depths are:

\[ \lambda_{ab} = \lambda_{ab} \sin(\omega_0 t) \]
\[ \lambda_c = \lambda_{ab} \sin(\omega_0 t) \]  

(26)

From which two values for \( \lambda_{ab} \) was obtained i.e

\[ \lambda_\perp = -\frac{4\pi \left( \frac{d^2}{d^2} \right) T_2}{\ln \left( \frac{d^2}{d^2} \right) \omega_0} \]

and \( \lambda_{\perp} = 0 \)

Therefore the penetration depth along the c-axis is given as

\[ \lambda_c = \frac{2\pi \left( \frac{d^2}{d^2} \right)^2 \left( \frac{1 - \frac{2}{\omega_0^2}}{1 - \frac{2}{\omega_0^2}} \right) \omega_0}{\ln \left( \frac{d^2}{d^2} \right)^2 \left( \frac{1 - \frac{2}{\omega_0^2}}{1 - \frac{2}{\omega_0^2}} \right) \omega_0} \cos (\omega_0 t) \]  

(27)

The minus sign shows the direction of the motion of the transverse magnetization or the reversible magnetization effects.

Figure 4: Analysis of penetration depth (\( \lambda_c \)) with respect to varying phase angles (\( \omega_0 t \)) when \( \frac{\Delta \omega}{\omega_0} = \frac{T}{T_2} \) provided \( \frac{T}{T_2} \leq 1 \).
Figure 2(a), Figure 3 and Figure 4, expresses the two critical fields i.e. $H_{c1}$ and $H_{c2}$ respectively. The superconducting material reaches the Meissner state under the critical field $H_{c1}$. When the field exceeds $H_{c1}$, the Abrikosov or mixed state is reached. Beyond the mixed state is the maximum critical field value $H_{c2}$. At any critical field values $H_{c1}$ and $H_{c2}$, these material exhibit a mixed behavior that allow magnetic flux to penetrate them by means of vortices of flux (as shown in figure 3 where varying magnetic fields between $2.1 \times 10^{-5}$T and $6.1 \times 10^{-5}$T were made to penetrate the system). The regions in between vortices are the superconducting part of this material. Each vortex in particular carries one fluxoid $\phi_0$ (the fundamental unit of quantized flux).

### 2.2 Application of Our Study to the Muon-Spin Relaxation Rate

The transverse field muon spin relaxation (TF-MSR) has good similarities with the present study in that

(i) It is operated at low magnetic fields for $H_{ext} \leq 0.3$T.
(ii) Each muon precess in its local magnetic field $B_1$ with the Larmor frequency $\omega_1 = \gamma B_1$
(iii) The function of the spin polarization function $P_\mu(t)$ is similar to the Lorentzian function $F(\Delta \omega)$ because they are both oscillatory in characteristics.
(iv) The magnetic moment is directly related to the magnetic penetration depth.

Past research work has shown that the c-axis oriented thin films (Pattanaik and Mudur, 1993) had given a sizeable anisotropy of the superconducting properties with anisotropy ratios ($\lambda_c / \lambda_{ab}$) ranging from about 1.6 to about 2.5. This gives equation (8) preference under this section. The motive is to inquire the relationship between the ratios of the penetrating velocities ($\lambda_c / \lambda_{ab}$) to other known quantities. Using the Fourier transform $P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_\mu(t) e^{-i\omega t} dt$ coupled to dynamic Kubo-Toyabe function (which expresses the spin polarization) of the form (Hayano et al., 1979).

$$P(\omega) = P_2(\omega) \exp \left( -\frac{1}{\tau_{g1}} \right) + \frac{1}{\tau_{g1}} \int_{-\infty}^{\infty} P_2(\omega) \exp \left( -\frac{1}{\tau_{g1}} \right) \cdot P_1(\tau) d\tau$$ (28)

Where $P_2(\omega) = \frac{1}{2} \int \frac{1}{2} \left( 1 - e^{-\omega t} \right) \exp \left( -\frac{1}{2} \frac{\omega^2}{\Delta^2} \right)$

