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SOME FIXED POINT THEOREMS FOR CONTRACTIVE MAPS IN FUZZY G-PARTIAL METRIC SPACES

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ABSTRACT

In this paper, we introduce the notion of fuzzy G-partial metric spaces and establish some fixed point theorems for contractive maps in this space. Examples are given to support our results. Our results extend and generalize some results in the literature.

Key words: fixed points; generalized contraction maps; fuzzy set; fuzzy G-partial metric space.

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1. INTRODUCTION

In Mathematics, fuzzy sets are sets whose elements have degrees of membership. This set was introduced by Zadeh[1] in 1965. Fuzzy set theory deals with neural nets and evolutionary programming which have be known as computional intelligence or soft computing. The theory is a tool to model reality better than traditional theories and an empirical validation is also very desirable. Many authors have extended the metric spaces by introducing fuzzy set to the notion of metric spaces and its generalizations (see; [2], [3], [4], [5], [6], [7]). Mustafa and Sims [8] generalized the notion of metric spaces by assigning triplets of real numbers to arbitrary sets. Eke and Olaleru [9] introduced the notion of G-partial metric spaces by generalizing the notion of G-metric spaces with the concept of partial metric. In this work, we extend the concept of G-partial metric spaces by introducing fuzzy set to it and term it, fuzzy G-partial metric spaces. Furthermore, we prove some fixed point results for contractive maps in this space.

2. PRELIMINARIES

Definition 2.1 [1]: Let X be a space of points, with a generic element of X denoted by x. A fuzzy set A in X is characterized by a membership function $f_{A(x)}$ which associates with each point in X a real number in the interval [0, 1], with the values of $f_{A(x)}$ at x representing the " grade of membership" of x in A. Thus, the nearer the value of $f_{A(x)}$ to unity, the higher the grade of membership of x in A.

Definition 2.2 [3] : A binary operation $*:[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norms if $*$ is satisfying conditions:

i) $*$ is an associative and commutative ;

ii) $*$ is continuous;

iii) $a * 1 = a$ for all $a \in [0, 1]$;

iv) $a*b \le c*d$ whenever $a \le c$ and $b \le d$ and $a, b, c, d \in [0, 1]$.

Definition 2.3 [2] : A 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and M is fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

i) $M(x, y,t) > 0$;

ii) $M(x, y,t) = 1$ iff $x = y$;

iii) $M(x, y,t) = M(y, x, t)$;

iv) $M(x, y,t) * M(y, z, s) \le M(x, z,t+s)$;

v) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

The following is the definition of partial fuzzy metric space given by Sedgi et al. [7].

Definition 2.4: Partial fuzzy metric on a nonempty set X is a function $P_M : X \times X \times (0, \infty) \to [0, 1]$ and $t, s > 0$

$P_{\{M1\}}$ $x = y$ iff $P_M(x, x, t) = P_M(x, y, t) = P_M(y, y, t)$ $P_{\{M2\}}$ $P_M(x, x, t) > P_M(x, y, t)$

$P_{\{M3\}}$ $P_M(x, y, t) = P_M(y, x, t)$

$P_{\{M4\}}$ $P_M(x, y, max\{t, s\}) * P_M(z, z, max\{t, s\})$

$> P_M(x, z, t) * P_M(z, y, s)$

$P_{\{M5\}}$ $P_M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous.

A partial fuzzy metric space is a 3-tuple $(X, P_M, *)$ such that X is nonempty set and P_M is a partial fuzzy metric on X .

The following also is the definition of fuzzy metric space in the setting of G-metric spaces.

