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ORIGINAL ARTICLE

Classical and Bayes estimation of reliability characteristics of the Kumaraswamy-Inverse Exponential distribution

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Abstract In this research, the Bayesian estimators of both the unknown model parameters, survivor (or reliability) function and failure rate of the three-parameter Kumaraswamy-Inverse Exponential distribution were obtained. The symmetric and asymmetric loss functions were used for the Bayesian estimations. Though, the Bayes estimators could not be obtained in explicit forms. Random samples were generated from the posterior distributions using the Metropolis Hastings algorithm procedure and the Bayes estimators were obtained. Comparison was made between the Bayes estimators and the maximum likelihood estimators using Monte Carlo simulations. In addition, the Bayes estimators of the reliability characteristics were all obtained whilst making use of both the symmetric and asymmetric loss functions. However, their performance was compared through their simulated risks. Furthermore, a numerical study was conducted in order to compare the proposed estimates using simulations while illustrative examples were also presented. Two real life data sets were analyzed for the case when all the three parameters are unknown.

Keywords Bayesian inference · Kumaraswamy Inverse Exponential distribution · Mathematical statistics · Maximum likelihood estimation · MH algorithm · Reliability analysis · Simulation

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1 Introduction

The three-parameter Kumaraswamy Inverse Exponential distribution (denoted by KIE distribution) was introduced by Oguntunde et al. (2014) using the Kumaraswamy generalized family of distributions due to Cordeiro and de Castro (2011). It has unimodal and decreasing shapes, various mathematical and structural properties of the distribution has be established and estimation of the model parameters has been attempted using the method of maximum likelihood estimation (MLE). The KIE distribution does not include any special function and thus, it is a viable alternative to its Beta counterpart distribution that was introduced by Singh and Goel (2015).

The inabilities of the Exponential distribution necessitated the introduction of the Inverse Exponential distribution. However, there is a need to extend the Inverse Exponential distribution so that it would be able to withstand and model data sets that are heavily or highly skewed. This principal reason was what birthed the KIE distribution (among several other extensions of the Inverse Exponential distribution like the Weibull Inverse Exponential distribution and Exponentiated Generalized Inverse Exponential by Oguntundeet al. (2017a, b) respectively). The KIE distribution is a good lifetime distribution as it has been compared with the Generalized Inverse Exponential (GIE) distribution and the Inverse Exponential (IE) distribution proposed by Abouammoh and Alshingiti (2009), Keller and Kamath (1982) respectively using six real lifetime data sets in Oguntunde et al. (2017c). It has successfully been used to analyze data sets relating to failure times in engineering and survival times in medicine. For this reason, extensions of the KIE distribution are considered in this present research.

Several compound distributions have been introduced in recent times but Bayesian inference of such distributions has not been rigorously considered. The Bayesian inference of the Kumaraswamy Inverse Weibull distribution has been attempted by Gusmao et al. (2017) using Gamma distribution as the prior distribution but in this present paper, the Bayes estimators of the KIE distribution are derived and comparisons are made with the ones obtained as the maximum likelihood estimates with the aid of simulation studies.

This paper is written and outlined in the following manner: the three-parameter KIE distribution is defined and its reliability characteristics are discussed in minute details in Sect. 2. The MLEs of the model parameters together with the reliability characteristics are derived and established in Sect. 3. After this, the Bayes estimators for both the parameters and the reliability characteristics are derived using the Metropolis Hastings (MH) algorithm approach in Sect. 4. Provided in Sect. 5 is a numerical study conducted between the proposed estimates both in terms of their mean square error and bias values. Real life applications are provided in Sect. 6 followed by a concluding remark.

2 Model

For a random variable denoted by X, the densities of a three-parameter KIE distribution are given by:

$$f_X(x) = \frac{a\,b\,\lambda}{x^2}\, e^{-a\,\lambda\,x^{-1}} \left(1 - e^{-a\,\lambda\,x^{-1}}\right)^{(b-1)}, \quad x > 0, a, b, \lambda > 0 \tag{2.1}$$

and

$$F_X(x) = 1 - \left(1 - e^{-a\,\lambda\,x^{-1}}\right)^{b}, \quad x > 0, a, b, \lambda > 0 \tag{2.2}$$

respectively.

$f_X(x)$ and $F_X(x)$ are the probability density and the cumulative distribution functions respectively. For notation purpose, one can say; $X \sim KIE(a, b, \lambda)$.

The Reliability or survival function of the $KIE(a, b, \lambda)$ distribution is of the form:

$$R(t) = \left(1 - e^{-a\,\lambda\,t^{-1}}\right)^{b}, \quad t > 0, \tag{2.3}$$

Its failure rate or hazard function is:

$$h(t) = \frac{a\,b\,\lambda}{t^2}\, e^{-a\,\lambda\,t^{-1}} \left(1 - e^{-a\,\lambda\,t^{-1}}\right)^{-1}, \quad t > 0. \tag{2.4}$$

We obtain maximum likelihood estimators of the parameters a, b and λ including the reliability characteristics in the next section.

