MIMO Deficiencies Due to Antenna Coupling

Charles Uzoanya Ndujiuba^{*}, Oluwadamilola Oshin, Nsikan Nkordeh

Electrical & Information Engineering, Covenant University, Ota, Nigeria

Abstract In wireless cellular radiocommunication systems, multiple-input and multiple-output, (MIMO) technology is commonly employed because of the enormous benefits it offers. It is the technique of using multiple antennas at the transmitter to propagate signal through multiple propagation paths to multiple antennas at the receiver. This can be used to significantly increase communication performance, measured by data throughput, and link reliability without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency, channel capacity and link reliability. These benefits are not without setbacks due to mutual coupling of the antennas, correlation of the signals and the degree of matching between the receiver and the load. In this paper, we discuss radiation pattern of antenna array configuration, mutual coupling between elements of antenna arrays, correlation between the antennas, and their impact on channel capacity. We then formulate general expressions showing the impact, on the capacity of the MIMO channel, by both antenna coupling and spatial correlation due to the propagation environment. We then provide simulation results to illustrate our theoretical analysis.

Keywords MIMO system, Antenna arrays, Mutual coupling, Spatial correlation, Channel matrix

1. Introduction

These days communication requires a very high rate with high reliability. Two major difficulties to obtain reliable communication via high rate wireless communication systems are bandwidth limitation of communication channels and multipath fading. To surmount these difficulties multiple antenna systems, which provide a transmit and/or receive diversity, can be used.

The increase in performance of the MIMO radio system can be measured by higher data rates, improved spectral efficiencies and the increase in channel capacity of the system.

In order to realize these advantages of MIMO, two conditions have to be satisfied. One requires the presence of a rich scattering environment, and the other one entails accurate channel state information (CSI) to be available at the receiver.

The rich scattering environment is necessary to support the formation of statistically independent virtual channels over which the parallel data transmission can take place. The lack of (spatial) correlation between the virtual channels leads to the increased MIMO capacity. The availability of accurate CSI is required to decode the received signal and to practically achieve the MIMO capacity [1-5]. In turn, an inaccurate CSI leads to an increased bit error rate (BER) that translates into a degraded capacity of the system [6-8].

It is generally accepted that correlation between the links of a MIMO channel reduces the capacity of the MIMO radio channel [1], [2]. In addition to the impact of spatial correlation due to the propagation environment on the capacity of the MIMO channel, the coupling between antenna elements of the transmitter and receiver also has impacts on the capacity of a given communication channel.

The electromagnetic interaction between the antenna elements in an antenna array results in mutual coupling. By its nature, mutual coupling exhibits differently in transmitting and receiving antenna arrays and therefore has to be treated differently. The effect of mutual coupling is serious if the element spacing is small. It will affect the antenna array mainly in the following ways:

- 1. Change the array radiation pattern
- 2. Change the received element voltages
- 3. Change the matching characteristic of the antenna elements (change the input impedances)

In this paper we mainly study the first two effects. We use capacity as a metric for comparing the performance of the MIMO systems with different coupling levels and correlation yield.

The analysis of mutually-coupled antennas in a multiple-input multiple-output (MIMO) system is performed in two folds; 1) mutual coupling between the elements of antenna array and 2) mutual coupling between the transmit and receive antennas. The approach uses network theory to formulate the transfer matrix relating the signals input to one element of the antenna array to the signals at the neighboring element in the first fold, and the input signal at the transmit

^{*} Corresponding author:

charles.ndujiuba@covenantuniversity.edu.ng (Charles Uzoanya Ndujiuba) Published online at http://journal.sapub.org/ijnc

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antennas to the signals at the output of the receiver front end, in the second fold. This transfer function includes the coupled transmit and receive antennas, and the multipath propagation channel, which describes the spatial correlation.

In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of all Mx N paths between the M transmit antennas at the transmitter and N receive antennas at the receiver. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information. Figure 1 is a model of a typical MIMO network.



Figure 1. Basic MIMO System

The core idea under the MIMO systems is the ability to turn multi-path propagation, which is typically an obstacle in conventional wireless communication, into a benefit for users.

