Coupled FCT-HP for Analytical Solutions of the Generalized Time-fractional Newell-Whitehead-Segel Equation

S. O. EDEKI
Department of Mathematics
Covenant University, Canaanland, Ota
NIGERIA
soedeki@yahoo.com

O. P. OGUNDILE
Department of Mathematics
Covenant University, Canaanland, Ota
NIGERIA
opeyemiogundile@yahoo.com

B. OSOBA
Department of Physical Sciences
Bells University of Technology, Ota
NIGERIA
benardomth@gmail.com

G. A. ADEYEMI
Department of Civil Engineering
Covenant University, Canaanland, Ota
NIGERIA
gideon.adewale@cu.edu.ng

F. O. EGARA
Department of Science Education
University of Nigeria, Nsukka
NIGERIA
felix.egara@unn.edu.ng

S. A. EJOH
Department of Biological Sciences
Covenant University, Canaanland, Ota
NIGERIA
samuel.ejoh@covenantuniversity.edu.ng

Abstract: - This paper considers the generalized form of the time-fractional Newell-Whitehead-Segel model (TFNWSM) with regard to exact solutions via the application of Fractional Complex Transform (FCT) coupled with He's polynomials method of solution. This is applied to two forms of the TFNWSM viz: nonlinear and linear forms of the time-fractional NWSM equation whose derivatives are based on Jumarie's sense. The results guarantee the reliability and efficiency of the proposed method with less computation time while still maintaining high level of accuracy.

Key-Words: Fractional calculus, Fractional complex transform, Adomian Decomposition method, Analytical solution; He's polynomials.

1 Introduction
Fractional calculus (FC) as a branch of applied mathematics deals with different possibilities of how the powers of operators being it differential or integral are defined in terms of real number or complex number. It generalizes the classical corresponding differential calculus of integer orders. Mathematical modelling in most cases, involves partial differential equations (PDEs) of nonlinear and/or linear forms. Meanwhile, obtaining solutions to these model-equations has been a great concern. Hence, the adoption and construction of semi-analytical, numerical methods, and their modified forms [1-14]. Therefore, this paper will be focusing on an effective coupled method applied to the time-fractional Newell-Whitehead-Segel equation of the form:

$$\begin{cases} \dfrac{\partial^{\alpha} w(x,t)}{\partial t^{\alpha}} = k \dfrac{\partial^{2} w(x,t)}{\partial x^{2}} + aw(x,t) - bw^{j}(x,t), \\ w(x,0) = f(x), \ \alpha \in (0,1] \end{cases}$$

(1)

where $a, b \in \mathbb{R}$, and $k, j \in \mathbb{Z}^{+}$.

Many researchers have considered solving (1) at $\alpha = 1$ via some solution techniques [15-18]. The aim of this paper is to obtain the solution of the *time-fractional* NWSE via the application of FCT coupled with He's polynomial solution method [19-24]. This coupled method involves less computational time and work.

2 Jumarie's Fractional Derivative
Jumarie's Fractional Derivative (JFD) [25, 26] is a modified form of the Riemann-Liouville derivatives.

Hence, the definition of JFD and its basic properties as follows:

Let $h(v)$ be a continuous real function of v (not necessarily differentiable), and $D_v^\alpha h = \dfrac{\partial^\alpha h}{\partial v^\alpha}$ denoting JFD of h, of order α w.r.t. v. Then,

$$D_v^\alpha h = \left(\left(\begin{array}{l} \dfrac{1}{\Gamma(-\alpha)}\dfrac{d}{dv}\displaystyle\int_0^v (v-\lambda)^{-\alpha-1}\big(h(\lambda)-h(0)\big)d\lambda, \\ \quad \text{for} \quad \alpha \in (-\infty,0) \end{array}\right)\left(\begin{array}{l} \dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dv}\displaystyle\int_0^v (v-\lambda)^{-\alpha}\big(h(\lambda)-h(0)\big)d\lambda, \\ \quad \text{for} \quad \alpha \in (0,1), \end{array}\right)\left(\big(h^{(\alpha-m)}(v)\big)^{(m)}, \ \alpha \in [m,m+1), \ m \geq 1\right)\right)$$
(2)

where $\Gamma(\cdot)$ denotes a gamma function. As summarized in [25], the basic properties of JFD are as stated via *P1-P5*:

P1: $D_v^\alpha k = 0, \ \alpha > 0$,

P2: $D_v^\alpha\big(kh(v)\big) = kD_v^\alpha h(v), \ \alpha > 0$,

P3: $D_v^\alpha v^\beta = \dfrac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)}v^{\beta-\alpha}, \ \beta \geq \alpha > 0$,

P4: $D_v^\alpha\big(h_1(v)h_2(v)\big) = \begin{cases} D_v^\alpha h_1(v)\big(h_2(v)\big) \\ +h_1(v)D_v^\alpha h_2(v) \end{cases}$,

P5: $D_v^\alpha\big(h(v(g))\big) = D_v^1 h \cdot D_g^\alpha v$,

where k is a constant.