Definition 2.5 [6] : A 3-tuple $(X, Q, *)$ is called a Q-fuzzy metric space if X is an arbitrary (nonempty) set, $*$ is a continuous t-norm, and Q is fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$;

Q_1 $Q(x, x, y, t) > 0$;

Q_2 $Q(x, x, y, t) \leq Q(x, y, z, t)$ $\forall\, x, y, z \in X$
with $z \neq y$;

Q_3 $Q(x, y, z, t) = 1$ if and only if $x = y = z$;

Q_4 $Q(x, y, z, t) = Q(P(x, y, z), t)$,
(Symmetry, where P is a permutation);

Q_5 $Q(x, a, a, t) * Q(a, y, z, s)$
$\leq Q(x, y, z, t+s)$;

Q_6 $Q(x, y, z, .) : (0, \infty) \to [0, 1]$ is continuous.

Definition 2.6 [9] : Let X be a nonempty set and let $G_p : X \times X \times X \to R^+$ be a function satisfying the following:

G_{p1} $G_p(x, y, z) \geq G_p(x, x, x) \geq 0$ $\forall\, x, y, z \in X$,

G_{p2} $G_p(x, y, z) = G_p(x, x, y) = G_p(y, y, z)$
$= G_p(z, z, x)$ if and only if $x = y = z$,

G_{p3} $G_p(x, y, z) = G_p(z, x, y) = G_p(y, z, x)$,

G_{p4} $G_p(x, y, z) \leq G_p(x, a, a) + G_p(a, y, z)$
$- G_p(a, a, a)$.

Then the function G_p is called a G-partial metric and the pair (X, G_p) is known as G-partial metric space.

3. FUZZY G-PARTIAL METRIC SPACE

In this section, we give the definition of fuzzy G-partial metric space and its motivations.

Definition 3.1 : A 3-tuple $(X, G_{pf}, *)$ is called a fuzzy G-partial metric space if X is an arbitrary (nonempty) set, $*$ is a continuous t-norm, and G_{pf} is fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$;

G_{pf1} $G_{pf}(x, y, z, t) = G_{pf}(x, x, y, t) = G_{pf}(y, y, z, t)$
$= G_{pf}(z, z, x, t)$=1 if an d only if $x = y = z$,

G_{pf2} $G_{pf}(x, y, z, t) \geq G_{pf}(x, x, y, t)$ $\forall\, x, y, z \in X$
and $y \neq z$,

G_{pf3} $G_{pf}(x, y, z, t) = G_{pf}(z, x, y, t) = G_{pf}(y, z, x, t)$,

G_{pf4} $G_{pf}(x, y, z, max\{t, s\}) \leq G_{pf}(x, a, a, t)$

$\quad * \, G_{pf}(a, y, z, s) \square G_{pf}(a, a, a, max\,\{t, s\})$

G_{pf5} $G_{pf}(x, y, z, .) : (0, \infty) \to [0, 1]$ is continuous.

Example 3.2: Let X be a nonempty set and G_p a G-partial metric on X. Denote $a * b \square c = \frac{ab}{c}$ for all $a, b, c \in [0, 1]$ for each $t > 0$, $x, y, z \in X$,

$$G_{pf}\ (x, y, z, t) = \frac{t}{t + \max\{x, y, z\}} \ .$$

Then $(X, G_{pf}, *)$ is a fuzzy G-partial metric space.

Example 3.3: Let (X, G_p) be a G-partial metric space and $G_{pf} : X^3 \times (0, \infty) \to [0, 1]$ be a mapping defined as

$$G_{pf}\ (x, y, z, t) = \ell^{-\frac{G_p\,(x, y, z)}{t}}$$

If $a * b \square c = \frac{ab}{c}$ for all $a, b, c \in [0, 1]$, then G_{pf} is a fuzzy G-partial metric .

Definition 3.4 : Let $(X, G_{pf}, *)$ be a fuzzy G-partial metric space. For $t > 0$, the open ball B(x, r, t) with centre $x \in X$ and radius $0 < r < 1$ is defined by
$B(x, r, t) = \{y \in X : G_{pf}(x, y, y, t) > 1 - r\}$.

A subset A of X is called an open set if for each $x \in A$ there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subseteq A$.