For a MIMO system with M number of transmit antennas and N number of receive antennas the channel coefficient matrix is given in Eq. (1)

$$\mathbf{H} = \begin{bmatrix} h_{11}h_{12} \dots & h_{1N} \\ h_{21}h_{22} \dots & h_{2N} \\ \vdots & \vdots & \vdots \\ h_{M1} & h_{M2} \dots & h_{MN} \end{bmatrix}$$
(1)

Here, it is assumed that all the elements of the channel matrix are independent and identically distributed (i.i.d.). If the input signals of the system are denoted as x and the Gaussian noise is represented as n then the output response of the MIMO system y is given as

$$y = xH + n \tag{2}$$

The channel is presented by an $N \ge M$ complex matrix **H**, whose elements h_{ij} are the channel coefficients between the j^{th} Tx antenna (j = 1, ..., M) and the i^{th} Rx antenna (i = 1, ..., N) [9].

If we assume that the average total power P_r received by each Rx antenna (regardless of noises) is equal to the average total transmitted power P from M Tx antennas, the Signal-to-Noise Ratio (SNR) at each Rx antenna is then

$$\rho = \frac{P_r}{\sigma^2} = \frac{P}{\sigma^2} \tag{3}$$

where σ^2 is noise power.

The *system capacity C*(*bits/s*) is defined as the maximum possible transmission rate such that the error probability is arbitrarily small.

In section II, at first, we derive the most general formula to calculate the channel capacity for both cases where channel coefficients are known as well as unknown at the transmitter. Based on this general formula, we will then derive the formulas for channel capacity in some particular cases.

The most general formula for calculating channel capacity in the case where channel coefficients are *either* known *or* unknown at the transmitter is the Shannon capacity formula [10]:

$$C = W \sum_{i=1}^{r} \log_2 \left(1 + \frac{P_{ri}}{\sigma^2} \right)$$
(4)

where *W* is the bandwidth of each sub-channel, *r* is the rank of the channel coefficient matrix H. P_{ri} is the received power at each Rx antenna from the i^{th} channel, for i = 1, ..., r, during the considered symbol time slot.

MIMO systems can provide a potentially huge capacity gain with the same requirements for power and bandwidth as the single antenna systems. In many cases, the capacity of channels is proved to increase linearly with the lower number among the number of transmitter antennas (Tx antennas) and that of receiver antennas (Rx antennas) [11].

One of the major problems in MIMO system is mutual coupling, which is mainly due to the smaller spacing between the elements of the antenna array. Another problem is accurate Channel State Information (channel coefficients). To correctly form a beam, the transmitter needs to understand the characteristics of the channel. Understanding the channel allows for manipulation of the phase and amplitude of each transmitter in order to form a beam. This process is called *channel sounding* or channel estimation [12]. A known signal is sent to the mobile device that enables it to build a picture of the channel environment. The mobile device sends back the channel characteristics to the transmitter. The transmitter can then apply the correct phase and amplitude adjustments to form a beam directed at the mobile device.

The benefits of beamforming are to increase the received signal gain - by making signals emitted from different antennas add up constructively, and to reduce the multipath fading effect. In line-of-sight propagation, beamforming results in a well-defined directional pattern.

In Long Term Evolution (LTE), MIMO technologies have been widely used to improve downlink peak rate, cell coverage, as well as average cell throughput [13]. To achieve this diverse set of objectives, LTE adopted various MIMO technologies including transmit diversity, single user (SU)-MIMO, multiuser (MU)-MIMO, closed-looprank-1 precoding, and dedicated beamforming [10-13]. The SU-MIMO scheme is specified for the configuration with two or four transmit antennas in the downlink, which supports transmission of multiple spatial layers with up to four layers to a given User Equipment (UE). The transmit diversity scheme is specified for the configuration with two or four transmit antennas in the downlink, and with two transmit antennas in the uplink. The MU-MIMO scheme allows allocation of different spatial layers to different users in the same time-frequency resource, and is supported in both uplink and downlink. The closed-loop rank-1 precodings cheme is used to improve data coverage utilizing SU-MIMO technology based on the cell-specific common reference signal while introducing a control signal message that has lower overhead. The dedicated beamforming scheme is used for data coverage extension when the data demodulation based on dedicated reference signal is supported by the UE.