Note: *P1, P2, P3, P4,* and *P5* are referred to as fractional derivative of: constant function, constant multiple function, power function, product function, and function of function respectively. P5 can be linked to Jumarie's chain rule of fractional derivative.

2.1 The He's Polynomials and the Generalized non-integer NWSE

Here, we make reference to [19, 27, 28] for details of the He's Polynomials method. Hence, presented below is the structure of the He's Polynomials solution method on the generalized version of NWSE in terms of integer order.

Consider (1) for $\alpha = 1$ where $I_0^t(\cdot)$ denotes an integral operator, then:

$$\begin{cases} w = w(0) + I_0^t\big(kw_{xx} + aw - bw^j\big), \\ w(0) = f(x), \ w = w(x,t), w(0) = w(x,0). \end{cases}$$
(3)

The series solution form can be expressed as:

$$w(x,t) = \sum_{n=0}^\infty p^n w_n,$$
(4)

which is calculated as $p \to \infty$. Hence, by the application of convex homotopy method, (3) becomes:

$$\sum_{\eta=0}^\infty p^\eta w_\eta = g(x) + I_0^t\left(\begin{array}{l} k\displaystyle\sum_{\eta=0}^\infty p^{\eta+1}w_{xx,\eta} \\ +a\displaystyle\sum_{\eta=0}^\infty p^{\eta+1}w_\eta - bH_\eta \end{array}\right),$$
(5)

where H_η, $\eta \in \{0\} \cup \mathbb{N}$ denotes He's polynomials for the concerned nonlinear term, $w^j(x,t)$.

So, by comparison of the $p's$ powers in (5), we have:

$$p^{(0)} : w_0 = g(x)$$

$$p^{(1)} : w_1 = I_0^t\big(kw_{xx,0} + aw_0 - bH_0\big)$$

$$p^{(2)} : w_2 = I_0^t\big(kw_{xx,1} + aw_1 - bH_1\big)$$

$$p^{(3)} : w_3 = I_0^t\big(kw_{xx,2} + aw_2 - bH_2\big)$$

$$\vdots$$

$$p^{(i)} : w_i = I_0^t\big(kw_{xx,i-1} + aw_{i-1} - bH_{i-1}\big), i \geq 1.$$

Hence, the solution: $w(x,t) = \displaystyle\sum_{n=0}^\infty p^n w_n \to \sum_{n=0}^\infty w_n$ as $p \to \infty$.

2.2 The Fractional Complex Transform

In general form, the fractional differential equation of the following form is considered:

$$\begin{cases} f\big(\varpi, D_t^\alpha \varpi, D_x^\beta \varpi, D_y^\lambda \varpi, D_z^\gamma \varpi\big) = 0, \\ \varpi = \varpi(t,x,y,z). \end{cases}$$
(6)

Then, the Fractional Complex Transform [24] is defined as follows:

$$\begin{cases} T = \dfrac{at^\alpha}{\Gamma(1+\alpha)}, \ \alpha \in (0,1], \\[3mm] X = \dfrac{bx^\beta}{\Gamma(1+\beta)}, \ \beta \in (0,1], \\[3mm] Y = \dfrac{cy^\lambda}{\Gamma(1+\lambda)}, \ \lambda \in (0,1], \\[3mm] Z = \dfrac{dz^\gamma}{\Gamma(1+\gamma)}, \ \gamma \in (0,1], \end{cases} \tag{7}$$

with a, b, c, and d as unknown constants.
From *P3*,

$$D_v^\alpha v^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} v^{\beta-\alpha}, \ \beta \ge \alpha > 0,$$

$$\therefore \ D_t^\alpha T = D_t^\alpha \left[\frac{at^\alpha}{\Gamma(1+\alpha)} \right] = \frac{a}{\Gamma(1+\alpha)} D_t^\alpha t^\alpha = a.$$
$$\tag{8}$$

