

Modification of Bingham Plastic Rheological Model for Better Rheological Characterization of Synthetic Based Drilling Mud

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Abstract: The Bingham plastic rheological model has generally been proved by rheologists and researchers not to accurately represent the behaviour of the drilling fluid at very low shear rates in the annulus and at very high shear rate at the bit. Hence, a dimensionless stress correction factor is required to correct these anomalies. Hence, this study is coined with a view to minimizing the errors associated with Bingham plastic model at both high and low shear rate conditions bearing in mind the multiplier effect of this error on frictional pressure loss calculations in the pipes and estimation of Equivalent Circulating Density (ECD) of the fluid under downhole conditions. In an attempt to correct this anomaly, the rheological properties of two synthetic based drilling muds were measured by using an automated Viscometer. A comparative rheological analysis was done by using the Bingham plastic model and the proposed modified Bingham plastic model. Similarly, a model performance analysis of these results with that of power law and Herschel Buckley Model was done by using statistical analysis to measure the deviation of stress values in each of the models. The results clearly showed that the proposed model accurately predicts mud rheology better than the Bingham plastic and the power law models at both high and low shear rates conditions.

Key words: Bingham plastic, modified Bingham plastic, Hershel Buckley, powerlaw, rheological properties, predicts

INTRODUCTION

In Petroleum Engineering, drilling fluids are complex fluid mixtures consisting of several additives which are added to enhance and/or control its rheological properties (Sarah and Isehunwa, 2015). These fluids are primarily concerned with the transportation of drill cuttings out of the hole in order to allow for separation of cuttings from the drilling fluids at the surface, formation of a tin filter cake on the walls of the wellbore to prevent fluid loss and prevention of inflow of formation fluids into the well bore (Falode *et al.*, 2008).

Several mathematical model have been developed to describe the shear stress/shear rate relationship of drilling fluids using viscosity because it is the most elementary property dealt with in rheology. These models are used to characterize flow properties in an effort to determine the ability of a fluid to perform specific functions (Khalili-Garakani *et al.*, 2011). There are two basic models for describing the rheology of drilling fluids viz: The Newtonian model where the shear stress (J) is directly proportional to the shear rate ($\dot{\gamma}$) and the constant of proportionality is the fluid viscosity (μ) as shown in Fig. 1a and the non-Newtonian model where the fluid

viscosity is not constant but a function of the shear stress and/or the prevailing shear rate or shear history as shown in Fig. 1b.

For non-Newtonian Model, there is usually a region at both low and high shear rate where the viscosity is independent or nearly independent of shear rate and a section in between that exhibits strong shear rate dependence (Steffe, 1996). Though, a fluid may also exhibit a linear relationship between shear stress and shear rate when plotted on a log-log paper and this is referred to as pseudoplastic fluid. Pseudo plastic fluids are shear thinning fluids which usually have less viscosity with higher shear rates. For pseudo plastic fluids the flow behaviour index is usually less than one, $n < 1$. Dilatant fluids on the other hand are shear thickening and less common than shear thinning fluids in nature. Dilatant fluids increase their viscosity exponentially when the shear force is increased, i.e., the flow behaviour index is $n > 1$ (Fig. 2). The non-Newtonian flow behaviour has been attributed to mechanisms in which the shear stress, transmitted through the continuous medium, orients or distorts the suspended particles in opposition to the randomizing effects of Brownian motion (Krieger and Dougherty, 1959).