A sequence $\{x_n\}$ in X converge x if and only if $G_{pf}(x_n, x_n, x, t) \to 1$ as $m, n \to \infty$ for for each $t > 0$. It is called Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that $G_{pf}(x_m, x_n, x_l, t) > 1 - \varepsilon$ for each $l, m, n \geq n_0$. The fuzzy G-partial metric space is called complete if every Cauchy sequence is convergent.

Example 3.5 : Let $X = [0, \infty)$ and $G_{pf}\ (x, y, z, t) = \frac{t}{t + max\{x, y, z\}}$ then $(X, G_{pf}, *)$ is a fuzzy G-partial metric space where $a * b \square c = \frac{ab}{c}$ for all $a, b, c \in [0, 1]$. Let $\{x_n\} = \{\cdots -1, 0, 1, \cdots\}$. Then the sequence is convergent and we have, $\lim_{n\to\infty} G_{pf}\ (x_n, x_n, x, t) = G_{pf}\ (x, x, x, t)$.

The sequence is also Cauchy since every convergent sequence is Cauchy. Therefore we obtain,

$$\lim_{n\to\infty} G_{pf}\ (x_n, x_n, x, t) = G_{pf}\ (x, x, x, t)$$
$$= \lim_{n\to\infty} G_{pf}\ (x_n, x_m, x_m, t).$$

Definition 3.6 : Let $(X, G_{pf}, *)$ be a fuzzy G-partial metric space. G_{pf} is said to be continuous function on $X^3 \times (0,\infty)$ if for every $x, y, z \in X$ and $t > 0$,
$\lim\limits_{n\to\infty} G_{pf}(x_n, y_n, z_n, t_n) = G_{pf}(x, y, z, t)$ if

$\lim\limits_{n\to\infty} x_n = x$, $\lim\limits_{n\to\infty} y_n = y$, $\lim\limits_{n\to\infty} z_n = z$ and $\lim\limits_{n\to\infty} G_{pf}(x, y, z, t_n) = G_{pf}(x, y, z, t)$.

Lemma 3.7 : If $(X, G_{pf}, *)$ is a fuzzy G-partial metric space then $G_{pf}(x, y, z, t)$ is non-increasing with respect to t for all $x, y, z \in X$.

Proof: By G_{pf4}, let $a = x$ and for $t > 0$ we obtain

$$G_{pf}(x, y, z, max\{t, s\}) \le G_{pf}(x, x, x, t)$$
$$* \ G_{pf}(x, y, z, s) \ \square G_{pf}(x, x, x, max\ \{t, s\}).$$

Let $t > s$ then we have

$$G_{pf}(x, y, z, t) \le G_{pf}(x, x, x, t) * G_{pf}(x, y, z, s)$$
$$\square G_{pf}(x, x, x, t)$$

This implies that ,

$$G_{pf}(x, y, z, t) \le G_{pf}(x, y, z, s).$$

This shows that $G_{pf}(x, y, z, t)$ is non-increasing with the assumption that $\lim\limits_{n\to\infty} G_{pf}(x, y, z, t) = 1$ and that $\square$ is the set of all natural numbers.

Lemma 3.8 : Let $(X, G_{pf}, *)$ be a fuzzy G-partial metric space. Then G_{pf} is a continuous function on $X^3 \times (0,\infty)$.

Proof: Since $\lim\limits_{n\to\infty} x_n = x$, $\lim\limits_{n\to\infty} y_n = y$, $\lim\limits_{n\to\infty} z_n = z$ and $\lim\limits_{n\to\infty} G_{pf}(x, y, z, t_n) = G_{pf}(x, y, z, t)$

then there is $n_0 \in N$ such that $|t - t_n| < \delta$ for $n \ge n_0$ and $\delta < \frac{t}{2}$.