Obviously in a similar manner, using properties *P1-P5*, and the FCT in (7), the following are easily obtained:

$$\begin{cases} D_t^\alpha T = \dfrac{\partial^\alpha T}{\partial t^\alpha} = a, \ D_x^\beta X = \dfrac{\partial^\beta X}{\partial x^\beta} = b, \\[3mm] D_y^\lambda Y = \dfrac{\partial^\lambda Y}{\partial y^\lambda} = c, \ D_z^\gamma Z = \dfrac{\partial^\gamma Z}{\partial z^\gamma} = d. \end{cases} \tag{9}$$

Hence,

$$\begin{cases} D_t^\alpha p(t,x,y,z) = D_t^\alpha p(T(t)) = D_T^1 p \cdot D_t^\alpha T = a\dfrac{\partial p}{\partial T}, \\[3mm] D_x^\beta p(t,x,y,z) = D_x^\beta p(X(x)) = D_X^1 p \cdot D_x^\beta X = b\dfrac{\partial p}{\partial X}, \\[3mm] D_y^\lambda p(t,x,y,z) = D_y^\lambda p(Y(y)) = D_Y^1 p \cdot D_y^\lambda Y = c\dfrac{\partial p}{\partial Y}, \\[3mm] D_z^\gamma p(t,x,y,z) = D_z^\gamma p(Z(z)) = D_Z^1 p \cdot D_z^\gamma Z = d\dfrac{\partial p}{\partial Z}. \end{cases} \tag{10}$$

Thus, for $p = p(t,x,y,z)$, we have:

$$D_t^\alpha p = a\frac{\partial p}{\partial T}, \ D_x^\beta p = b\frac{\partial p}{\partial X}, \ D_y^\lambda p = c\frac{\partial p}{\partial Y}, \ D_z^\gamma p = d\frac{\partial p}{\partial Z}. \tag{11}$$

3 Applications and Test Cases

Here, the proposed method is applied to both linear and nonlinear NWSE as follows:

Case Ia: Consider the following linear NWSE [16, 17]:

$$\begin{cases} w_t^\alpha - w_{xx} + 3w = 0, \\ w(x,0) = e^{2x}, \end{cases} \tag{12}$$

with an exact solution:

$$w(x,t) = e^{2x+t}. \tag{13}$$

Procedure:
By FCT,

$$T = \frac{at^\alpha}{\Gamma(1+\alpha)}, \ \text{which according to section 3}$$

gives $\ D_t^\alpha w = \dfrac{\partial w}{\partial T} = w_T \ $ for $\ a = 1.$ So, (12) becomes:

$$\begin{cases} w_T - w_{xx} + 3w = 0, \\ w(x,0) = e^{2x}. \end{cases} \tag{14}$$

Therefore, using the outlined procedures in section 2.1, we obtained the following:

$$p^{(0)} : w_0 = e^{2x},$$

$$\left. \begin{aligned} p^{(1)} : w_1 &= I_0^T \left(w_{xx,0} - 3w_0 \right) \\ &= Te^{2x} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(2)} : w_2 &= I_0^T \left(w_{xx,1} - 3w_1 \right) \\ &= \frac{T^2 e^{2x}}{2!} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(3)} : w_3 &= I_0^T \left(w_{xx,2} - 3w_2 \right) \\ &= \frac{T^3 e^{2x}}{3!} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(4)} : w_4 &= I_0^T \left(w_{xx,3} - 3w_3 \right) \\ &= \frac{T^4 e^{2x}}{4!} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(5)} : w_5 &= I_0^T \left(w_{xx,4} - 3w_4 \right) \\ &= \frac{T^5 e^{2x}}{5!} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(6)} : w_6 &= I_0^T \left(w_{xx,5} - 3w_5 \right) \\ &= \frac{T^6 e^{2x}}{6!} \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(7)} : w_7 &= I_0^T \left(w_{xx,6} - 3w_6 \right) \\ &= \frac{T^7 e^{2x}}{7!} \end{aligned} \right\},$$

S. O. Edeki, O. P. Ogundile, B. Osoba,
G. A. Adeyemi, F. O. Egara, S. A. Ejoh

$$p^{(8)} : w_8 = I_0^T \left(w_{xx,7} - 3w_7 \right)$$
$$= \frac{T^8 e^{2x}}{8!}$$

$$p^{(9)} : w_9 = I_0^T \left(w_{xx,8} - 3w_8 \right)$$
$$= \frac{T^9 e^{2x}}{9!}$$

$$p^{(10)} : w_{10} = I_0^T \left(w_{xx,9} - 3w_9 \right)$$
$$= \frac{T^{10} e^{2x}}{10!}$$

$$\vdots$$

$$p^{(m)} : w_m = I_0^T \left(w_{xx,m\text{-}1} - 3w_{m-1} \right)$$
$$= \frac{T^m e^{2x}}{m!}, m \in N$$