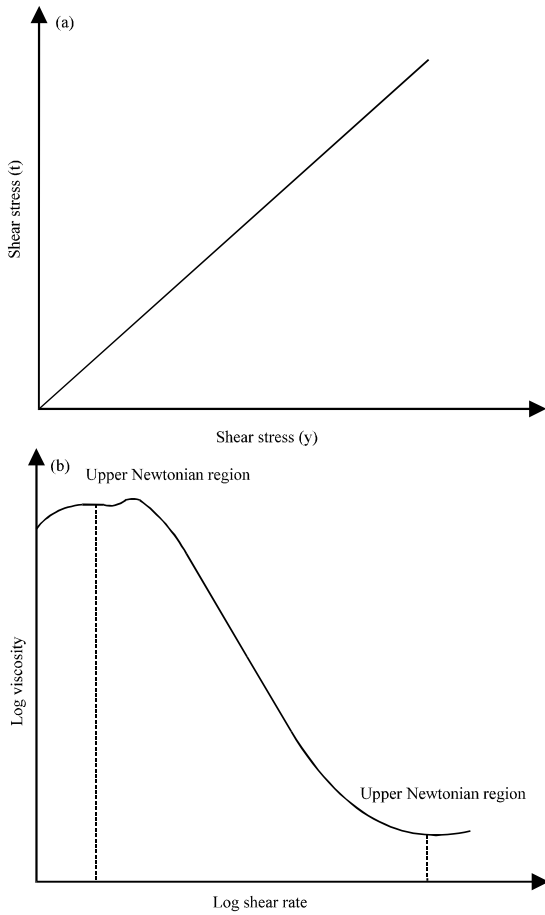


Fig. 1: Viscosity profile of non-Newtonian fluid

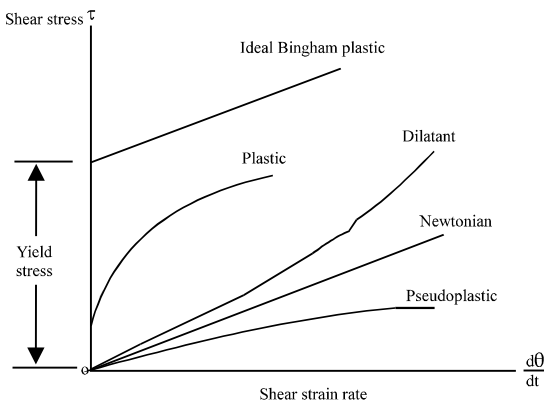


Fig. 2: Shear stress vs. shear rate for pseudoplastic and dilatant fluids

Depending on how the fluid structure responds to the applied shear forces, one can observe different types of macroscopic flow behaviour in drilling fluids such as shear thinning, yield stress, thixotropic, rheopectic and viscoelasticity as shown in Fig. 3 (Coussot *et al.*, 2002).

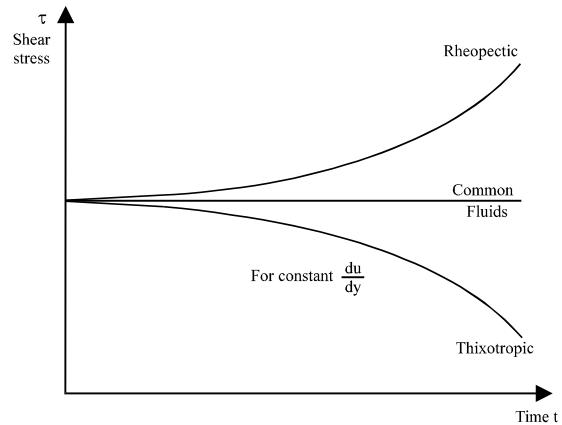


Fig. 3: Shear stress versus shear rate for thixotropic and rheopectic fluids

Thixotropic is generally understood as time dependent decrease in viscosity of a fluid due to finite, measurable reversible change of the fluid microstructure during shear. But Rheopectic fluid increases in viscosity as stress increases over time.

The rheological model for non-Newtonian fluids may be grouped under three categories. These are the empirical model which are derived from examination of experimental data and an example is power law rheological model (Reiner, 1926), the structure model such as the Casson model (Casson, 1926) and the Hershel Buckley model (Hershel and Buckley, 1926). Also, there is theoretical model which indicates factors that influences a rheological parameter and examples are, the Krieger-Dougherty Model (Krieger, 1959) for relative viscosity and the Bingham plastic model (Bingham, 1922). These models are graphically represented in Fig. 4.

However, the Bingham plastic model does not accurately represent the behavior of drilling fluid at very low shear rate in the annulus and at very high shear rate at the bit (Lauzon and Reid, 1979), hence, a modification of the model in terms of introducing stress correction factor is not only necessary but of utmost importance. This is because accurate determination of drilling fluid rheological parameters is important for the following applications, calculating frictional pressure loss in annuli and pipe, estimating Equivalent Circulating Density (ECD) of the fluid under downhole conditions, determination of flow regimes in the annulus, estimation of hole-cleaning efficiency, estimating surge and swab pressures and optimization of the circulating system for improved drilling efficiency (Anonymous, 2010).