From Lemma 3.7 , we prove that $G_{pf}(x, y, z, t)$ is non-increasing with respect to t, hence we have

$$G_{pf}(x_n, y_n, z_n, t_n) \le G_{pf}\left(x_n, x, x, \frac{\delta}{3}\right)$$
$$* G_{pf}\left(x, y_n, z_n, t + \frac{4\delta}{3}\right) \square G_{pf}\left(x, x, x, \frac{\delta}{3}\right)$$
$$\le G_{pf}\left(x_n, x, x, \frac{\delta}{3}\right) * G_{pf}\left(y_n, y, y, \frac{\delta}{3}\right)$$
$$\square G_{pf}\left(y, y, y, \frac{\delta}{3}\right)$$

$$* \, G_{pf}\left(x,\, y,\, z_n,\, t+\frac{5\delta}{3}\right) \square G_{pf}\left(x,\, x,\, x,\, \frac{\delta}{3}\right)$$

$$G_{pf}\left(x_n,\, y_n,\, z_n,\, t_n\right) \le \, G_{pf}\left(x_n,\, x,\, x,\, \frac{\delta}{3}\right)$$

$$* G_{pf}\left(y_n,\, y,\, y,\, \frac{\delta}{3}\right) * \, G_{pf}\left(z_n,\, z,\, z,\, \frac{\delta}{3}\right)$$

$$* \, G_{pf}\left(x,\, y,\, z,\, \frac{t+2}{\delta}\right) \square G_{pf}\left(x,\, x,\, x, \frac{\delta}{3}\right)$$

$$\square G_{pf}\left(y,\, y,\, y,\, \frac{\delta}{3}\right) \square G_{pf}\left(z,\, z,\, z,\, \frac{\delta}{3}\right)$$

And

$$G_{pf}\left(x,\, y,\, z,\, \frac{t-2}{\delta}\right) \le G_{pf}\left(x,\, y,\, z,\, \mathrm{t}_n - \frac{2\delta}{3}\right)$$

$$\le G_{pf}\left(x,\, x_n,\, x_n,\, \frac{\delta}{3}\right) * G_{pf}\left(x_n,\, y,\, z,\, \mathrm{t}_n - \frac{2\delta}{3}\right)$$

$$\square G_{pf}\left(x_n, x_n, x_n, \frac{\delta}{3}\right)$$

$$\le \, G_{pf}\left(x,\, x_n, x_n, \frac{\delta}{3}\right) * \, G_{pf}\left(y,\, y_n,\, y_n, \frac{\delta}{3}\right)$$

$$* G_{pf}\left(x_n,\, y_n,\, z,\, \mathrm{t}_n - \frac{\delta}{3}\right) \square G_{pf}\left(x_n,\, x_n,\, x_n,\, \frac{\delta}{3}\right)$$

$$\square G_{pf}\left(y_n,\, y_n,\, y_n,\, \frac{\delta}{3}\right)$$

$$\le G_{pf}\left(x,\, x_n,\, x_n,\, \frac{\delta}{3}\right) * \, G_{pf}\left(y,\, y_n,\, y_n,\, \frac{\delta}{3}\right)$$

$$* \, G_{pf}\left(z,\, z_n,\, z_n,\, \frac{\delta}{3}\right) * G_{pf}\left(x_n,\, y_n,\, z_n,\, t_n\right)$$

$$\square G_{pf}\left(x_n,\, x_n,\, x_n,\, \frac{\delta}{3}\right) \square G_{pf}\left(y_n,\, y_n,\, y_n,\, \frac{\delta}{3}\right)$$

$$\square G_{pf}\left(z_n,\, z_n,\, z_n,\, \frac{\delta}{3}\right)$$

Let $n \to \infty$ and by continuity of the function G_{pf} with respect to t,

$$G_{pf}\left(x,\, y,\, z,\, t+2\delta\right) \ge \mathrm{G}_{pf}\left(x,\, y,\, z,\, t\right)$$
$$\ge \, G_{pf}\left(x,\, y,\, z,\, t-\delta\right)$$
.

Thus G_{pf} is a continuous function on $X^3 \times (o, \infty)$.