In a normal communication system, usually a single antenna at the transmitter and another single antenna at the receiver is employed. The signal reaching the receiver has to travel through various paths, affected by noise in the path and finally reaches the receiver. In a system with Mtransmit and N receive antennas (Figure 1), assuming the path gains between individual antenna pairs are independent and identically distributed (i.i.d.) Rayleigh faded, the maximal diversity gain is MN, which is the total number of fading gains that one can average over [14]. Usually, multipath effects are drawback for a normal system, whereas in MIMO system, this multipath propagation is taken as advantage for transmitting multiple data streams. Essentially, if the path gains between individual transmit-receive antenna pairs fade independently, the channel matrix is well conditioned with high probability, in which case multiple parallel spatial channels are created, thus improving the channel capacity [15-18]. By transmitting independent information streams in parallel through the spatial channels, the data rate can be increased.

2. Channel Correlation

The performance of a multiple-input multiple-output (MIMO) is critically dependent on the availability of independent multiple channels. It is well known that channel correlation will downgrade the performance of a MIMO system, especially its capacity. Channel correlation is a measure of similarity or likeliness between the channels. In the extreme case that if the channels are fully correlated, then the MIMO system will have no difference from a single-antenna communication system. The channel correlation of a MIMO system is mainly due to two components:

(1) spatial correlation

(2) antenna mutual coupling

We consider some particular cases as follows:

A) Unknown Channel Coefficients at the Transmitter

• Single antenna channel: In this case, we have r = M = N = 1 the channel capacity is calculated

As

$$C = W \log_2 \left[\det \left(1 + \frac{P}{\sigma^2} \right) \right]$$
(5)

At SNR $p = \frac{P}{\sigma^2} = 20 dB$, for instance, the normalized capacity of the single antenna channel is C/W = 6.658 bits/s/Hz.

• **Receive diversity**: In this case, M = 1, $N \ge 2$ and $H = (h_1, \dots, h_N)^T$ where $(.)^T$ denotes the transposition operation. The channel capacity is calculated as

$$C = W \log_2 \left(1 + \frac{P}{\sigma^2} \sum_{i=1}^N |h_i|^2 \right)$$
(6)

Assuming that $|h_i|^2 = 1$, for i = 1, ..., N, then we have

$$C = W \log_2 \left(1 + \frac{PN}{\sigma^2} \right)$$

For N = 2 and SNR p = 2dB, we have C/W = 7.6511 bits/s/Hz.

We can see that the normalized capacity in this case is larger than that in the case of channels with single Tx and Rx antennas.

• **Transmit diversity**: In this case, $M \ge 2$, N = 1, and H = (h_1, \dots, h_M) the channel capacity is calculated as

$$C = W \log_2 \left(1 + \frac{P}{M\sigma^2} \sum_{i=1}^{M} |h_i|^2 \right)$$

Assuming that $|h_i|^2 = 1$, for i = 1, ..., M, then we have

$$C = W \log_2 \left(1 + \frac{P}{\sigma^2} \right)$$

We see that the capacity of the channel where channel coefficients are fixed and unknown at the transmitter is the same as that of the single antenna channel regardless of the number Mof Tx antennas.

Hence, for M = 2, N = 1 and SNR p = 2dB, we have C/W = 6.658 bits/s/Hz.

B) Known Channel Coefficients at the Transmitter

The channel capacity can be increased if channel coefficients are known at the transmitter. In this case, the transmitted power is assigned unequally to the Tx antennas, such that a larger power is assigned to a better sub-channel and vice versa.