3 Maximum likelihood estimators

Considering a random sample say, $(X_1, X_2, \ldots, X_n)$ drawn from the model in (2.1), the likelihood function (L) is obtained as:

$$L(a, b, \lambda) = \prod_{i=1}^{n} \frac{a\,b\,\lambda}{x_i^2}\, e^{-a\,\lambda\,x_i^{-1}} \left(1 - e^{-a\,\lambda\,x_i^{-1}}\right)^{(b-1)} \tag{3.1}$$

and the log-likelihood function $(\log L)$ is obtained as:

$$\begin{aligned} \log L = {} & n \log a + n \log b + n \log \lambda - a\,\lambda \sum_{i=1}^{n} x_i^{-1} \\ & + (b-1) \sum_{i=1}^{n} \log\left(1 - e^{-a\,\lambda\,x_i^{-1}}\right) \end{aligned} \tag{3.2}$$

Now, we differentiate the log-likelihood function with respect to parameters a, b and λ respectively and the results are equated to zero as follows:

$$\frac{\partial \log L}{\partial a} = \frac{n}{a} - \lambda \sum_{i=1}^{n} x_i^{-1} + \lambda\,(b-1) \sum_{i=1}^{n} \frac{e^{-a\,\lambda\,x_i^{-1}}}{x_i\left(1 - e^{-a\,\lambda\,x_i^{-1}}\right)} = 0, \tag{3.3}$$

$$\frac{\partial \log L}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} \log\left(1 - e^{-a\,\lambda\,x_i^{-1}}\right) = 0, \tag{3.4}$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} - a \sum_{i=1}^{n} x_i^{-1} + a\,(b-1) \sum_{i=1}^{n} \frac{e^{-a\,\lambda\,x_i^{-1}}}{x_i\left(1 - e^{-a\,\lambda\,x_i^{-1}}\right)} = 0. \tag{3.5}$$

The simultaneous solutions of the Eqs. (3.3), (3.4) and (3.5) are therefore the maximum likelihood estimates; $\hat{a}$, $\hat{b}$ and $\hat{\lambda}$ of parameters a, b and λ respectively. The solutions can not be obtained in closed forms, hence, some other techniques can be adopted for this purpose. Suitable methods like the Newton-Raphson can be used to obtain the desired MLEs of the unknown model parameters. In this research however, the use of R software is adopted for necessary computations and particularly, we used the the *nleqslv* package in R to solve the nonlinear equations.

Now, expressions for the $\hat{R}(t)$ and $\hat{h}(t)$ which are the MLEs of $R(t)$ and $h(t)$ respectively are derived as:

$$\hat{R}(t) = \left(1 - e^{-\hat{a}\hat{\lambda}t^{-1}}\right)^{\hat{b}}, \quad \hat{h}(t) = \frac{\hat{a}\hat{b}\hat{\lambda}}{t^2}\, e^{-\hat{a}\hat{\lambda}t^{-1}} \left(1 - e^{-\hat{a}\hat{\lambda}t^{-1}}\right)^{-1}.$$

Bayes estimators of the unknown parameters and reliability characteristics are derived in the next section.

4 The Bayes estimators

The Bayesian estimates of $a, b, \lambda, R(t)$ and $h(t)$ under the symmetric and asymmetric loss functions are obtained in this section. We consider loss functions like LINEX, square error and entropy. Useful details about these loss functions are available in Varian (1975), Zellner (1986), Rastogi and Merovci (2017) and the references therein. It is worthy to note that the squared error loss function is a special case of the Entropy loss function.

The Bayesian estimates with respect to these loss functions are expressed as:

$$L_S(\hat{\mu}(\theta), \mu(\theta)) = (\hat{\mu}(\theta) - \mu(\theta))^2,$$
$$L_L(\hat{\mu}(\theta), \mu(\theta)) = e^{h(\hat{\mu}(\theta)-\mu(\theta))} - h(\hat{\mu}(\theta) - \mu(\theta)) - 1, \quad h \neq 0,$$
$$L_E(\hat{\mu}(\theta), \mu(\theta)) \propto \left(\frac{\hat{\mu}(\theta)}{\mu(\theta)}\right)^w - w \log\left(\frac{\hat{\mu}(\theta)}{\mu(\theta)}\right) - 1, \quad w \neq 0,$$

respectively.

where $\hat{\mu}(\theta)$ is an estimate of $\mu(\theta)$.

The Bayesian estimate of $\mu(\theta)$ under the loss function L_L is derived as:

$$\hat{\mu}_{BL} = -\frac{1}{h} \log\left\{E_\theta\left(e^{-h\theta}|\underline{x}\right)\right\}$$

Meanwhile, the corresponding estimate under the loss function L_E is:

$$\hat{\mu}_{BE} = (E_\theta(\theta^{-w} \mid \underline{x}))^{\frac{-1}{w}}$$

provided that the corresponding $E_\theta(.)$ exists. Further, we note that the Bayes estimate $\hat{\mu}_{BS}$ of μ under the L_S loss is given by $\hat{\mu}_{BE}$ with $w = -1$

Now, the Bayesian estimates of parameters a, b, λ, the survivor function $R(t)$ and the failure rate $h(t)$ under loss functions L_S, L_L and L_E are derived respectively. Meanwhile, the likelihood function involving a, b and λ is as presented in (3.1).