4. FIXED POINT RESULTS

In this section, we establish some fixed point theorems for contractive maps in fuzzy G-partial metric spaces.

Theorem 4.1: Let $(X, G_{pf}, *)$ be a complete fuzzy G-partial metric space. If the mapping $T: X \to X$ satisfies the contractive condition;

$$G_{pf}(Tx, Ty, Ty, t) \leq a_1 G_{pf}(x, y, y, t)$$
$$* a_2\ G_{pf}(x, Tx, Tx, t) * a_3 G_{pf}(y, Ty, Ty, t) \qquad \text{and}$$
$$* a_4 G_{pf}(x, Ty, Ty, t) * a_5 G_{pf}(y, Tx, Tx, t) \qquad (1)$$

$$G_{pf}(Tx, Tx, Ty, t) \leq a_1 G_{pf}(x, x, y, t)$$
$$* a_2\ G_{pf}(x, x, Tx, t) * a_3 G_{pf}(y, y, Ty, t)$$
$$* a_4 G_{pf}(x, x, Ty, t) * a_5 G_{pf}(y, y, Tx, t) \qquad (2)$$

for all $x, y \in X$. If $a_i \in (0, 1)$ $(i = 1, 2, 3, 4, 5)$ is constant with $a_1 + a_2 + a_3 + a_4 + a_5 < 1$, $a_4 < a_5$ and $a_2 + a_5 < 1$. Then T has a unique fixed point in X

Proof: Let $x_0 \in X$. Choose $x_1 \in X$ such that $x_1 = Tx_0$. In general, we can choose $x_n \in X$ such that $x_n = Tx_{n-1}$. Using (1) and letting $x = x_n$ and $y = x_{n+1}$ we have,

$$G_{pf}(x_n, x_{n+1}, x_{n+1}, t) = G_{pf}(Tx_{n-1}, T\,x_n, Tx_n, t)$$
$$\leq a_1 G_{pf}(x_{n-1}, x_n, x_n, t) * a_2 G_{pf}(x_{n-1}, T\,x_{n-1}, Tx_{n-1}, t)$$
$$* a_3 G_{pf}(x_n, T\,x_n, Tx_n, t) * a_4 G_{pf}(x_{n-1}, Tx_n, Tx_n, t)$$
$$* a_5 G_{pf}(x_n, Tx_{n-1}, Tx_{n-1}, t)$$
$$= a_1 G_{pf}(x_{n-1}, x_n, x_n, t) * a_2 G_{pf}(x_{n-1}, x_n, x_n, t)$$
$$* a_3 G_{pf}(x_n, x_{n+1}, x_{n+1}, t) * a_4 G_{pf}(x_{n-1}, x_{n+1}, x_{n+1}, t)$$
$$* a_5 G_{pf}(x_n, x_n, x_n, t)$$
$$\leq a_1 G_{pf}(x_{n-1}, x_n, x_n, t) * a_2 G_{pf}(x_{n-1}, x_n, x_n, t)$$
$$* a_3 G_{pf}(x_n, x_{n+1}, x_{n+1}, t) * [a_4 G_{pf}(x_{n-1}, x_n, x_n, t)$$
$$* G_{pf}(x_n, x_{n+1}, x_{n+1}, t) \square G_{pf}(x_n, x_n, x_n, t)]$$
$$* a_5 G_{pf}(x_n, x_n, x_n, t).$$

