Implementing Orthogonal Frequency Division Multiplexing Using IFFT/FFT

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Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is a modulation system that offers many advantages over other modulation schemes. OFDM is a particular form of Multi-carrier transmission and is suited for frequency selective channels and high data rates; it overcomes the Inter Symbol Interference (ISI) problem by modulating multiple narrow-band sub-carriers in parallel. In this paper, analysis of OFDM is carried out with emphasis on the implementation using IFFT/FFT as against multiple Oscillators and demodulators. The concept of orthogonality of carriers is used to explain the ability to transmit multi-carriers without interference and the ease of decoupling the signal information at the receivers. Matlab simulations were carried out to show these concepts.

Index Term—FFT, IFFT, Modulation, Multi-carrier, OFDM, Orthogonality

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a technique that involves transmitting signal information using smaller subcarriers, instead of a single large carrier. OFDM is a form of multi-carrier modulation. The basic principle of multicarrier modulation is to divide the data stream, $d$, into $N$ parallel data streams with a reduced data rate of $d/N$; low rate data streams are then modulated on a separate narrow band subcarriers and summed together for transmission, thereby providing the same data rate as an equivalent single large carrier system [3]. At the receiver a set of filter banks separate the wide-band signal into the original narrowband subcarriers for demodulation. In other words, OFDM involves dividing the available spectrum into several narrow sub-channels/sub carriers which experience differential flat fading as they propagate; this make equalization at the receiver end simple.

Robert W. Chang was the first person to show in 1966 theoretically the principle of operation of OFDM; he obtained a US patent on OFDM in 1970. Chang showed theoretically how to transmit simultaneous data stream through linear band limited channel without Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI)
For two exponential signals \( \psi_n(t) = e^{j2\pi f_k t} \) and \( \psi^*_{-n}(t) = e^{-j2\pi f_k t} \) that make up the sub-carrier of an OFDM system at \( f_k = k/T \), these signals are defined to be orthogonal if the integral of the product of the signal over a period is zero that is, orthogonality condition is given as:

\[
\int_0^T \psi_n(t) \psi^*_{-n}(t) dt = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \\ \end{cases} \tag{1}
\]

Or

\[
\frac{1}{T} \int_0^T e^{j2\pi f_k t} e^{-j2\pi f_{-k} t} dt = \begin{cases} 0, & k = i \\ 1, & k \neq i \\ \end{cases} \tag{2}
\]

Orthogonality can also be shown for discrete signal; taking samples of equation (2) at \( t = nT_s = \frac{nT}{N} \) for \( n = 0, 1, 2, 3, ..., N-1 \)

Taking equation (2) into discrete domain, we have

\[
\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi \frac{k}{N} nT} e^{-j2\pi \frac{1}{N} nT} = \sum_{n=0}^{N-1} e^{j2\pi \frac{k-1}{N} nT} = \begin{cases} 1, & k = i \\ 0, & k \neq i \end{cases} \tag{3}
\]

If the available bandwidth for transmission is given by \( \Delta W \); in an OFDM this bandwidth will be split into \( N \) sub-channels. The information signal to be transmitted is first converted to digital bits using an Analogue-to-Digital (A/D) converter before being transmitted. The OFDM transmitter maps the information signal bits into a sequence of PSK or QAM symbols which are subsequently converted into \( N \) parallel streams. The serially modulated digital information signals are then passed through a Serial-to-Parallel (S/P) device which are converted to \( N \) parallel streams and are transmitters through different sub-carriers [5]. If \( X_i[k] \) denote the \( l \)th transmit symbol at the \( k \)th carrier, \( l = 0, 1, 2, ..., \infty \), \( k = 0, 1, 2, 3, ..., N-1 \).

The Serial-to-Parallel conversion causes an extension in the transmission time for \( N \) symbols to \( NT_s \) which forms a single OFDM symbol with a length \( T = NT_s \).

Let \( \psi_{l,k}(t) \) denote the \( l \)th OFDM signal at the \( K \)th sub-carrier, which is given as:

\[
\psi_{l,k}(t) = \begin{cases} e^{j2\pi f_k (t-lT)} & \forall \ 0 < t < T \\ 1, & \text{elsewhere} \end{cases} \tag{4}
\]

According to [5], the carrier and information (baseband)OFDM signals respectively can be mathematically represented as follow in the continuous-time domain:

\[
x_i(t) = Re \left\{ \frac{1}{T} \sum_{l=0}^{\infty} \left( \sum_{k=0}^{N-1} X_i[k] \psi_{l,k}(t) \right) \right\} \tag{5}
\]

and

\[
x_i(t) = \sum_{l=0}^{\infty} \sum_{k=0}^{N-1} X_i[k] e^{j2\pi f_k (t-\frac{lT}{N})} \tag{6}
\]

If equation (5) is sampled at \( t = lT + nT_s \) with \( T_s = T/N \) and \( f_k = k/T \), the corresponding discrete-time OFDM is

\[
x_i[k] = \sum_{k=0}^{N-1} X_i[k] e^{j2\pi kn/N} \forall n = 0, 1, ..., N-1 \tag{7}
\]

A closer look at equation (7) reveals that the process of OFDM modulation is effectively an \( N \)-point Inverse Direct Fourier Transform (IDFT), and this can be efficiently computed using the IFFT (Inverse Fast Fourier Transform) algorithm.