Thus, the analytical solution of (14) according to He's polynomial with reference to (1) where $k = 1$, $a = -3$, $b = 0$, and $g(x) = e^{2x}$ gives:

$$w(x,T) = \left(1 + T + \frac{T^2}{2} + \frac{T^3}{6} + \frac{T^4}{24} + \cdots \right) e^{2x} \quad (15)$$
$$= e^{2x+T}.$$

Whence, the exact solution of (12) is:

$$w(x,t) = e^{2x+T}$$
$$= \exp(2x) \exp\left(\frac{t^\alpha}{\Gamma(1+\alpha)} \right) \quad (16)$$
$$= e^{2x+\left(\frac{t^\alpha}{\Gamma(1+\alpha)} \right)}$$

Note: When $\alpha = 1$, we have $w(x,t) = e^{2x+t}$ which corresponds to the exact solution of the classical NWSE of integer order [16, 17]. The exact and approximate solutions of case 1a are graphically presented in Fig. 1 and Fig. 1 respectively.
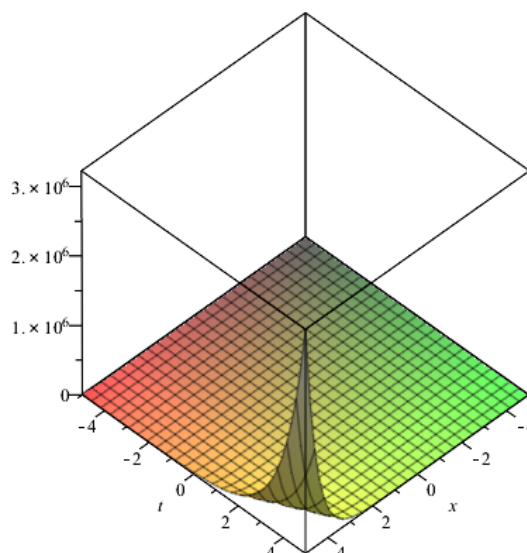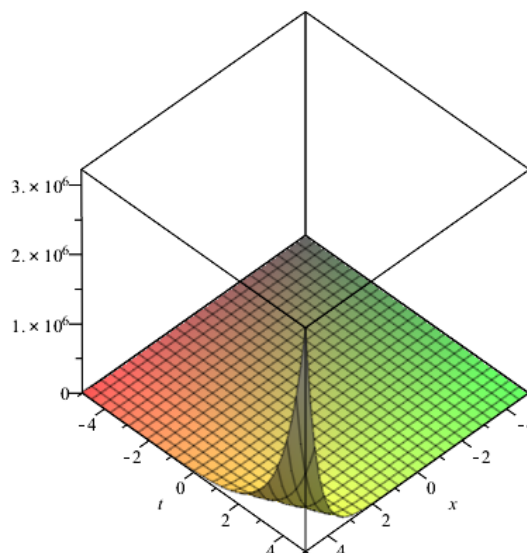

Fig. 1: Exact Solution w.r.t. *Case 1a*

Fig. 2 Approximate Solution w.r.t. *Case 1a*

Case 1b: Consider the following linear NWSE [18]:

$$\begin{cases} w_t^\alpha = w_{xx} - 2w, \\ w(x,0) = e^x, \end{cases} \quad (17)$$

with an exact solution:

$$w(x,t) = e^{x-t}. \quad (18)$$

Procedure w.r.t case 1b:
By FCT,

$$T = \frac{at^\alpha}{\Gamma(1+\alpha)}, \text{ which according to section 3}$$

gives $D_t^\alpha w = \frac{\partial w}{\partial T} = w_T$ for $a = 1$. Hence, (17) becomes:

S. O. Edeki, O. P. Ogundile, B. Osoba,
G. A. Adeyemi, F. O. Egara, S. A. Ejoh

$$\begin{cases} w_T = w_{xx} - 2w = 0, \\ w(x,0) = e^x, \end{cases} \qquad (19)$$