Since $a_4 < a_5$ then we obtain,

$$G_{pf}(x_n, x_{n+1}, x_{n+1}, t) \leq a_1 G_{pf}(x_{n-1}, x_n, x_n, t)$$
$$* a_2 G_{pf}(x_{n-1}, x_n, x_n, t) * a_3 G_{pf}(x_n, x_{n+1}, x_{n+1}, t)$$
$$* a_5 G_{pf}(x_{n-1}, x_n, x_n, t) * a_5 G_{pf}(x_n, x_{n+1}, x_{n+1}, t)$$
$$\square a_5 G_{pf}(x_n, x_n, x_n, t) * a_5 G_{pf}(x_n, x_n, x_n, t)$$

$$\le \left(a_1 + a_2 + a_5\right) G_{pf}\left(x_{n-1}, x_n, x_n, t\right)$$
$$* \left(a_3 + a_5\right) G_{pf}\left(x_n, x_{n+1}, x_{n+1}, t\right)$$
$$\le \frac{a_1 + a_2 + a_5}{1 - a_3 - a_5} G_{pf}\left(x_{n-1}, x_n, x_n, t\right)$$
$$= bG_{pf}\left(x_{n-1}, x_n, x_n, t\right) \text{ where } b = \frac{a_1 + a_2 + a_5}{1 - a_3 - a_5} < 1.$$

According to the definition of convergent sequence, we have,

$$\lim_{n\to\infty} G_{pf}\left(x_n, x_{n+1}, x_{n+1}, t\right) \le 1*1 = 1.$$

Let m, n be arbitrary with $m > n$. Using rectangle inequality we have,

$$G_{pf}\left(x_n, x_m, x_m, t\right) \le G_{pf}\left(x_n, x_{n+1}, x_{n+1}, t\right)$$
$$*G_{pf}\left(x_{n+1}, x_{n+2}, x_{n+2}, t\right) * \cdots * G_{pf}\left(x_{m-1}, x_m, x_m, t\right)$$
$$\square G_{pf}\left(x_{n+1}, x_{n+1}, x_{n+1}\ t\right) \square G_{pf}\left(x_{n+2}, x_{n+2}, x_{n+2}, t\right)$$
$$\square \cdots \square G_{pf}\left(x_{m-1}, x_{m-1}, x_{m-1}, t\right)$$
$$\le G_{pf}\left(x_n, x_{n+1}, x_{n+1}, t\right) * G_{pf}\left(x_{n+1}, x_{n+2}, x_{n+2}, t\right)$$
$$* \cdots * G_{pf}\left(x_{m-1}, x_m, x_m, t\right).$$

Taking the limit as $n \to \infty$ yields,

$$\lim_{n\to\infty} G_{pf}\left(x_n, x_m, x_m, t\right) \le 1*1*\cdots*1 = 1.$$

Thus $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists $z \in X$ such that $x_n \to z$ as $n \to \infty$. Also

$$G_{pf}\left(z, z, z, t\right) = \lim_{n\to\infty} G_{pf}\left(x_n, z, z, t\right)$$
$$= \lim_{m,n\to\infty} G_{pf}\left(x_n, x_m, x_m, t\right) = 1.$$

We show that z is the fixed point of T . Using (1) we get ,

$$G_{pf}\left(Tz, z, z, t\right) \le G_{pf}\left(Tz, x_n, x_n, t\right)$$
$$* G_{pf}\left(x_n, z, z, t\right) \square G_{pf}\left(x_n, x_n, x_n, t\right)$$
$$\le G_{pf}\left(Tz, x_n, x_n, t\right) * G_{pf}\left(x_n, z, z, t\right)$$
$$= G_{pf}\left(Tz, Tx_{n-1}, Tx_{n-1}, t\right) * G_{pf}\left(x_n, z, z, t\right)$$
$$\le a_1 G_{pf}\left(z, x_{n-1}, x_{n-1}, t\right) * a_2 G_{pf}\left(z, Tz, Tz, t\right)$$
$$* a_3 G_{pf}\left(x_{n-1}, Tx_{n-1}, Tx_{n-1}, t\right) * a_4 G_{pf}\left(z, Tx_{n-1},, Tx_{n-1}, t\right)$$
$$* a_5 G_{pf}\left(x_{n-1}, Tz, Tz, t\right) * G_{pf}\left(x_n, z, z, t\right)$$
$$= a_1 G_{pf}\left(z, x_{n-1}, x_{n-1}, t\right) * a_2 G_{pf}\left(z, Tz, Tz, t\right)$$
$$* a_3 G_{pf}\left(x_{n-1}, x_n, x_n, t\right) * a_4 G_{pf}\left(z, x_n, x_n, t\right)$$
$$* a_5 G_{pf}\left(x_{n-1}, Tz, Tz, t\right) * G_{pf}\left(x_n, z, z, t\right)$$