At the receiver where demodulation takes place, the transmitted signal \( X_i[k] \) can be decoupled from the receive OFDM.

\[
y_i(t) = \sum_{k=0}^{N-1} X_i[k] e^{j2\pi f_k (t-lT)} (lT < t < lT + nT_s) \tag{8}
\]
y exploiting the orthogonality among the sub-carriers as stated in Equation(2), as follow:

\[
Y_i[k] = \frac{1}{T} \int_{-\infty}^{\infty} y_i(t) e^{-j2\pi k f_k (t-T)} dt
\]

\[
= \frac{1}{T} \int_{-\infty}^{\infty} \left( \sum_{i=0}^{N-1} X_i[i] e^{-j2\pi i f_k (t-T)} \right) e^{-j2\pi k f_k (t-T)} dt
\]

\[
= \sum_{i=0}^{N-1} \left( \frac{1}{T} \int_{0}^{T} X_i[i] e^{-j2\pi (f_i - f_k) (t-T)} dt \right) = X_i[k] \quad (8)
\]

The sampled discrete time representation OFDM signal, \(y_i(t)\) at \(t = lN + nT_s\) is given as

\[
Y_i[k] = \sum_{n=0}^{N-1} y_i[n] e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} \left( \frac{1}{N} \sum_{i=0}^{N-1} X_i[i] e^{j2\pi in/N} \right) e^{-j2\pi kn/N}
\]

\[
= \sum_{n=0}^{N-1} \sum_{i=0}^{N-1} X_i[i] e^{j2\pi i (n-k) N} = X_i[k] \quad (9)
\]

The left hand side of equation (9) computes the N-point DFT of \(y_i[n]\) (for \(n = 0,1,2, \ldots , N-1\)); this can be efficiently computed using the FFT (Fast Fourier Transform) algorithm.

Equations (8) and (9) show that OFDM process can be wholly implemented digitally by employing an IFFT (Inverse Fast Fourier Transform) algorithm at the transmitter and a FFT (Fast Fourier Transform) algorithm at the receiver, as a second method of implementation. In the first method, multiple oscillators are needed at the transmitter end, while at the receiver many coherently matched demodulators are required. The complexity of the oscillators and demodulator increases as the number of sub-carriers increase. The implementation of OFDM through the use of IFFT/FFT algorithm greatly reduces the cost, size of the system and complexity; the IFFT/FFT has the computational capability to handle as many sub-carriers as possible.

The simulation was done using Matlab; a realization of what takes place at the transmitter (modulator) and receiver (demodulator) was carried out. OFDM is the modulation standard, and OFDMA the access method for both WiMAX and LTE. The simulation was carried out for WiMAX 2.5GHz, of channel bandwidth 10MHz and OFDM FFT size of 8192.
OFDMA is becoming increasingly popular in the telecoms and Signal processing industries because of the inherent capacity increase it offers. From the analysis in this paper, it has been shown that by mathematical analysis and MATLAB simulation that OFDM helps in 'compressing' many orthogonal signals into one main carrier; thereby increasing transmission bandwidth. The graphs shows the process through which IFFT/FFT is used in realizing OFDM.

REFERENCES
[1] Ramjee Prasad and Fernando J. Velez, WiMAX Networks @ Springer Science
[2] Ramjee Prasad and Fernando J. Velez, WiMAX Networks @ Springer Science
[12] Maneesha Sharma “Effective channel state information (CSI) feedback for MIMO Systems in Wireless Broadband communication” M.Eng Thesis to School of Electrical Engineering and Computer Science and Engineering Faculty, Queensland University of Technology
[14] Characterization of MIMO Antennas with Multiplexing Efficiency by Ruyuan Tian, BaoKong Lau, and Zhihong Ying ; Electromagnetic Theory Department of Electrical and Information Technology Lund University Sweden
[19] VinuaykNagalCooperative multiplexing in Wireless Relay Network
[24] Characterization of MIMO Antennas with Multiplexing Efficiency by Ruyuan Tian, BaoKong Lau, and Zhihong Ying ; Electromagnetic Theory Department of Electrical and Information Technology Lund University Sweden

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