$$p^{(0)} : w_0 = e^{2x},$$

$$\left. \begin{array}{l} p^{(1)} : w_1 = I_0^T \left(w_{xx,0} - 2w_0 \right) \\ \qquad = -Te^x \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(2)} : w_2 = I_0^T \left(w_{xx,1} - 2w_1 \right) \\ \qquad = \dfrac{T^2 e^x}{2!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(3)} : w_3 = I_0^T \left(w_{xx,2} - 2w_2 \right) \\ \qquad = -\dfrac{T^3 e^x}{3!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(4)} : w_4 = I_0^T \left(w_{xx,3} - 2w_3 \right) \\ \qquad = \dfrac{T^4 e^x}{4!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(5)} : w_5 = I_0^T \left(w_{xx,4} - 2w_4 \right) \\ \qquad = -\dfrac{T^5 e^x}{5!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(6)} : w_6 = I_0^T \left(w_{xx,5} - 2w_5 \right) \\ \qquad = \dfrac{T^6 e^x}{6!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(7)} : w_7 = I_0^T \left(w_{xx,6} - 2w_6 \right) \\ \qquad = -\dfrac{T^7 e^x}{7!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(8)} : w_8 = I_0^T \left(w_{xx,7} - 2w_7 \right) \\ \qquad = \dfrac{T^8 e^x}{8!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(9)} : w_9 = I_0^T \left(w_{xx,8} - 2w_8 \right) \\ \qquad = -\dfrac{T^9 e^x}{9!} \end{array} \right\},$$

$$\left. \begin{array}{l} p^{(10)} : w_{10} = I_0^T \left(w_{xx,9} - 2w_9 \right) \\ \qquad = \dfrac{T^{10} e^x}{10!} \end{array} \right\},$$

$$\vdots$$

$$\left. \begin{array}{l} p^{(m)} : w_m = I_0^T \left(w_{xx,m\text{-}1} - 2w_{m-1} \right) \\ \qquad = \left(-1\right)^m \dfrac{T^m e^{2x}}{m!}, m \in N \end{array} \right\},$$

Therefore, the analytical solution of (19) according to He's polynomial is:

$$w(x,T) = \left(1 - T + \frac{T^2}{2!} - \frac{T^3}{3!} + \frac{T^4}{4!} + \cdots \right) e^x \qquad (20)$$

$$= e^{x-T}.$$

Whence, the exact solution of (17) is:

$$\left. \begin{array}{l} w(x,t) = \exp(x)\exp\left(-\dfrac{t^\alpha}{\Gamma(1+\alpha)} \right) \\[4mm] \qquad = e^{x - \left(\frac{t^\alpha}{\Gamma(1+\alpha)} \right)} \end{array} \right\} \qquad (21)$$

Note: When $\alpha = 1$, we have $w(x,t) = e^{x-t}$ which corresponds to the exact solution of the classical NWSE of integer order [16]. The exact and the approximate solutions of *Case 1b* are graphically presented in Fig. 3 and Fig. 4 respectively.
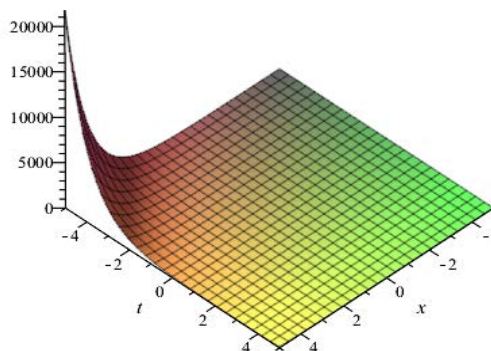

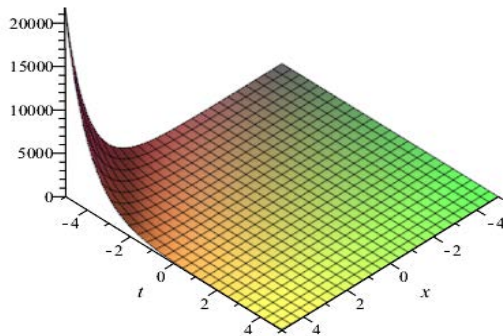Fig. 3: Exact Solution w.r.t. *Case 1b*

S. O. Edeki, O. P. Ogundile, B. Osoba,
G. A. Adeyemi, F. O. Egara, S. A. Ejoh

Fig 4. Approximate Solution w.r.t. *Case 1b*

Case 2: Consider the following nonlinear (NL) NWSE

$$\begin{cases} w_t^\alpha = w_{xx} + 2w - 3w^2, \\ w(x,0) = \eta, \end{cases} \tag{22}$$

with the exact solution at $\alpha = 1$:

$$w(x,t) = \frac{2\eta e^{2t}}{2 - 3\eta + 3\eta e^{2t}} \tag{23}$$

Procedure w.r.t Case2:

By the FCT,

$$T = \frac{at^\alpha}{\Gamma(1+\alpha)}, \text{ which according to section 3}$$

gives $D_t^\alpha w = \dfrac{\partial w}{\partial T} = w_T$ for $a = 1$. Hence, (22)

becomes:

$$\begin{cases} w_T = w_{xx} + 2w - 3w^2, \\ w(x,0) = \eta. \end{cases} \tag{24}$$