Consider a complete random sample say, $X_1, X_2, \ldots, X_n$ each drawn and distributed according to the $KIE(a, b, \lambda)$ distribution, we therefore derive the corresponding Bayesian estimates for all the unknowns. Of course, the assumption that parameters a, b and λ are statistically independent holds. Also, we assume that $Gamma\,(p_1, q_1)$, $Gamma\,(p_2, q_2)$ and $Gamma\,(p_3, q_3)$ distributions are the priors for these parameters. As a consequence, the prior distribution of a, b and λ is:

$$\pi(a, b, \lambda) \propto a^{p_1-1}\, e^{-a\,q_1}\, b^{p_2-1}\, e^{-b\,q_2}\, \lambda^{p_3-1}\, e^{-q_3\,\lambda}, \qquad (4.1)$$

all $a > 0, p_1 > 0,\ q_1 > 0, b > 0,\ p_2 > 0,\ q_2 > 0 \lambda > 0,\ p_3 > 0,\ q_3 > 0.$

Then, the corresponding posterior distribution of a, b and λ is obtained and expressed as:

$$\pi(a,\, b,\, \lambda |\, \underline{x}) = \frac{1}{k}\, a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3 + a\sum_{i=1}^n x_i^{-1}\right\}}\, e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}, \qquad (4.2)$$

where $\underline{x} = (x_1, x_2, \ldots, x_n)$ and

$$k = \int_0^\infty \int_0^\infty \int_0^\infty a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}}\, e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

Firstly, the Bayesian estimate of parameter a due to the loss function L_S with respect to the posterior distribution $\pi(a,\, b,\, \lambda|\, \underline{x})$ is obtained. The estimate is obtained as:

$$\tilde{a}_{BS} = \frac{1}{k}\int_0^\infty \int_0^\infty \int_0^\infty a^{(n+p_1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}}\, e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

When the loss function is L_L, then the Bayesian estimate of a is:

$$\tilde{a}_{BL} = -\frac{1}{h} \log\left\{E\left(e^{-h\,a} \mid \underline{x}\right)\right\}, \quad h \neq 0,$$

where

$$E[e^{-h\,a}|\underline{x}] = \frac{1}{k}\int_0^\infty \int_0^\infty a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a(h+q_1)}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}}\, e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

Finally, considering the loss function L_E, then:

$$\tilde{a}_{BE} = \left\{E\left(a^{-w} \mid \underline{x}\right)\right\}^{\frac{-1}{w}},$$

where

$$E[a^{-w}|\underline{x}] = \frac{1}{k}\int_0^\infty \int_0^\infty a^{(n+p_1-w-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}}\, e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

Similarly, we consider the derivation of the Bayesian estimates of parameters b and λ with respect to the aforementioned loss functions.

Assuming parameters a, b and λ are unknown, the expressions for the Bayesian estimates of the survival function $R(t)$ are derived and obtained in like manner. Particularly, if we consider the loss function L_S for instance, we have:

$$\tilde{R}(t)_{BS} = \frac{1}{k}\int_0^\infty \int_0^\infty a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1} \\ e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}} \\ \left(1-e^{-a\,\lambda\,t^{-1}}\right)^b e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\,\lambda\,x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

When the loss function is L_L, then:

$$\tilde{R}(t)_{BL} = -\frac{1}{h}\, log\{E(e^{-hR(t)}|\underline{x})\}, \quad h \neq 0,$$

where

$$E[e^{-hR(t)}\,|\,\underline{x}] = \frac{1}{k}\int_0^\infty \int_0^\infty a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)}\, e^{-a\,q_1} \\ e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}} \\ e^{-h\left(1-e^{-a\lambda t^{-1}}\right)^b} e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda,$$

Also, When the loss function is L_E, we obtain the Bayesian estimate of $R(t)$ as:

$$\tilde{R}(t)_{BE} = \left[E((R(t))^{-w}\,|\,\underline{x})\right]^{\frac{-1}{w}}$$

where

$$E[(R(t))^{-w}\,|\,\underline{x}] = \frac{1}{k}\int_0^\infty \int_0^\infty a^{(n+p_1-1)}\, b^{(n+p_2-1)}\, \lambda^{(n+p_3-1)} \\ e^{-a\,q_1}\, e^{-b\,q_2}\, e^{-\lambda\left\{q_3+a\sum_{i=1}^n x_i^{-1}\right\}} \\ \left(1-e^{-a\,\lambda\,t^{-1}}\right)^{-w\,b} e^{(b-1)\sum_{i=1}^n \log\left(1-e^{-a\lambda x_i^{-1}}\right)}\, da\, db\, d\lambda.$$

In the next sub-section, we use the Metropolis-Hasting (MH) algorithm and compute some more estimates of the unknown parameters including the reliability characteristics.

4.1 MH algorithm

The Metropolis-Hastings algorithm is widely and basically used to generate random samples from probability distributions. It is of course an alternative method for obtaining Bayes estimates for unknown parameters. Details about this algorithm and its applications are available in Metropolis et al. (1953) and Hastings (1970). The procedure for generating the random samples from prescribed posterior distribution is as follows:

Step 1 Select an initial value of (a, b, λ) and denote it as (a_0, b_0, λ_0)

Step 2 Obtain $b^{'}$ by making use of the proposal $N(b_{n-1}, \sigma^2)$ distribution and $\lambda^{'}$ by making use of the proposal $N(\lambda_{n-1}, \sigma^2)$ distribution, then $b^{'}$ from $G_{b|(a,\lambda)}\left(n+p_2, q_2 - \sum_{i=1}^n \log\left(1-e^{-a_{n-1}\,\lambda_{n-1}\,x_i^{-1}}\right)\right)$

Step 3 Compute $h = \frac{\pi(a',b',\lambda'|x)}{\pi(a_{n-1},b_{n-1},\lambda_{n-1}|x)}$

Step 4 Then, obtain a sample denoted by u from the uniform $U(0, 1)$ distribution

Step 5 If $u \leq h$ then set

$a_n \leftarrow a'$;
$b_n \leftarrow b'$;
$\lambda_n \leftarrow \lambda'$;

otherwise

$a_n \leftarrow a_{n-1}$;
$b_n \leftarrow b_{n-1}$;
$\lambda_n \leftarrow \lambda_{n-1}$;

Step 6 Repeat the procedure in (2-5) in say Q number of times using a number of replicates that is large enough.