$\Delta$ is the RMS width of the field distribution, $\tau_g$ is the jump rate i.e. the residence time in each potential well. At resonance, the two jump rates equalize i.e. $\tau_g = \tau_{g1}$, therefore at this point, $\tau_g = \tau_{g1}$. Equation (28) becomes

$$P(\omega) = P_2(\omega) \exp \left( -\frac{1}{\tau_{g1}} \right) + \frac{1}{\tau_{g1}} \int_{-\infty}^{\infty} P_2(\omega) \exp \left( -\frac{1}{\tau_{g1}} \right) \cdot P_1(\tau) d\tau$$ (29)

The result is thus

$$P(\omega) = 2\pi \left( \frac{e^{-\Delta t_0^2} \tau_{g1}}{\tau_{g1}^2} \right) \exp \left( -\frac{1}{\tau_{g1}^2} \right)$$ (30)

Equating equation (30) to its equivalent in eqn. (8) gives the solutions

$$\frac{\tau_{g1}}{\tau_{g2}} = \frac{\Delta t_0^2}{\tau_{g1}^2}$$ (31)

or

$$\frac{\tau_{g1}}{\tau_{g2}} = -2\pi \Delta \omega \left( \frac{e^{-\Delta t_0^2} \tau_{g1}}{\tau_{g1}^2} \right) \exp \left( \frac{e^{-\Delta t_0^2} \tau_{g1}}{\tau_{g1}^2} \right)$$ (32)

An assumption was made at the low limit when $\exp \left( \frac{e^{-\Delta t_0^2} \tau_{g1}}{\tau_{g1}^2} \right) \ll 1$, leaving the answers as

$$\frac{\tau_{g1}}{\tau_{g2}} = -\frac{1}{2\pi \Delta \omega} \left( \frac{\tau_{g1}}{\tau_{g1}^2} \right)$$ (33)
Barford (2009) showed that the measured effective penetration depth $\lambda_{\perp}$ is independent of the anisotropy ratio because it solely dependent on the in-plane penetration depth $\lambda_{ab}$

$$\lambda_{\perp} = f_{\text{anisotropy}} \cdot \lambda_{ab}$$

In furtherance with the investigation on the relational dependence of the anisotropy ratio and the phase angles ($\omega t$) as shown in the Figure 5.

![Figure 5: The effect of the phase angle on the anisotropic ratio](image)

### 3.0 RESULTS AND DISCUSSION

The Gaussian events have been proven to initiate the computation of penetration length and vortex size of cylindrical HTS samples. The temperature dependence of the in-plane and out-plane penetration depth around $T_c$ shown in homogeneous nanoscale superconducting domains was proven in Figure 2, Figure 3 and equation (27). Generally, the power law variation was observed which may signify the presence of nodes in the superconducting gap. Equation (27) reveals the possible effects of reversible magnetization as discussed by Prozorov and Giannette (2006). The inter and intra band scattering which was believed to strongly affect the superconducting anisotropy of two-band superconductor (Kogan et al., 2003) was visited in Figure 65. It was shown that, beyond the explanation of Kogan, the phase angle variation of the transverse magnetic field could also be a reason for the changes observed in the superconducting anisotropy of two-band superconductor and the temperature dependence of the anisotropy ratio in the inter-band scattering. At the superconducting state, the jump rate increases as temperature, beyond which, it has been established in equation (33) that an increase in the jump rates fosters an increase in the penetration velocity ratio. The same equation (33) reveals that the penetration velocity ratio decreases as the transverse magnetization increases as well as the root mean square of the field distribution. The penetration velocity can be extracted (like the penetration depth) from the traditional resonant circuit measurement technique with no additional devices. Figure 3 elaborates what happens at the mixed state (in between the first and second critical fields of the Meissner state). Different magnetic fields were analyzed within the mixed state. It was observed that though the magnitude of the superconducting state changes, its properties does not change. The penetration depth analysis under the changes of the phase angles (Figure 4) shows a Gaussian distribution which can be used to compute the penetration depths of other HTS shapes other cylindrical shapes.

### 4.0 CONCLUSION

At the superconducting state (under the muon spin relaxation rate), the jump rate increases as temperature, beyond which, it was establish that an increase in the jump rates fosters an increase in the penetration velocity ratio. Another way of measuring the magnetic penetration depth has been introduced-penetration velocity ratio which is subject to further investigation. The restriction of the Gaussian distribution to computing penetration depths of cylindrical HTS has been proven for other shapes of HTS.
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