As $n \to \infty$ we get,

$$
\begin{aligned}
&G_{pf}(Tz,\ z,\ z,\ t) \le a_2 G_{pf}(z,\ Tz,\ Tz,\ t) \\
&* a_5 G_{pf}(z,\ Tz,\ Tz,\ t) \\
&\le (a_2\ +\ a_5) G_{pf}(z,\ Tz,\ Tz,\ t). \qquad (3)
\end{aligned}
$$

Using (2) we have,

$$
\begin{aligned}
&G_{pf}(Tz,\ Tz,\ z,\ t)\ \le\ G_{pf}(Tz,\ Tz,\ x_n,\ t) \\
&\quad *\ G_{pf}(x_n, x_n,\ z,\ t) \square G_{pf}(x_n, x_n, x_n,\ t) \\
&\le G_{pf}(Tz,\ Tz,\ x_n,\ t) *\ G_{pf}(x_n, x_n,\ z,\ t) \\
&=\ G_{pf}(Tz,\ Tz,\ Tx_{n-1}\ t)\ *\ G_{pf}(x_n, x_n,\ z,\ t) \\
&\le a_1 G_{pf}(z,\ z,\ x_{n-1},\ t) * a_2 G_{pf}(z,\ z,\ Tz,\ t) \\
&\quad * a_3 G_{pf}(x_{n-1}, x_{n-1},\ Tx_{n-1},\ t) * a_4 G_{pf}(z,\ z,\ Tx_{n-1},\ t) \\
&\quad * a_5 G_{pf}(x_{n-1}, x_{n-1},\ Tz,\ t) *\ G_{pf}(x_n, x_n,\ z,\ t) \\
&=\ a_1 G_{pf}(z,\ z,\ x_{n-1},\ t) * a_2 G_{pf}(z,\ z,\ Tz,\ t) \\
&\quad * a_3 G_{pf}(x_{n-1}, x_{n-1}, x_n,\ t) * a_4 G_{pf}(z,\ z,\ x_n,\ t) \\
&\quad * a_5 G_{pf}(x_{n-1}, x_{n-1},\ Tz,\ t) *\ G_{pf}(x_n, x_n,\ z,\ t)
\end{aligned}
$$

As $n \to \infty$ we get,

$$
\begin{aligned}
&G_{pf}(Tz,\ Tz,\ z,\ t) \le a_2 G_{pf}(z,\ z,\ Tz,\ t) \\
&\quad * a_5 G_{pf}(z,\ z,\ Tz,\ t) \\
&\le (a_2\ +\ a_5) G_{pf}(z,\ z,\ Tz,\ t) \qquad (4)
\end{aligned}
$$

Combining (3) and (4) yields

$$G_{pf}(Tz,\ z,\ z,\ t) \le\ (a_2\ +\ a_5) G_{pf}(Tz,\ z,\ z,\ t).$$

Since $a_2\ +\ a_5 < 1$ then we have $Tz = z$. Thus z is the fixed point of T.