• **Transmit diversity**: In this case, $M \ge 2$, N = 1, and H = (h_1, \dots, h_M) the channel capacity is calculated as

$$C = W \log_2 \left(1 + \frac{P}{M\sigma^2} \sum_{i=1}^M |h_i|^2 \right)$$

Assuming that $|h_i|^2 = 1$, for i = 1, ..., M, then we have

$$C = W \log_2 \left(1 + \frac{PM}{\sigma^2} \right)$$

Hence, for M = 2, N = 1 and SNR p = 2dB, we have C/W = 7.6511 bits/s/Hz which is larger than the channel capacity when the channel coefficients are unknown at the transmitter (C/W = 6.658 bits/s/Hz).

2.1. Impact of Antenna Mutual Coupling on Array Radiation Pattern

The main effect of antenna mutual coupling is to change the signal correlation from that caused by spatial correlation alone. An important question is how to model this effect so that it can be correctly built into the correlation structure of channel matrix \mathbf{H} given by equation (1).



Figure 2. Two-element antenna array

Assume $I_2 = I_1 \angle \alpha$

$$\varphi = \beta dcos\theta + a$$



Figure 3(a). Equivalent circuit of two-element antenna array with no mutual coupling



 V_{sl} , V_{s2} =Excitation voltage source

 Z_{gl} , Z_{g2} = Source internal impedance

 $Z_{11}, Z_{22} =$ Antenna self-impedance

 $I_1, I_2 = \text{Terminal current}$

 V_{12} , V_{21} = Coupled voltage



$$Z_{12} = \frac{V_{12}}{I_2} = -\frac{1}{I_1 I_2} \int_{r_{11}}^{r_{12}} E_{12}(r_1) J_1(r_1) dr_1$$

= Mutual impedance with antenna 2 excited (7)

$$Z_{21} = \frac{V_{21}}{I_1} = -\frac{1}{I_1 I_2} \int_{r_{21}}^{r_{22}} E_{21}(r_2) J_2(r_2) dr_2$$

= Mutual impedance with antenna 1 excited (8)

where $J_2(r_2)$ is the current distribution on antenna 2, and $E_{21}(r_2)$ is the electric field produced by the current distribution $J_1(r_1)$ on antenna 1 along antenna 2.

From the antenna equivalent circuits

$$Z_{21} = -\frac{1}{I_1 I_2} \int_{r_{11}}^{r_{12}} \int_{r_{21}}^{r_{22}} k(r_1 r_2) J_1(r_1) J_2(r_2) dr_1 dr_2 \quad (9)$$

where k is the constant function of the conductivity of the medium.

For an N-element antenna array, the mutual impedances can be obtained by considering two antennas at a time. The total mutual impedances of the array, Zij (i,j=1,2,...,N) will then be the set of two-antenna mutual impedances for all possible pair of antennas in the array.

$$I_{s1} = \frac{V_{s1}}{Z_{g1} + Z_{11}}, \qquad I_{s2} = \frac{V_{s2}}{Z_{g2} + Z_{22}}$$
(10)

 I_{s1} and I_{s2} are the terminal currents at the antennas when there is no mutual coupling effect.

$$I_{1} = I_{s1} - \frac{V_{12}}{Z_{g1} + Z_{11}} = I_{s1} - \frac{I_{2}Z_{12}}{Z_{g1} + Z_{11}}$$
$$I_{2} = I_{s2} - \frac{V_{21}}{Z_{g2} + Z_{22}} = I_{s2} - \frac{I_{1}Z_{21}}{Z_{g2} + Z_{22}}$$

Array Factor,

$$AF = \frac{1}{I_1} \left[I_1 + I_2 e^{j\beta d\cos\theta} \right]$$

= $\frac{1}{I_1 K} \left\{ (I_{s1} - Z'_{12} I_{s2}) + (I_{s2} + Z'_{21} I_{s1}) e^{j\beta d\cos\theta} \right\}$ (11)

where

$$Z'_{12} = \frac{Z_{12}}{Z_{g1} + Z_{11}}, \quad Z'_{21} = \frac{Z_{21}}{Z_{g2} + Z_{22}}$$
$$K = 1 - \frac{Z_{12}Z_{21}}{\left(Z_{g1} + Z_{11}\left(Z_{g2} + Z_{22}\right)\right)} \tag{12}$$