The corresponding Bayes estimates of a under the loss function L_S is:

$$\tilde{a}_{mh} = \frac{1}{Q-Q_0}\sum_{i=Q_0+1}^{Q} a_i,$$

where Q and Q_0 are the total number of generated samples and initial burn-in samples respectively.

The approximate Bayes estimate for a under LINEX loss function is:

$$\hat{a}_{mh} = \frac{-1}{h}\log\left[\frac{1}{Q-Q_0}\sum_{i=Q_0+1}^{Q} e^{-h\,a_i}\right].$$

The Bayes estimate of θ with respect to the loss function L_e is:

$$\hat{a}_{mh} = \left[\frac{1}{Q-Q_0}\sum_{i=Q_0+1}^{Q} a_i^{-w}\right]^{\frac{-1}{w}}.$$

Following the same procedure, the Bayesian estimates of $b, \theta, R(t)$ and $h(t)$ under all the three loss functions can be obtained.

5 Numerical comparisons

After the derivations in Sects. 3 and 4, it is important to compare the performance of these estimates. However, this comparison is made with respect to the mean square errors (MSEs) and average values (or means) of these estimates in this section.

In this present research, the MSEs and means were evaluated after generating 10,000 random sample of size n from the $KIE(a, b, \lambda)$ distribution. We assume the true

Table 1 Average and MSE values of all estimates of a, b and λ for different values of n

n	$\hat{a}$	$\tilde{a}_{BS}$	$\tilde{a}_{BL}$			$\tilde{a}_{BE}$		
	$\hat{b}$	$\tilde{b}_{BS}$	$\tilde{b}_{BL}$			$\tilde{b}_{BE}$		
	$\hat{\lambda}$	$\tilde{\lambda}_{BS}$	$\tilde{\lambda}_{BL}$			$\tilde{\lambda}_{BE}$		
			$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
20	0.217963	0.193908	0.193945	0.193719	0.193531	0.192846	0.19068	0.189589
	0.002586	0.002611	0.002611	0.002612	0.002613	0.002653	0.002753	0.002809
	0.450228	0.398268	0.398484	0.397192	0.396124	0.395704	0.390568	0.387997
	0.019991	0.008009	0.008027	0.007922	0.007839	0.007917	0.007775	0.007725
	0.540998	0.495787	0.49583	0.495573	0.49536	0.495345	0.494459	0.494016
	0.015334	0.003298	0.003297	0.0033	0.003303	0.003311	0.00334	0.003356
40	0.208767	0.19709	0.197125	0.196919	0.196747	0.196181	0.194338	0.193412
	0.00987	0.002075	0.002075	0.002074	0.002073	0.002091	0.002133	0.002157
	0.418254	0.397872	0.397989	0.397291	0.396711	0.396457	0.393623	0.392206
	0.00653651	0.004631	0.004636	0.004604	0.004578	0.004603	0.004559	0.004543
	0.52113	0.497441	0.497483	0.497228	0.497015	0.497005	0.496131	0.495694
	0.00619679	0.003183	0.003183	0.003184	0.003184	0.003192	0.003212	0.003223
60	0.205072	0.199185	0.199217	0.199025	0.198866	0.198362	0.196694	0.195855
	0.00616	0.001867	0.001868	0.001865	0.001863	0.001874	0.001895	0.001908
	0.416274	0.400792	0.400874	0.400378	0.399966	0.399785	0.39777	0.396763
	0.004384	0.003391	0.003394	0.003375	0.00336	0.003371	0.003338	0.003325
	0.512588	0.49937	0.499411	0.499162	0.498954	0.498945	0.498092	0.497665
	0.004862	0.003237	0.003237	0.003237	0.003237	0.003245	0.003263	0.003273
80	0.203026	0.200697	0.200726	0.200551	0.200404	0.199951	0.198442	0.197684
	0.0042123	0.001657	0.001658	0.001655	0.001653	0.00166	0.00167	0.001677
	0.409066	0.399354	0.399418	0.399036	0.398718	0.398572	0.397007	0.396225
	0.002851	0.002652	0.002654	0.002643	0.002635	0.002642	0.002626	0.00262
	0.507224	0.499131	0.499171	0.498928	0.498724	0.498718	0.49789	0.497475
	0.004145	0.002974	0.002974	0.002973	0.002973	0.00298	0.002993	0.003001
100	0.202604	0.201137	0.201164	0.201	0.200863	0.200448	0.199056	0.198357
	0.003072	0.001431	0.001431	0.001429	0.001426	0.001431	0.001434	0.001438
	0.408132	0.399626	0.399679	0.399364	0.399102	0.39898	0.397688	0.397041
	0.002248	0.002135	0.002136	0.002129	0.002123	0.002127	0.002116	0.002111
	0.506525	0.498553	0.498594	0.498352	0.498151	0.498144	0.497322	0.496911
	0.003888	0.002985	0.002985	0.002985	0.002985	0.002991	0.003007	0.003015

value of (a, b, λ) to be (0.2, 0.4, 0.5) and hyper-parameters take the values; $a = 1, b = 5, c = 2, d = 5, p = 4, q = 8$. For each of the cases, the Bayes estimates under the loss function L_L is obtained for three different values of p, that is, -0.1, 0.5, 1. In the same way, under the loss function L_E, estimates are computed for the values of $w = -0.5, 0.5, 1$. Furthermore, the MSEs and average values of $R(t)$ are obtained for two distinct values of t, namely; 0.2, 1, and $h(t)$ is computed for two different values of t, that is; 0.5, 1.

