ENGINEERING MATHEMATICS EDUCATION WITH COMPUTER ALGEBRA: THE MATLAB ALTERNATIVE

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Abstract

Computer algebra systems have become an important tool for many engineering and technical professionals. There is a growing need to incorporate such tools into the education of such professionals. This paper discusses these systems and their role within engineering mathematics in higher education. Some advantages and problems associated with computer algebra are highlighted and illustrated using MATLAB.

1. Introduction

In the early 1980s there were predictions that computer algebra systems (also called computer based symbolic manipulation systems and abbreviated to SAMs) would bring about a revolution. Stern [1] asserted that SAMs would do for mathematical analysis what the hand-held calculator had done for calculation and arithmetic. However, the revolutionary change to practice in higher education that many have predicted has not yet happened. Lawson [2] suggested two main reasons for this: one related to teaching staff and the other to students. The first is the innate conservatism of many mathematicians in higher education coupled with the fact that there were a number of failings in most of the early packages [3].

The second reason is one of access. Students readily use pocket calculators because of familiarity and access. The same has not been true of SAMs. Students have to visit computer laboratories in order to be able to use a SAM. Consequently, SAMs have not yet shared the universal availability, which the calculator enjoys.

However, SAMs are an increasingly important tool for professionals in a wide variety of technical and scientific occupations. It was recognized early that those responsible for designing and providing the education, both initial formation and continuing professional development, of these professionals must ensure that they acquire appropriate knowledge of the capabilities and limitations of such systems and learn to incorporate their use into the overall pattern of their professional activities [4]

This paper briefly reviews what a SAM is, and outlines some of the potentials and problems which attend the use of SAMs in undergraduate engineering mathematics. Illustrations are presented using the programming language MATLAB whose Symbolic Math Toolbox utilizes tools built on portions of one of the most popular SAMs, MAPLE.

3. SAMs and Their Uses

Basically, a SAM is a piece of software, which is capable of working symbolically as well as numerically. In principle it is a program that does on a computer the manipulation that has traditionally been done with pencil and paper. So, for example, if requested to expand cos(7x) a SAM will effortlessly return the answer:

$64\cos^7(x)-112\cos^5(x)+56\cos^3(x)-7\cos(x)$

Similarly, a SAM command to differentiate the function $\sin(E^x/(x^2+1)$ with respect to x produces the answer (almost instantly):

$$\cos\left(\frac{E^x}{x^2+1}\right)\left(\frac{E^x}{x^2+1}-2\frac{E^x x}{\left(x^2+1\right)^2}\right)$$

Early SAMs such as, MACSYMA and REDUCE, were large, required considerable computer power and had complex syntax. However, hardware and software advances over recent years completely altered this situation. Modern programs that incorporate SAM capabilities include *Derive*, *Maple, Mathematica* and *Matlab*. They run on PCs, have graphical user interfaces and although there is still a reasonable amount of syntax required for more complicated operations, can be used to perform routine manipulation without the need for lengthy study of manuals. In addition these packages have progressed enormously beyond simply being manipulators of complicated symbolic strings. They are now complete mathematical environments with algebraic, numerical, graphical and programming facilities.

3. The MATLAB SAM

MATLAB (*Mat*rix *Lab*oratory) is a high performance interactive software tool for technical computing. Typical uses include general-purpose numeric computation; algorithm prototyping; modelling and simulation; data analysis, exploration and visualisation; scientific and engineering graphics; a teaching aid in linear algebra and other topics; and solution of special purpose matrix formulations such as those associated with automatic control, statistics and signal processing [5,6,7].

MATLAB features a family of application-specific solutions called *toolboxes*, which allow the user to learn and apply specialised technology. Toolboxes are specialised collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include: symbolic maths; analogue and digital signal processing; analogue and digital control systems; neural networks; fuzzy logic; simulation; system identification; and many others.