For passive antennas $Z'_{12} = Z'_{21}$

$$AF = \frac{1}{I_1 K} \{ (I_{s1} + I_{s2} e^{j\beta d\cos\theta}) - Z'_{12} (I_{s2} + I_{s1} e^{j\beta d\cos\theta}) \}$$
$$= \frac{I_{s1}}{I_1 K} \{ (1 + e^{jp} e^{j\beta d\cos\theta}) - Z'_{12} (e^{jp} + e^{j\beta d\cos\theta}) \}$$
$$= \frac{I_{s1}}{I_1 K} \{ \underbrace{(1 + e^{j(\beta d\cos\theta + p)})}_{\text{original pattern}} - Z'_{12} e^{jp} \underbrace{(1 + e^{j(\beta d\cos\theta - p)})}_{\text{additional pattern}} \}$$

where

$$e^{jp} = \frac{I_{s2}}{I_{s1}}$$

It can be seen that the radiation pattern with antenna coupling, consists of two parts: the original radiation pattern (without antenna coupling) plus an additional pattern (due to antenna coupling):

$$Z_{12}^{\prime}e^{jp}\left(1+e^{j\left(\beta dcos\theta-p\right)}\right)$$

which modifies (reduces) the amplitude of the original radiation pattern, hence reduces the received power at the receiver.

We demonstrate this result with the following case:

Find the normalized array pattern $|\mathbf{E}_n|$ on the horizontal plane $(\theta = \pi/2)$ of a two-monopole array with the following parameters with mutual coupling taken into account:

$$\begin{split} I_{s1} &= 1, \qquad I_{s2} = e^{j\beta}, \qquad \beta = 150^{\circ} \\ d &= {}^{\lambda}/_{4}, \quad l = {}^{\lambda}/_{4} \\ Z_{12} &= Z_{21} = 21.8 - j21.9\Omega \\ Z_{11} &= Z_{22} = 47.3 + j22.3\Omega \\ Z_{g1} &= Z_{g2} = 50\Omega \\ kd &= \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \\ Z'_{21} &= \frac{Z_{12}}{Z_{g1} + Z_{11}} = \frac{Z_{21}}{Z_{g2} + Z_{22}} = 0.6 - j0.26 \\ D &= 1 - \frac{Z_{12}Z_{21}}{(Z_{11} + Z_{g1})(Z_{22} + Z_{g2})} = 1.042 + j0.09 \\ |E| &= |AF| \\ &= \frac{I_{s1}}{I_{1}D} \{ [1 + e^{j(kd\cos\varphi + \beta)}] - Z'_{12}e^{j\beta} [1 + e^{j(kd\cos\varphi - \beta)}] \} \\ &= \left| \frac{0.9 - j0.37}{I_{1}} \right| |1 + (-1.14 + j0.4)e^{j\pi/2\cos\varphi} | \\ |E|_{\varphi = 180^{\circ}} &= \frac{1.83}{I_{1}} \\ |E_{n}| &= \frac{|E|}{|E|_{\varphi = 180^{\circ}}} = 0.52 |1 + (-1.14 + j0.4)e^{j(\pi/2)\cos\varphi} | (13) \end{split}$$

The pattern of $f = |1 + (-1.14 + j0.4)e^{j(\pi/2)cos\phi}|$ is shown in Figure 4.



Figure 4. Radiation pattern of coupled antennas

$$|E_n|_{\text{no mutual coupling}} = 0.52 | [1 + e^{j2.62} e^{j(\pi/2) \cos\varphi}] | 0.52 | 1 + (-0.866 + j0.5) e^{j(\pi/2) \cos\varphi} |$$
(14)

The pattern of $f = |1 + (-0.866 + j0.5)e^{j(\pi/2)cos\varphi}|$ is shown in Figure 5.