Presented in Table 1 are the MSEs and means of the estimators $\hat{a}, \tilde{a}_{SB}, \tilde{a}_{LB}, \tilde{a}_{EB}, \hat{b}, \tilde{b}_{SB}, \tilde{b}_{LB}, \tilde{b}_{EB}$ and $\hat{\lambda}, \tilde{\lambda}_{SB}, \tilde{\lambda}_{LB}, \tilde{\lambda}_{EB}$ for different values of n. Under the MSE criterion, it was discovered that the Bayes estimates are far better than their corresponding MLEs. Also, for the estimation of the unknown parameters, the choice of $h = 1$ appears better under the L_L loss while in case of L_E loss, the choice of $w = 1$ appears better.

Presented in Tables 2 and 3 are the MSEs and means of the estimates $\hat{R}(t)$, $\tilde{R}_{SB}(t)$, $\tilde{R}_{LB}(t)$ and $\tilde{R}_{EB}(t)$ of the survival function $R(t)$ for different values of t. It was however discovered that the Bayes estimators appear better when compared to their MLE counterpart. In addition, among all the other estimates of $R(t)$, the squared error Bayes estimates of $R(t)$ has the lowest MSE values. Of all the estimators obtained from the linex loss function, the choice of $h = -0.5$ appears better.

Presented in Tables 4 and 5 are the MSEs and means of all the estimates of $h(t)$. It was again noticed that the Bayes

Table 2 Average and MSE values of all estimates of $R(t)$ for different values of n and $t = 0.2$

n	$\hat{R}(t)$	$\tilde{R}(t)_{BS}$	$\tilde{R}(t)_{BL}$			$\tilde{R}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.695666	0.681246	0.681256	0.681195	0.681143	0.681168	0.681012	0.680934
	0.006993	0.004053	0.004052	0.004056	0.004058	0.004058	0.004068	0.004073
40	0.696353	0.684715	0.684725	0.684667	0.684619	0.684644	0.6845	0.684427
	0.002959	0.002399	0.002399	0.0024	0.002401	0.002401	0.002406	0.002408
60	0.689716	0.685291	0.685304	0.685226	0.685161	0.685195	0.685002	0.684906
	0.002138	0.00195	0.00195	0.001951	0.001952	0.001952	0.001957	0.00196
80	0.689535	0.687892	0.687905	0.687825	0.687758	0.687794	0.687599	0.687501
	0.001536	0.001333	0.001333	0.001333	0.001333	0.001334	0.001334	0.001335
100	0.689374	0.687528	0.687542	0.687457	0.687386	0.687424	0.687216	0.687112
	0.001144	0.00123	0.00123	0.001231	0.001231	0.001231	0.001234	0.001235

estimates are better than the MLEs. The choice of $h = 1$ under the L_L loss gives the best estimate of $h(t)$.

Generally, it can be said that the values of the mean squared error for all the estimates decreases as the sample size n increases.

In Tables 6, 7 and 8, the true parameter values of (a, b, λ) are assumed to be (1.5, 1.5, 1.5) and the hyperparameters are given the following values: $a = 6, b = 4, c = 6, d = 4, p = 6, q = 4$. A Similar result with other true parameter values of (a, b, λ) which has been discussed above is obtained.

6 Data analysis

For illustration purpose, two real life examples are presented in this section.

Example 1 This first data set is an uncensored data which was obtained from Nichols and Padgett (2006), Mead and Abd-Eltawab (2014). It consists of 100 observations relating to the breaking stress of carbon fibres (in Gba). The observations are as follows:

It is important to verify if the KIE distribution is appropriate for this data set or not. Therefore, we consider two other related and important distributions namely, the Kumaraswamy Inverse Rayleigh distribution(K-IRD) and Inverse Exponential distribution (IED) are fitted and compared with KIE distribution. The MLEs of the parameters and the Akaike Information Criterion (AIC), Negative log-likelihood criterion (NLC), Bayesian Information Criterion (BIC), Corresponding Second order Information Criterion (AICc) are obtained and are subsequently used to select the best distribution among the competing distributions. It is well known that the lower the values of these selection criteria, the better the model. The goodness-of-fit statistics and parameter estimates are obtained and provided in Table 9. The results show that the KIE distribution fits the data set better than both the K-IRD and IED.