Therefore, using the outlined procedures in section 2.1, we obtained the following:

$$p^{(0)} : w_0 = \eta,$$

$$\left. \begin{aligned} p^{(1)} : w_1 &= I_0^T \left(w_{xx,0} + 2w_0 - 3(w_0)^2 \right) \\ &= \left(-3\eta^2 + 2\eta \right) T \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(2)} : w_2 &= I_0^T \left(w_{xx,1} + 2w_1 - 3(2w_0 w_1) \right) \\ &= \left(9\eta^3 - 9\eta^2 + 2\eta \right) T^2 \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(3)} : w_3 &= I_0^T \left(w_{xx,2} + 2w_2 - 3(2w_0 w_2 + w_1 w_1) \right) \\ &= \left(-27\eta^4 + 36\eta^3 - 14\eta^2 + \frac{3}{4}\eta \right) T^3 \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(4)} : w_4 &= I_0^T \left(w_{xx,3} + 2w_3 - 3(2w_0 w_3 + 2w_1 w_2) \right) \\ &= \left(81\eta^5 - 135\eta^4 + 75\eta^3 - 15\eta^2 + \frac{2}{3}\eta \right) T^4 \end{aligned} \right\},$$

$$\left. \begin{aligned} p^{(5)} : w_5 &= I_0^T \left(w_{xx,4} + 2w_4 - 3\left(\begin{matrix} 2w_0 w_4 \\ +2w_1 w_3 + w_2 w_2 \end{matrix} \right) \right) \\ &= \left(\begin{matrix} -243\eta^6 + 486\eta^5 - 351\eta^4 \\ +108\eta^3 - \frac{62}{5}\eta^2 + \frac{4}{15}\eta \end{matrix} \right) T^5 \end{aligned} \right\},$$

$$\vdots \ .$$

Therefore, the analytical solution of (24) according to He's polynomial is:

$$w(x,T) = \begin{cases} \eta + \left(2\eta - 3\eta^2\right)T + \left(2\eta - 9\eta^2 + 9\eta^3\right)T^2 \\ + \left(\frac{3}{4}\eta - 14\eta^2 + 36\eta^3 - 27\eta^4\right)T^3 \\ + \left(\begin{matrix} \frac{2}{3}\eta - 15\eta^2 + 75\eta^3 \\ -135\eta^4 + 81\eta^5 \end{matrix} \right) T^4 \\ + \left(\begin{matrix} \frac{4}{15}\eta - \frac{62}{5}\eta^2 + 108\eta^3 \\ -351\eta^4 + 486\eta^5 - 243\eta^6 \end{matrix} \right) T^5 + \cdots \end{cases} \tag{25}$$

Hence, the solution of (22) is:

$$w(x,t) = w\left(x, \left(T = \frac{t^\alpha}{\Gamma(1+\alpha)} \right) \right)$$

$$= \begin{cases} \eta + \left(2\eta - 3\eta^2\right)T + \left(2\eta - 9\eta^2 + 9\eta^3\right)T^2 \\ + \left(\frac{3}{4}\eta - 14\eta^2 + 36\eta^3 - 27\eta^4\right)T^3 \\ + \left(\frac{2}{3}\eta - 15\eta^2 + 75\eta^3 - 135\eta^4 + 81\eta^5\right)T^4 \\ + \left(\begin{matrix} \frac{4}{15}\eta - \frac{62}{5}\eta^2 + 108\eta^3 - 351\eta^4 \\ +486\eta^5 - 243\eta^6 \end{matrix} \right) T^5 + \cdots \end{cases} \tag{26}$$

Note: In Table 1 and Table 2, the exact solution *(Soln)*, approximate *(Appr.)* solution and relative absolute error *(Rel. Abs. Error)* are considered for

$\alpha = 1$. For Table 1, $\eta = 0.01$ through 0.11 at $t = 1$ is used while $0.01 \leq \eta \leq 0.10$ and $0 \leq t \leq 1$ are used for Table 2. Figure 2 shows the exact and FCT-HP solutions for *Case 2.*

Table 1: Solutions of *Case 2* NL at $t = 1$

Exact Soln	Appr. Soln	Rel. Abs. Error
0.067428494	0.06754959	0.00179274
0.124011608	0.12571705	0.01356573
0.172171175	0.17569407	0.02005131
0.213657904	0.21856245	0.02244003
0.249768793	0.25530047	0.02166734
0.281485162	0.28678908	0.01849415
0.309563187	0.3138179	0.01355792
0.334594967	0.33709110	0.00740492
0.357050732	0.35723301	0.00051024
0.377308671	0.37479367	0.00671037
0.395676392	0.39025411	0.01389424