Figure 5. Radiation pattern of antennas with no coupling

2.2. Impact of Antenna Mutual Coupling on Channel Correlation

In the transmitter antenna array, antenna mutual coupling causes the input signals being coupled into neighbouring antennas. This effect can be represented by a mutual coupling impedance matrix \mathbf{Z}_t

$$\mathbf{V}_{tot} = \mathbf{Z}_t^{-1} \mathbf{V}_s \tag{15}$$

where \mathbf{V}_s is the excitation voltage vector with mutual coupling not taken into account, \mathbf{V}_{tot} is the excitation voltage vector when mutual coupling is taken into account and

$$\mathbf{Z}_{t} = \begin{bmatrix} 1 & \frac{Z_{12}}{Z_{g2} + Z_{22}} & \cdots & \frac{Z_{1M}}{Z_{gM} + Z_{MM}} \\ \frac{Z_{21}}{Z_{g1} + Z_{11}} & 1 & \cdots & \frac{Z_{2M}}{Z_{gM} + Z_{MM}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{M1}}{Z_{g1} + Z_{11}} & \frac{Z_{M2}}{Z_{g2} + Z_{22}} & \cdots & 1 \end{bmatrix}$$
(16)

Similarly, for the output signals, they are also modified by the antenna mutual coupling effect in the receiving antenna arrays. The actual output coupled voltage vector \mathbf{V}_c is related to the uncoupled output voltage vector \mathbf{V}_u as [19]:

$$\mathbf{V}_c = \mathbf{Z}_r^{-1} \mathbf{V}_u \tag{17}$$

where \mathbf{Z}_r is the mutual impedance matrix containing the receiving mutual impedances

$$\mathbf{Z}_{r} = \begin{bmatrix} 1 & -\frac{Z_{t}^{12}}{Z_{L}} & \dots & -\frac{Z_{t}^{1N}}{Z_{L}} \\ -\frac{Z_{t}^{21}}{Z_{L}} & 1 & \dots & -\frac{Z_{t}^{2N}}{Z_{L}} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{Z_{t}^{N1}}{Z_{L}} & -\frac{Z_{t}^{N2}}{Z_{L}} & \dots & 1 \end{bmatrix}$$
(18)

In Equation (17), \mathbf{V}_c and \mathbf{V}_u are terminal voltage vectors across the antenna terminal loads. If the uncoupled output voltages refer to the open-circuit voltages, then \mathbf{V}_u is related

to the open-circuit voltage vector \mathbf{V}_{oc} as:

$$\mathbf{V}_{u} = \frac{Z_{L}}{Z_{in} + Z_{L}} \mathbf{V}_{oc} \tag{19}$$

In Equation (19), it is assumed that all the antenna elements have the same internal impedance Z_{in} and terminal impedance Z_L . Equation (17) then becomes:

$$\mathbf{V}_{c} = \frac{Z_{L}}{Z_{in} + Z_{L}} \mathbf{Z}_{r}^{-1} \mathbf{V}_{oc}$$
(20)

But in order for comparison with the performance of the uncoupled system whose output is expressed as open-circuit voltages, we need to change the terminal coupled voltage vector \mathbf{V}_c to the open-circuit coupled voltage vector \mathbf{V}'_{oc} . That is:

$$\mathbf{V}_{oc}^{'} = \frac{Z_{in} + Z_L}{Z_L} \mathbf{V}_c \tag{21}$$

Combining (17), (19), and (21), we have the signal model for a MIMO system under both spatial correlation and antenna mutual coupling as well as channel noise as:

$$\mathbf{V}_{oc}' = \mathbf{Z}_r^{-1} \mathbf{H} \mathbf{Z}_t^{-1} \mathbf{V}_s + \mathbf{V}_n \tag{22}$$

where V_n is the vector of noise voltages which are assumed to be not affected by antenna mutual coupling. Note that the spatial correlation is included inside the channel matrix **H** while the antenna mutual coupling is included\ inside the matrices \mathbf{Z}_t and \mathbf{Z}_r .