The MLEs and Bayes estimates for all the unknown model parameters and reliability characteristics are computed, the MH estimates are obtained with respect to all the three loss functions against a non-informative prior where each hyperparameters approach zero. Table 10 shows all the estimates of a, b and λ while the results for the reliability characteristics are given in Tables 11 and 12. The

```
3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19,
3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96,
2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56,
3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68,
2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73,
1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69,
1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57,
1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65.
```

Table 3 Average and MSE values of all estimates of $R(t)$ for different values of n and $t = 1$

n	$\hat{R}(t)$	$\tilde{R}(t)_{BS}$	$\tilde{R}(t)_{BL}$			$\tilde{R}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.384454	0.390902	0.390954	0.390639	0.390377	0.390199	0.388784	0.388071
	0.007954	0.005329	0.005329	0.00533	0.005331	0.005352	0.005403	0.00543
40	0.386871	0.392888	0.392906	0.392797	0.392706	0.392649	0.392169	0.391929
	0.003756	0.002889	0.002889	0.00289	0.00289	0.002894	0.002904	0.002909
60	0.388489	0.390432	0.390441	0.390386	0.39034	0.390311	0.390069	0.389948
	0.002269	0.002197	0.002197	0.002198	0.002198	0.002199	0.002204	0.002206
80	0.387484	0.39102	0.391026	0.390992	0.390965	0.390948	0.390804	0.390731
	0.001614	0.001599	0.001599	0.001599	0.001599	0.0016	0.001602	0.001603
100	0.389716	0.393229	0.393233	0.393209	0.393189	0.393177	0.393072	0.39302
	0.001391	0.001324	0.001324	0.001324	0.001324	0.001325	0.001326	0.001326

Table 4 Average and MSE values of all estimates of $h(t)$ for different values of n and $t = 0.5$

n	$\hat{h}(t)$	$\tilde{h}(t)_{BS}$	$\tilde{h}(t)_{BL}$			$\tilde{h}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.789733	0.716978	0.717529	0.714242	0.711533	0.713325	0.705979	0.702285
	0.050901	0.024351	0.024427	0.02399	0.023653	0.02415	0.02383	0.023713
40	0.744883	0.72115	0.721426	0.719777	0.718409	0.71929	0.715556	0.713682
	0.016733	0.012467	0.012486	0.012372	0.012283	0.012405	0.012302	0.012261
60	0.745667	0.718908	0.719091	0.717998	0.71709	0.717663	0.715164	0.713911
	0.011840	0.00891	0.008919	0.008869	0.008831	0.008887	0.008851	0.008838
80	0.735145	0.716167	0.716301	0.715499	0.714831	0.715245	0.713394	0.712466
	0.007897	0.006954	0.006957	0.006936	0.006919	0.006947	0.006941	0.00694
100	0.734156	0.726156	0.726266	0.725608	0.725061	0.72541	0.723913	0.723163
	0.006277	0.005646	0.00565	0.005623	0.005601	0.005627	0.005593	0.005577

Table 5 Average and MSE values of all estimates of $h(t)$ for different values of n and $t = 1$

n	$\hat{h}(t)$	$\tilde{h}(t)_{BS}$	$\tilde{h}(t)_{BL}$			$\tilde{h}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.423027	0.381406	0.381583	0.380524	0.379648	0.379213	0.374815	0.372608
	0.016255	0.007464	0.007479	0.007387	0.007312	0.007373	0.007222	0.007162
40	0.398539	0.379129	0.379218	0.37868	0.378233	0.377977	0.375669	0.374513
	0.006354	0.003751	0.003755	0.003734	0.003717	0.003731	0.003699	0.003688
60	0.393447	0.381401	0.381463	0.381091	0.380782	0.380605	0.37901	0.378212
	0.003386	0.002893	0.002895	0.002883	0.002873	0.002879	0.002855	0.002845
80	0.390353	0.379845	0.379893	0.379609	0.379374	0.379233	0.378008	0.377394
	0.002303	0.002106	0.002107	0.0021	0.002095	0.002099	0.002089	0.002085
100	0.386843	0.378087	0.378124	0.3779	0.377713	0.377599	0.376621	0.376131
	0.001882	0.001819	0.00182	0.001816	0.001813	0.001816	0.001813	0.001812

reliability function and the failure rate are computed for $t = 1.5$ and $t = 3$.

Example 2 The second data set relates to the strengths of 1.5 cm glass fibres measured at the National Physical Laboratory, England. The data has been used previously by Smith and Naylor (1987). The observations are as follows:

The Goodness of fit test is provided in Table 13. For this second example, the KIE distribution also provides a

```
0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00,
0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01,
0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24,
0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.84,
1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.70, 1.78, 1.89.
```

Table 6 Average and MSE values of all estimates of parameters a, b and λ for different values of n