Table 2: Solutions of *Case 2* NL at $t \in [0,1]$

| η	t	Exact Soln	Appr. Soln	Rel. Abs Err
0.01	0.0	0.010000000	0.0100000	1.7347E-16
0.02	0.2	0.029402665	0.0294027	1.8769E-06
0.03	0.3	0.052713413	0.0527153	3.486E-05
0.04	0.4	0.082924030	0.0829416	0.00021159
0.05	0.5	0.120398234	0.1204862	0.00073028
0.06	0.6	0.164795898	0.1650817	0.00173143
0.07	0.7	0.214918881	0.2155659	0.0030016
0.08	0.8	0.268754950	0.2697255	0.00359832
0.09	0.9	0.323760179	0.3242452	0.00149576
0.10	1.0	0.377308671	0.3747937	0.00671037
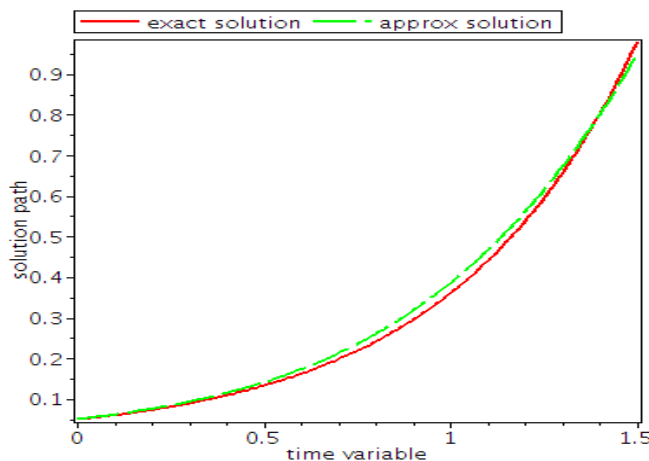
Figure 2: Exact and FCT-HP solutions for case 2

4 Concluding Remarks

Fractional Complex Transform (FCT) coupled with He's polynomials method was successfully applied for the time-fractional Newell-Whitehead-Segel model (TFNWSM) with regard to exact and approximate solutions. The associated derivatives were defined in terms of Jumarie's sense. Based on the solved linear and nonlinear problems, reliability and efficiency of the proposed solution method are assured by the obtained results as less computational time is involved. In addition, high basic knowledge of fractional calculus is not necessarily required. The method is therefore, recommended for space-fractional derivatives of higher-ordered problems arising from other areas of pure and applied sciences. This can also be benchmarked with the Restarted Adomian decomposition Method and other approaches.

Acknowledgement

Conflict of Interests

The authors declare that they have no conflict of interest regarding the publication of this paper.

References
[1] A.M. Wazwaz, M.S. Mehanna, The combined Laplace–Adomian method for handling singular integral equation of heat transfer, *Int.. J Nonlinear Sci.* 10, 2010, 248-52.
[2] M.M. Rashidi, The modified differential transform method for solving MHD boundary-layer equations, *Comput Phys Commun,* 180, 2009, 2210–7.
[3] J.H. He, A coupling method of homotopy techniques and perturbation technique for nonlinear problems, *International Journal of Non-Linear Mechanics*, 35 (1), 2000, 37-43
[4] S. O. Edeki, G. O. Akinlabi, and S. A. Adeosun, On a modified transformation method for exact and approximate solutions of linear Schrödinger equations, *AIP Conference Proceedings*, 1705, 020048, 2016, doi: 10.1063/1.4940296.
[5] G.O. Akinlabi and S. O. Edeki, On Approximate and Closed-form Solution Method for Initial-value Wave-like Models,

International Journal of Pure and Applied Mathematics, 107 (2), 2016, 449-456.

[6] N. E. Mastorakis, An Extended Crank-Nicholson Method and its Applications in the Solution of Partial Differential Equations: 1-D and 3-D Conduction Equations, *WSEAS Transactions on Mathematics*, 6 (1), 2007, 215-225.

[7] H. K. Mishra and A. K. Nagar, He-Laplace Method for Linear and Nonlinear Partial Differential Equations, *Journal of Applied Mathematics*, 2012, 2012, 1-16.

[8] G.O. Akinlabi and S. O. Edeki, The solution of initial-value wave-like models via perturbation iteration transform method, *Lecture Notes in Engineering and Computer Science,* 2228, 2017, 1015-1018.