3. Propagation Channel Model of MIMO Systems with Coupled Antennas

The correlation between the links of a MIMO channel has a detrimental effect on the MIMO capacity. Among the several reasons for correlation are the propagation environment, and the coupling between transmit and receive antennas, which in turn has impact on the capacity of the communication channel. Whereas the correlation caused by coupling between antenna elements can be computed or measured spatial correlation is not known at the transmitter and must be provided by means of Channel State Information. A known signal is sent by the transmitter to the mobile device that enables it to build a picture of the channel environment. The mobile device sends back the channel characteristics to the transmitter. The transmitter can then apply the correct phase and amplitude adjustments to form a beam directed at the mobile device.

In this section, using Z-matrix formulation, we show how coupling between antennas affects spatially correlated channel and the MIMO channel capacity. We give the channel's correlation matrix as a composition of both spatial correlation and mutual coupling.

3.1. Spatial Correlation and Its Impact on Channel Capacity

In a practical multipath wireless communication environment, the wireless channels are not independent from

each other but due to scatterings in the propagation paths, the channels are related to each other with different degrees. This kind of correlation is called spatial correlation. For a given channel matrix \mathbf{H} , the spatial correlation coefficient between the channels are defined as [20]:

$$\rho_{ij,pq} = \frac{E\{h_{ij} h_{pq}^*\}}{\sqrt{E\{h_{ij} h_{ij}^*\}E\{h_{pq} h_{pq}^*\}}} \begin{cases} i, p = 1, 2, \dots N\\ j, q = 1, 2, \dots M \end{cases}$$
(23)

In a MIMO system with arbitrary numbers of transmitting (M) and receiving (N) dipole antennas and the antenna separations are d_t in the transmitter and d_r in the receiver, the correlation coefficients can be calculated two-by-two at a time. The general formula is:

$$\rho_{ij,pq} = J_0(kd_t|q-j|)J_0(kd_t|p-i|)$$
(24)

where J_0 stands for the zero-order Bessel function, k ia a wave number $= \frac{2\pi}{\lambda}$, and d_{ij} is the distance between elements *i* and *j* of the uniform array antenna.

In the undertaken investigations, the Kronecker channel model [21, 22] is postulated to construct the channel matrix H. In this model, the transmitter and receiver correlation matrices are assumed to be separable and the channel matrix is represented by:

$$H = R_r^{1/2} G_H R_t^{1/2}$$
(25)

where G_H is the matrix including identical independent distributed (i.i.d) Gaussian entries with the zero mean and the unit variance, and R_r and R_t are the spatial correlation matrices at the receiver and transmitter, respectively. Here, it is assumed that the transmitting and receiving sides of MIMO system are equipped with vertically polarized wire dipole antennas. The scattering environment is represented by circles of uniformly distributed scattering objects surrounding the transmitting and receiving nodes.

$$C = E\{Wlog_2[\det(I_M + \rho H H^T)]\}$$
(26)

where ρ is the signal-to noise ratio.

$$\mathbf{R}_{H} = E\{\operatorname{vec}[\mathbf{H}]\operatorname{vec}(\mathbf{H})^{H}\} = \mathbf{\rho}_{t} \otimes \mathbf{\rho}_{r} \qquad (27)$$

This is the Kronecker product of ρ_t and ρ_r .

To demonstrate this analysis, we can obtain the channel matrix of a 3x3MIMO system equipped with dipole antennas aligned as uniform linear arrays (ULAs). The antenna separations at the transmitter and receiver are 0.2 λ and 0.15 λ , respectively. The Angle of Departure (AOD) at the transmitter and the Angle of Arrival (AOA) at the receiver of the multipath signals are all 360°. Assume that the channels are Gaussian random channels with a unit variance and a zero mean, and the antenna mutual coupling can be ignored. Calculate the channel capacity when the SNR = 20dB.

$$d_t = 0.2\lambda, \qquad d_r = 0.15\lambda$$
$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$