If w is another fixed point of T, then

$$
\begin{aligned}
&G_{pf}(z,\ w,\ w,\ t)\ = G_{pf}(Tz,\ Tw,\ Tw,\ t) \\
&\le a_1\ G_{pf}(z,\ w,\ w,\ t)\ *\ a_2 G_{pf}(z,\ Tz,\ Tz,\ t) \\
&\quad * a_3 G_{pf}(w,\ Tw,\ Tw,\ t) * a_4 G_{pf}(z,\ Tw,\ Tw,\ t) \\
&\quad * a_5 G_{pf}(\ w, Tz,\ Tz,\ t) \\
&=\ a_1\ G_{pf}(z,\ w,\ w,\ t)\ *\ a_2 G_{pf}(z,\ z,\ z,\ t) \\
&\quad * a_3 G_{pf}(w,\ w,\ w,\ t) * a_4 G_{pf}(z,\ w,\ w,\ t) \\
&\quad * a_5 G_{pf}(\ w,\ z,\ z,\ t) \\
&= (a_1\ +\ a_4) G_{pf}(z,\ w,\ w,\ t) * a_5 G_{pf}(\ w,\ z,\ z,\ t) \\
&\le\ \frac{a_5}{1 - a_1\ -\ a_4} G_{pf}(\ w,\ z,\ z,\ t) \qquad (5)
\end{aligned}
$$

Similarly, using (2) and following the same argument in getting (5) yields,

$G_{pf}(z, z, w, t) \le \frac{a_5}{1-a_1-a_4} G_{pf}(w, w, z, t). \quad (6)$ Combining (5) and (6) yields,

$$G_{pf}(z, w, w, t) \le \left(\frac{a_5}{1-a_1-a_4}\right)^2 G_{pf}(z, w, w, t).$$

Since $a_5 + a_1 + a_4 < 1$ then it is a contradiction. Hence the fixed point is unique.

Corollary 4.2: Let $(X, G_{pf}, *)$ be a complete fuzzy G-partial metric space. If the mapping $T: X \to X$ satisfies the contraction condition,

$$G_{pf}(Tx, Ty, Ty, t) \le aG_{pf}(x, y, y, t)$$

and

$$G_{pf}(Tx, Tx, Ty, t) \le aG_{pf}(x, x, y, t),$$

for all $x, y \in X$. If $a \in (0, 1)$. Then has a unique fixed point in X. For any $x \in X$, iterative sequence $\{T^{nx}\}$ converges to the fixed point.

Corollary 4.3: Let $(X, G_{pf}, *)$ be a complete fuzzy G-partial metric space. If the mapping $T: X \to X$ satisfies the contractive condition,

$$G_{pf}(Tx, Ty, Ty, t) \le k\begin{pmatrix} G_{pf}(x, Tx, Tx, t) \\ * G_{pf}(y, Ty, Ty, t) \end{pmatrix}$$

and

$$G_{pf}(Tx, Tx, Ty, t) \le k\begin{pmatrix} G_{pf}(x, x, Tx, t) \\ * G_{pf}(y, y, Ty, t) \end{pmatrix}$$

for all $x, y \in X$. where $k \in \left(0, \frac{1}{2}\right)$ is constant. Then T has a unique fixed point in X.

Remark 4.4: Corrollary 4.2 and corollary 4.3 are analogue results of ([10] Theorem 4.1 and Theorem 4.5 respectively) in the setting of fuzzy G-partial metric spaces.

Example 4.5: Let $X = R^+$. Define $G_{pf}: X^3 \times [0, \infty) \to [0, 1]$ by,

$$G_{pf}(x, y, y, t) = \ell^{-\frac{G_p(x,y,y)}{t}}$$

for all $x, y \in X$ and $t > 0$. Then $(X, G_{pf}, *)$ is a complete fuzzy G-partial metric space where $a * b * c = \frac{ab}{c}$. Define map $T: X \to X$ by $Tx = \frac{x}{3}$ for $x \in X$. Thus T satisfy all the conditions of corollary 4.2. The fixed point is 0 and unique.

5. CONCLUSIONS

In this work, an idea of fuzzy set is introduce into the notion of G-partial metric space. Fuzzy G-partial metric space is a generalization of fuzzy metric space introduced by George and Veeramani [2] and extension of G-partial metric spaces. We also prove that fixed point of contractive mappings in this space is unique. I think that more fixed point theorems of contractive and expansive mappings develop in this context will further enchance the scope of fuzzy functional analysis.

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