The Symbolic Toolbox deals with symbols, formulas, and equations. It features in-built functions for: differentiation; integration; solution of differential equations; Laplace and Fourier transformations; linear algebra; graphics; etc. It is employed in the following sections to illustrate some advantages and drawbacks associated with use of a SAM in mathematics education.

Clicking the MATLAB (version 5.3) icon in the WINDOWS – 95/98/NT/Me environment opens the command window, which is the main window for communicating with the MATLAB interpreter. The command window prompt is "»". To find out about the symbolic computation functions available in MATLAB, execute the command:

» **help symbolic.**

4. Uses of SAMs in Education

A strong case has been presented for the widespread use of computer algebra in mathematics education. Some benefits resulting from using computer algebra have been enumerated [8]. The key advantages include the following:

- Computer algebra allows the focus to be on concepts and understanding principles without the distraction of large amounts of 'routine' manipulation.
- Motivation can be enhanced because more realistic problems can be tackled.
- Students can cover many more examples allowing them to be more active participants in discovery learning.

These are expanded in the following with illustrative examples in MATLAB. Although all the examples of computer algebra commands given in this section are for MATLAB, all SAM packages can easily do the things shown here. Where MATLAB sessions are shown, lines beginning with '»' show inputs to MATLAB and other lines show answers MATLAB returns. The bold typeface shows MATLAB in-built functions or constants.

(a) SAMS reduce the premium on pure manipulative ability. Students may use a SAM to assist in or confirm their own calculations. As a result weaker students can be enabled to follow more sophisticated or complex arguments. In the following MATLAB session a student enters the expression, f = sin (yx^2), and differentiates it with respect to x and y in turn.

```
»f = sym ( 'sin(y*x^2)');
»diff (f)     % derivative with respect to x
   ans =
   2*cos(y*x^2)*y*x
»diff(f,'y')   %derivative with respect to y
   ans =
    cos(y*x^2)*x^2
```

(b) SAMs enable students to concentrate on concepts, not detail. A case in point is that of differentiation. In undergraduate engineering mathematics, students learn a range of techniques of differentiation including such things as the product rule, the quotient rule, and the function of a function rule, implicit differentiation and logarithmic differentiation. A SAM will return the derivative of a function without any explicit indication that a particular technique or rule has been used. For example, differentiation of the function in the above MATLAB session requires use of both the function of a function and the product rule. However, as far as the user of a SAM is concerned, there is no difference between differentiating this function and differentiating x^2 as illustrated in the following MATLAB session. Note that in both cases MATLAB returns the answer (*ans*) almost instantaneously.

```
»x=sym ('x');
»diff(x^2)
    ans =
    2*x
```

This can be viewed as a great advantage as it makes the point that differentiation is the same process no matter how complex the function being differentiated. The technique employed to determine the derivative does not alter what the derivative means.

Consequently students need only to know that there is a product rule, what the rule is and where it comes from but need not necessarily complete large numbers of ever more (algebraically) complicated examples. In the time that is freed by this reduction in “drill” exercises, key concepts such as the derivative being a rate of change, qualitative information such as a positive derivative indicating an increasing function and using derivatives to represent physical quantities (such as velocity and acceleration) are all topics which might be more heavily emphasized than at present.

(c) A SAM acts as a computational assistant for routine procedures. Once a procedure has been understood at a basic level a better understanding can sometimes be achieved by the study of a range of examples. However, the labour of repetitive computations often inhibits such learning. A SAM can reduce the labour and also allow lecturers to set a wider range of illustrative work in assignments. Fourier series offer a good example. To illustrate the more subtle points of the topic requires a wide range of examples to be completed. The labour of calculating the Fourier coefficients for that large number of examples discourages students from completing them and thus inhibits understanding. The following MATLAB session evaluates the coefficients b_n for the first five terms of a Fourier series for the function f(x) = x(1-x), 0≤x≤1. The Fourier coefficients are given by:

$$b_n = \int_0^1 x(1-x)\sin(\pi n x)\,dx$$

With pencil and paper this would require integration by parts twice.

```
» syms x n bn
»bn = int(x*(1-x)*sin(pi*n*x),0,1)

bn =
-(pi*n*sin(pi*n)+2*cos(pi*n))/pi^3/n^3+
+2/pi^3/n^3

» pretty(bn)

     pi n sin(pi n) + 2 cos(pi n)        2
   - ---------------------------------- + ------
                  3  3                     3  3
                pi  n                    pi  n

» subs(bn,n,{1,2,3,4,5})

bn =

[ 4/pi^3,    0,    4/27/pi^3,    0,     4/125/pi^3]
```

The result is actually

$$b_n = -\frac{\pi n\sin(\pi n) + 2\cos(\pi n)}{\pi^3 n^3} + \frac{2}{\pi^3 n^3}$$

$$= -2\frac{(-1)^n}{\pi^3 n^3} + \frac{2}{\pi^3 n^3}$$

The first five coefficients are:

$$b_n = [\ \frac{4}{\pi^3}, 0, \frac{4}{27\pi^3}, 0, \frac{4}{125\pi^3}\ ]$$

Majority of engineering undergraduates would make at least one minor error in completing the above integration. This would simply obscure any learning, which was intended to follow from a consideration of the (correct) Fourier series representation of the waveform. Using a SAM, students can be expected to complete a wider range of examples more quickly and more accurately, and when guided appropriately, to gain thus an enhanced understanding of the topic.

(d) SAMs act as an expert system. A student may know, in principle, that a manipulation is possible but may not have the knowledge, skill or persistence to carry it through in practice. Trigonometric identities provide good examples of this. Having met the identities for cos(2x) and cos(3x) as polynomials in cos(x), a student might perhaps know, in theory, that cos(6x) could also be expressed as a polynomial in cos(x) but be unable to complete the computation quickly or accurately. Below is a MATLAB session showing how this may be done with a SAM.

```
» x=sym ('x');
» expand (cos(2*x))
 ans =
 2*cos(x)^2-1
```

» **expand** (**cos**(3*x))
ans =
4*cos(x)^3-3*cos(x)

» **expand** (**cos**(6*x))
ans =
32*cos(x)^6-48*cos(x)^4+18*cos(x)^2-1

Note that MATLAB's response is almost instantaneous.

(e) SAMs make graph drawing and visualisation easier. The sophisticated graph plotting facilities of most SAMs can help to give insight into mathematical ideas through visualisation. For example the function f(x) =x(1-x) in (c) may be visualised using the command:

» **ezplot**('x*(1-x)',0,1)

The resulting plot is shown in Fig.1.

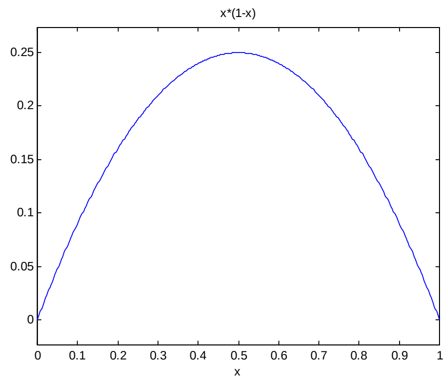

Fig. 1: A visualisation of the function whose Fourier coefficients were obtained in (c).

Three- dimensional visualizations are supported. The following example illustrates the creation of a 3-D plot of the *sinc* function, sin(r)/r.

» [X,Y]=**meshgrid**(-9:.5:9);
» R=**sqrt**(X.^2+Y.^2)+**eps**;
» Z=**sin**(R)./R;
» **mesh**(X,Y,Z)

The plot is shown in Fig. 2. In this example, R is the distance from the origin, which is the centre of the matrix. Adding **eps** avoids the indeterminate 0/0 at the origin.