Figure 6. cdf of Channel Capacity

The channel correlation matrix, $\mathbf{\rho}_r$ at the receiver,

$$\boldsymbol{\rho}_{r} = \begin{bmatrix} E\{h_{11}h_{11}^{*}\} & E\{h_{11}h_{21}^{*}\} & E\{h_{11}h_{31}^{*}\} \\ E\{h_{21}h_{11}^{*}\} & E\{h_{21}h_{21}^{*}\} & E\{h_{21}h_{31}^{*}\} \\ E\{h_{31}h_{11}^{*}\} & E\{h_{31}h_{21}^{*}\} & E\{h_{31}h_{31}^{*}\} \end{bmatrix}$$
(28)

$$= \begin{bmatrix} 1 & J_{0}(0.3\pi) & J_{0}(0.6\pi) \\ J_{0}(0.3\pi) & 1 & J_{0}(0.3\pi) \\ J_{0}(0.6\pi) & J_{0}(0.3\pi) & 1 \end{bmatrix}$$
(29)

Similarly, he channel correlation matrix, $\boldsymbol{\rho}_{t}$ at the transmitter

$$\boldsymbol{\rho}_{t} = \begin{bmatrix} E\{h_{11}h_{11}^{*}\} & E\{h_{11}h_{12}^{*}\} & E\{h_{11}h_{13}^{*}\} \\ E\{h_{12}h_{11}^{*}\} & E\{h_{12}h_{12}^{*}\} & E\{h_{12}h_{13}^{*}\} \\ E\{h_{13}h_{11}^{*}\} & E\{h_{13}h_{12}^{*}\} & E\{h_{13}h_{13}^{*}\} \end{bmatrix}$$
(30)

$$= \begin{bmatrix} 1 & J_0(0.4\pi) & J_0(0.8\pi) \\ J_0(0.4\pi) & 1 & J_0(0.4\pi) \\ J_0(0.8\pi) & J_0(0.4\pi) & 1 \end{bmatrix}$$
(31)

Then

$$\mathbf{R}_H = \mathbf{\rho}_r \otimes \mathbf{\rho}_t \tag{32}$$

Find the eigenvalues and eigenvectors of \mathbf{R}_{H} . Then the channel matrix \mathbf{H} can be expressed as:

$$\operatorname{vec}(\mathbf{H}) = \mathbf{V} \mathbf{D}^{1/2} \mathbf{r}$$
(33)

where **r** (*NM* x1) is a vector containing i.i.d. complex Guassian random numbers with a unitvariance and a zero mean, **V** is the matrix whose column vectors are the eigenvectors of **R**_{*H*}, and **D** is a diagonal matrix whose diagonal elements are the eigenvalues of **R**_{*H*}. A Matlab program is then used to obtain the system channel capacity as in Equation (26). The equation suggests that the channel capacity increases with increasing the SNR(ρ) and no. of antennas up to certain level.

As previously mentioned, MIMO systems perform best when it can answer to the issues related to antenna theory such as array configuration, radiation pattern, type of polarization and mutual coupling. Here, various interesting concepts of antenna design for a MIMO system are listed briefly.

4. Conclusions

As analyzed in this paper, MIMO systems potentially possess a high capacity, which is a desired property for the current communication needs requiring a very high data rate and high reliability, such as multimedia communication services, cellular mobile, and the Internet. In many cases the capacity of MIMO systems is approximately linearly proportional to the number of antennas.

In this work, we used a detailed network model of a MIMO system to realistically account for mutual coupling on the overall capacity. In conjunction with a path-based channel model, this formulation constructed the channel matrix relating the signals input to the transmit antennas to those at the output of the receiver front end and uses this result to compute the MIMO system capacity. Computational examples using coupled dipoles characterized using full-wave electromagnetic analysis

reveal that mutual coupling between antennas significantly reduced the radiation efficiency of the antennas.

We also calculated the channel capacity for both cases where channel coefficients are known as well as unknown at the transmitter, which revealed an increased capacity when channel coefficients are known than unknown in the transmitter.

Finally, it was established that the impact of channel correlation (spatial correlation plus antenna coupling) is to reduce channel capacity, hence suggesting that a lot of research is required to be done in antenna design for the better performance of MIMO systems, which form a main part for the future 4G communications.

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