n	$\hat{a}$	$\tilde{a}_{BS}$	$\tilde{a}_{BL}$			$\tilde{a}_{BE}$		
	$\hat{b}$	$\tilde{b}_{BS}$	$\tilde{b}_{BL}$			$\tilde{b}_{BE}$		
	$\hat{\lambda}$	$\tilde{\lambda}_{BS}$	$\tilde{\lambda}_{BL}$			$\tilde{\lambda}_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	1.56891	1.49875	1.49777	1.49738	1.49706	1.49749	1.49706	1.49684
	0.049867	0.002519	0.002518	0.002522	0.002524	0.002522	0.002528	0.002532
	1.80579	1.50914	1.51151	1.49743	1.48597	1.50157	1.48639	1.47878
	0.562905	0.073307	0.073835	0.070908	0.068889	0.072477	0.07117	0.070694
	1.56891	0.495787	0.49583	0.495573	0.49536	0.495345	0.494459	0.494016
	0.049871	0.002754	0.002753	0.002758	0.002762	0.002758	0.002767	0.002771
40	1.53309	1.49992	1.49999	1.49962	1.49932	1.49972	1.49931	1.49911
	0.022143	0.002399	0.002398	0.0024	0.002402	0.002401	0.002406	0.002408
	1.61846	1.50817	1.50952	1.50147	1.49485	1.5038	1.49506	1.49068
	0.178107	0.046633	0.046834	0.045699	0.044879	0.046297	0.04574	0.045521
	1.53312	1.49781	1.49787	1.49747	1.49714	1.49758	1.49714	1.49691
	0.022134	0.002742	0.002741	0.002744	0.002747	0.002745	0.002751	0.002755
60	1.52107	1.49984	1.4999	1.49951	1.49919	1.49962	1.49918	1.49897
	0.015324	0.002451	0.002451	0.002451	0.002452	0.002452	0.002455	0.002457
	1.58196	1.51561	1.51659	1.51078	1.50599	1.51247	1.50618	1.50303
	0.098658	0.037012	0.037145	0.036384	0.035814	0.036755	0.036303	0.036107
	1.52107	1.49896	1.49903	1.49864	1.49832	1.49875	1.49831	1.4981
	0.015324	0.00245	0.00245	0.002447	0.002445	0.002449	0.002449	0.002449
80	1.51475	1.49932	1.49938	1.499	1.49869	1.49911	1.49869	1.49848
	0.011301	0.002508	0.002508	0.002508	0.002508	0.002509	0.002512	0.002513
	1.55704	1.5032	1.50394	1.49947	1.49577	1.50074	1.49584	1.49339
	0.072125	0.028288	0.028355	0.027975	0.027694	0.028176	0.027988	0.027912
	1.51475	1.49519	1.49525	1.49488	1.49457	1.49498	1.49457	1.49436
	0.011301	0.002427	0.002427	0.002431	0.002434	0.002431	0.002438	0.002442
100	1.51331	1.49754	1.4976	1.49723	1.49692	1.49733	1.49692	1.49671
	0.008191	0.002547	0.002546	0.00255	0.002554	0.00255	0.002558	0.002562
	1.53763	1.49915	1.49977	1.49608	1.49303	1.49713	1.49307	1.49104
	0.047640	0.022794	0.022835	0.022603	0.022435	0.022729	0.022626	0.022586
	1.51331	1.50071	1.50077	1.50039	1.50007	1.50049	1.50006	1.49985
	0.008191	0.002539	0.00254	0.002539	0.002539	0.00254	0.002542	0.002544

better fit. In Table 14, the MLEs and Bayes estimates of the unknown parameters are obtained and reported. Parameter estimates of the survival and hazard functions are also provided in Tables 15 and 16. However, the survival and hazard functions are calculated at $t = 1$ and $t = 1.5$.

Table 7 Average and MSE values of all estimates of $R(t)$ for different values of n and $t = 1.5$

n	$\hat{R}(t)$	$\tilde{R}(t)_{BS}$	$\tilde{R}(t)_{BL}$			$\tilde{R}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.68887	0.685631	0.685659	0.685491	0.68535	0.685421	0.685001	0.684791
	0.007357	0.003387	0.003386	0.003389	0.003392	0.003393	0.003408	0.003415
40	0.689863	0.681313	0.681353	0.681113	0.680914	0.681014	0.680417	0.680118
	0.003454	0.002376	0.002376	0.002381	0.002385	0.002385	0.002404	0.002414
60	0.686493	0.681316	0.681359	0.6811	0.680884	0.680996	0.680354	0.680034
	0.002510	0.001599	0.001598	0.001602	0.001605	0.001605	0.001618	0.001626
80	0.686199	0.683018	0.683064	0.682785	0.682553	0.682675	0.681988	0.681645
	0.001886	0.001373	0.001373	0.001375	0.001377	0.001377	0.001386	0.001391
100	0.687737	0.682746	0.682795	0.682501	0.682256	0.682385	0.681661	0.681299
	0.001422	0.001144	0.001143	0.001145	0.001147	0.001147	0.001156	0.001162

Table 8 Average and MSE values of all estimates of $h(t)$ for different values of n and $t = 1.5$

n	$\hat{h}(t)$	$\tilde{h}(t)_{BS}$	$\tilde{h}(t)_{BL}$			$\tilde{h}(t)_{BE}$		
			$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
20	0.448917	0.421794	0.421822	0.421657	0.421519	0.421479	0.420837	0.420508
	0.0138514	0.005898	0.005899	0.005891	0.005883	0.005891	0.005877	0.00587
40	0.434121	0.426697	0.426711	0.426631	0.426565	0.426544	0.426233	0.426076
	0.005273	0.003898	0.003898	0.003896	0.003894	0.003897	0.003895	0.003894
60	0.435016	0.429695	0.429706	0.429637	0.429579	0.42956	0.429288	0.42915
	0.003522	0.002495	0.002495	0.002494	0.002493	0.002495	0.002494	0.002493
80	0.433366	0.428684	0.428696	0.428625	0.428567	0.428545	0.428265	0.428124
	0.002673	0.002068	0.002068	0.002068	0.002068	0.002069	0.002072	0.002074
100	0.430468	0.428593	0.428606	0.42853	0.428466	0.428443	0.428139	0.427987
	0.001872	0.001624	0.0016	0.001601	0.001601	0.001602	0.001607	0.001609