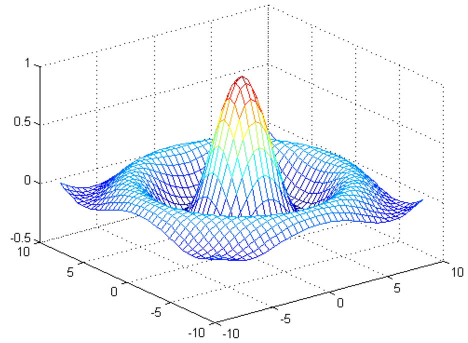

Fig 2. 3-D visualisation of the *sinc* function

(f) SAMs encourage exploration and experiment. The many features of SAMs can be used to encourage students to be more open to mathematical exploration and experimentation. The SAM removes the drudgery of repetitive calculation, provides standard tools, eliminates much of the need for routine reference to tables and textbooks and generally makes experimentation easier, more exciting and more rewarding.

(g) Lecturers can construct complex/animated visualisations and improve instruction. The sophisticated graphical features of many SAMs engender the construction of visualisations and animations, which can motivate or clarify complex mathematical ideas. For example a SAM may be used as a virtual laboratory where students perform computer-based experiments on animated mass-spring-damper systems [8]. They can then relate the results of their virtual experiments to solutions of the differential equations that model such systems.

5. Some problems with use of a SAM

(a) Additional time is needed to learn SAM usage. This would inevitably take time from other subjects in the curriculum. However, skill with a SAM will ultimately improve the student's efficiency in learning other material, so that part, at least, of the time spent on learning a SAM would be recovered.

(b) Undermining development of manipulative skills. Just as the pocket calculator is considered to have led to the atrophy of arithmetic skills, so might SAMs probably lead to a reduction in purely manipulative algebra and calculus skills? However, once SAMS become available on affordable pocket calculators, it may be argued that this loss is unimportant. Indeed Lawson [9] argues that computer algebra ought not to be seen as a de-skilling tool but rather as a re-

skilling one that gives access to higher level skills (such as formulation and interpretation) instead of being focussed on lower level skills of manipulation and techniques.

(c) Problems of remembering the SAM language. The rich vocabulary of SAM commands contains a number of possibilities. To become truly adept in a SAM language requires routine everyday use. Occasional users would quickly lose their 'edge'. The following MATLAB session illustrates what can easily happen in a SAM if an occasional user attempts to combine sin(A+B) and sin(A-B). The student has an idea of the form the answer takes but wants MATLAB to confirm the correct details. Among the many related commands of MATLAB which one is correct? The approach adopted here is to try any possibility until the correct answer is obtained. Obviously this can be time consuming and frustrating for a novice or occasional user. Note that the command **simple** (not used here) gives even intermediate steps used in the process.

```
» syms a b x
» x=sin(a+b)+sin(a-b)
 x =
 sin(a+b)+sin(a-b)
 » simplify (x)
 ans =
 sin(a+b)+sin(a-b)
 » factor (x)
 ans =
 sin(a+b)+sin(a-b)
 » expand (x)
 ans =
 2*sin(a)*cos(b)
```

(d) Course examination. Examination of a SAM-based course is a serious problem. With the ever-increasing mathematics classes, it is impractical to consider the option of providing a microcomputer at each exam desk. All forms of non-examination assessment (e.g. continuous assessment only) might not be acceptable, especially in a society in which traditional values of 'fair play' and honesty can no longer be taken for granted.

6. Conclusion

In the face of the demonstrated versatility of SAMs there is a need to change either the engineering mathematics curriculum or the approach to the curriculum in such a way as to exploit the numerous advantages they (SAMs) offer. Engineering mathematics teachers should not only use SAMs as a tool to free students from the drudgery of routine algebra, calculus and graph drawing, but should deploy SAMs to help them (students) to understand at a deeper level, the nature and structures of mathematics and to appreciate the ways SAMs can be used to model the operation of those physical and other real-world systems that are of interest to engineers and technologists.

7. References

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