[9] R. Vilums, A. Buikis, Conservative Averaging and Finite Difference Methods for Transient Heat Conduction in 3D Fuse, *WSEAS Transactions on Heat and Mass Transfer*, 3 (1), 2008, 111-124.

[10] S.O. Edeki, G.O. Akinlabi and S.A. Adeosun, Analytic and Numerical Solutions of Time-Fractional Linear Schrödinger Equation, *Communications in Mathematics and Applications*, 7 (1), 2016, 1–10.

[11] J. G. Oghonyon, S.A. kunuga, S. A. Bishop, A 5-step block predictor and 4-step corrector methods for solving general second order ordinary differential equations, *Global Journal of Pure and Applied Mathematics*, 11 (5), 2015, 3847-386.

[12] S.O. Edeki, G.O. Akinlabi, He-Laplace Method for the Solutions of the Navier-Stokes Model, *Proceedings of the World Congress on Engineering*, London, UK, 2017.

[13] S.O. Edeki, G.O. Akinlabi, Zhou Method for the Solutions of System of Proportional Delay Differential Equations, *MATEC Web of Conferences* 125, 02001, (2017) doi: 10.1051/matecconf/201712502001.

[14] J. G. Oghonyon, N.A. Omoregbe, S. A. Bishop, Implementing an order six implicit block multistep method for third order ODEs using variable step size approach, *Global Journal of Pure and Applied Mathematics*, 12 (2), 2016, 1635-1646.

[15] G. O. Akinlabi, S. O., Edeki, Perturbation Iteration Transform Method for the Solution of Newell-Whitehead-Segel Model Equations, *Journal of Mathematics and Statistics*. (2017), https://arxiv.org/abs/1703.06745

[16] P. Pue-on, Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation, *Applied Mathematical Sciences,* **7** (132), 2013, 6593 – 6600.

[17] J. Patade, and S. Bhalekar, Approximate Analytical solutions of Newell-Whitehead-Segel equation using a new iterative method. *World J. of Modelling and Simulation,* 11(2), 2015, 94–103.

[18] A. Saravanan, N. Magesh, A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell–Whitehead–Segel equation, *Journal of the Egyptian Mathematical Society*, 21, 2013, 259–265.

[19] S.O. Edeki, O. O. Ugbebor, E. A. Owoloko, He's polynomials for analytical solutions of the Black-Scholes pricing model for stock option valuation, *Lecture Notes in Engineering and Computer Science*, 2224, 2016, 632-634.

[20] S.O. Edeki, G.O. Akinlabi, and M.E. Adeosun, Analytical solutions of the Navier-Stokes model by He's polynomials, *Lecture Notes in Engineering and Computer Science*, 2223, 2016, 16-19.

[21] A. Ghorbani and J. S. Nadjfi, He's homotopy perturbation method for calculating Adomian's polynomials*, Int. J. Nonlin. Sci. Num. Simul*. 8 (2), 2007, 229-332.

[22] J.H. He, Homotopy perturbation method: A new nonlinear analytical technique, *Appl. Math. Comput.* 135, 2003, 73-79.

[23] J. Saberi-Nadjafi, A. Ghorbani, He's homotopy perturbation method: an effective tool for solving nonlinear integral and integro-differential equations, *Computers & Mathematics with Applications*, 58, 2009, 1345–1351.

[24] S. O. Edeki, and G. O. Akinlabi, Coupled method for solving time-fractional navier-stokes equation, *International Journal of Circuits, Systems and Signal Processing*, 12, 2017, 27-34.

[25] G. Jumarie, Modified Riemann-Liouville Derivative and Fractional Taylor series of Non-differentiable Functions Further Results, *Computers and Mathematics with Applications*, 51, (9-10), 2006, 1367-1376.

[26] G. Jumarie, Cauchys integral formula via the modified Riemann- Liouville derivative for analitic functions of fractional order, *Appl. Math. Lett*, 23, 2010, 1444-1450.

[27] J. Singh, D. Kumar and S. Rathore, Application of Homotopy Perturbation

S. O. Edeki, O. P. Ogundile, B. Osoba,
G. A. Adeyemi, F. O. Egara, S. A. Ejoh

Transform Method for Solving Linear and Nonlinear Klein-Gordon Equations, *Journal of Information and Computing Science,* 7 (2), 2012, 131-139.

[28] H. M. A. Rahman The Homotopy Perturbation Sumudu Transform Method for Solving the Nonlinear Partial Differential Equations, *WSEAS Transactions on Mathematics,* 16, 2017, 421-429.