Table 9 Test of Goodness of fit for the three distributions

	$(\hat{a}, \hat{b}, \hat{\lambda})$	NLC	AIC	AICc	BIC
K-IED	(2.34427, 9.06008, 2.64365)	151.241	308.482	308.732	316.298
K-IRD	(1.35384, 1.13974 1.62724)	174.8405	194.7447	194.9352	199.124
IED	(2.13994)	199.3955	400.791	400.832	403.396

Table 10 Parameter estimates of a, b and λ

$\hat{a}$	$\tilde{a}_{BS}$	$\tilde{a}_{BL}$			$\tilde{a}_{BE}$		
$\hat{b}$	$\tilde{b}_{BS}$	$\tilde{b}_{BL}$			$\tilde{b}_{BE}$		
$\hat{\lambda}$	$\tilde{\lambda}_{BS}$	$\tilde{\lambda}_{BL}$			$\tilde{\lambda}_{BE}$		
		$h = -0.1$	$h = 0.5$	$h = 1.0$	$w = -0.5$	$w = 0.5$	$w = 1.0$
2.34148	2.48838	2.48876	2.48646	2.48453	2.48761	2.48606	2.48528
9.06086	9.04928	9.04932	9.04911	9.04894	9.04926	9.04923	9.04921
2.64689	2.62044	2.62054	2.61995	2.61946	2.62025	2.61988	2.61969

Table 11 Parameter estimates of $R(t)$ for different values of t

n	$\hat{R}(t)$	$\tilde{R}(t)_{BS}$	$\tilde{R}(t)_{BL}$			$\tilde{R}(t)_{BE}$		
			$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
1.5	0.863597	0.857124	0.857135	0.857068	0.857011	0.857058	0.856925	0.856858
3	0.292994	0.274672	0.27472	0.274431	0.27419	0.273783	0.271984	0.271079

Table 12 Parameter estimates of $h(t)$ for different values of t

n	$\hat{h}(t)$	$\tilde{h}(t)_{BS}$	$\tilde{h}(t)_{BL}$			$\tilde{h}(t)_{BE}$		
			$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
1.5	0.407233	0.422666	0.422731	0.422341	0.422021	0.421919	0.420474	0.419776
3	0.905301	0.927876	0.927992	0.927296	0.926718	0.927252	0.926013	0.925396

Table 13 Test of Goodness of fit for all the three distributions

	$(\hat{a}, \hat{b}, \hat{\lambda})$	NLC	AIC	AICc	BIC
K-IED	(2.69486, 163.19999, 3.02456)	22.06055	50.1211	50.3711	57.9366
K-IRD	(1.53364, 5.79740 1.71581)	33.6669	73.3338	73.5838	81.1493
IED	(1.4084)	89.439	180.878	180.919	183.483

Table 14 Parameter estimates of a, b and λ

$\hat{a}$	$\tilde{a}_{BS}$	$\tilde{a}_{BL}$			$\tilde{a}_{BE}$		
$\hat{b}$	$\tilde{b}_{BS}$	$\tilde{b}_{BL}$			$\tilde{b}_{BE}$		
$\hat{\lambda}$	$\tilde{\lambda}_{BS}$	$\tilde{\lambda}_{BL}$			$\tilde{\lambda}_{BE}$		
		$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
2.65773	2.63725	2.6373	2.63703	2.6368	2.63717	2.637	2.63691
163.207	162.969	162.969	162.969	162.969	162.969	162.969	162.969
3.06684	2.91055	2.91059	2.91033	2.91011	2.91047	2.91032	2.91024

Table 15 Parameter estimates of $R(t)$ for different values of t

n	$\hat{R}(t)$	$\tilde{R}(t)_{BS}$	$\tilde{R}(t)_{BL}$			$\tilde{R}(t)_{BE}$		
			$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
1	0.954	0.949637	0.949646	0.94959	0.949543	0.949587	0.949488	0.949438
1.5	0.489614	0.463104	0.463199	0.462628	0.462155	0.462083	0.46006	0.45906

Table 16 Parameter estimates of $h(t)$ for different values of t

n	$\hat{h}(t)$	$\tilde{h}(t)_{BS}$	$\tilde{h}(t)_{BL}$			$\tilde{h}(t)_{BE}$		
			$h=-0.1$	$h=0.5$	$h=1.0$	$w=-0.5$	$w=0.5$	$w=1.0$
1	0.383889	0.415192	0.415706	0.412651	0.410165	0.409294	0.398182	0.393054
1.5	2.5927	2.76527	2.76903	2.74636	2.72739	2.75838	2.74444	2.73742

7 Conclusion

The estimation of the unknown model parameters and reliability characteristics of the Kumaraswamy Inverse Exponential distribution (K-IED) have been successfully obtained and reported in this research. Explicit expressions for both the classical and Bayes estimation of the unknown parameters and reliability characteristics have been provided. The Newton-Raphson method was been considered since analytical expressions for the proposed estimators could not be obtained. The Bayes estimates of the parameters, survival and hazard functions under the square error, Linex and entropy loss functions have been successfully derived. Since the Bayes estimates could not be obtained in explicit forms, the MH alogritm was considered and adopted. The proposed estimates have been explicitly compared numerically, appropriate and adequate comments have also been made. However, the results from the analysis indicate that the Bayesian estimation is more accurate than the maximum likelihood estimation.

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