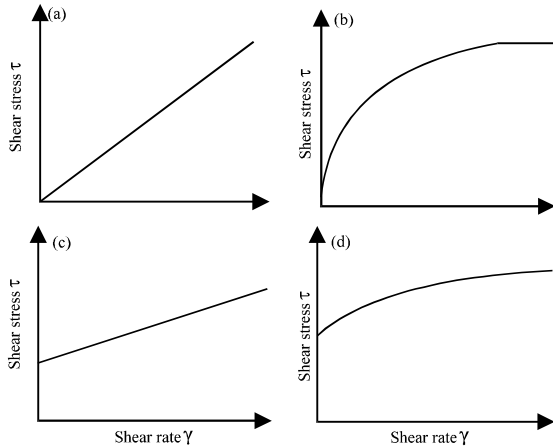


Fig. 4: Shear stress vs. shear rate for drilling fluid rheological models (Amoco Production Company, Manual 2001): a) Newtonian Modles; b) Power law model; c) Bingham plastic model and d) Herschel-Bulkely Model

MATERIALS AND METHODS

Two synthetic base fluid samples were used to prepare synthetic based mud with the same mud components throughout. These are: Trans-Esterified Palm Kernel Oil (TRANSPKO) and Inter-Esterified Palm Kernel Oil (ITERPKO).

Basic rheological concepts: Viscosity is the resistance offered by a fluid to deformation when it is subjected to a shear stress. If the viscosity is independent of the shear rate, the fluid is called a Newtonian fluid. If the viscosity of a fluid is a function of shear stress (or equivalently of shear rate) such a fluid is called non-Newtonian fluid. This is illustrated in Fig. 1a, b, respectively. The unit of viscosity can be expressed as Newton seconds/m² or Pascal seconds or poise.

Shear stress: Force per unit area and is expressed as a function of the velocity gradient of the fluid as:

$$\tau = -\mu \frac{dv}{dr} \tag{1}$$

Where:

- μ = The fluid viscosity
- dv/dr = The velocity gradient

The unit is N/m², Pascal or Dynes/cm². The negative sign in Eq. 1 arises because momentum flux flows in the direction of negative velocity gradient. That is, the momentum tends to go in the direction of decreasing velocity.

Shear rate: Defined as the absolute value of velocity gradient and it is expressed mathematically as:

$$\gamma = \left| \frac{dv}{dr} \right| \tag{2}$$

It is expressed in sec⁻¹ (reciprocal seconds). Equation 1 can also be written as:

$$\tau = \mu\gamma \tag{3}$$

Yield point: A measure of the electrochemical or attractive force in a fluid and it is that part of resistance to flow that may be controlled by proper chemical treatment. Mathematically, it is expressed as:

$$YP = \theta_{300} - PV \tag{4}$$

The unit is lb./100ft² or Pa.s. Where PV is the plastic viscosity in lb./100ft².

Plastic viscosity: Described as that part of resistance to flow caused by mechanical friction. It is expressed as:

$$PV = \theta_{600} - \theta_{300} \tag{5}$$

The unit is centipoise (cp).

Non-Newtonian Model fluid rheology: The following mathematical models are used to describe the rheology of non-Newtonian fluids. These are:

- C Power law model
- C Bingham plastic model
- C Hershel Buckley Model
- C Casson Model
- C API Model (RP 13D)
- C Unified Model
- C Robertson and Stiff Model

The Bingham plastic model: The Bingham plastic model is a two parameter model that is widely used in the drilling fluid industry to describe the flow characteristics of many type of muds. These fluids require a finite shear stress, Ty below that, they will not flow. Above this finite shear stress, referred to as yield point, the shear rate is linear with shear stress, just like a Newtonian fluid. Bingham fluids behave like a solid until the applied pressure is high enough to break the shear stress. The shear stress-shear rate graphical representation is shown in Fig. 4. The shear stress can be written as:

$$\tau = \tau_y + \mu_p \gamma \tag{6}$$

Where:

J_y = The Yield Point (YP) and the unit is lb./100ft² or Pa.sⁿ

μ_p = The referred to as the Plastic Viscosity (PV) of the fluid and the unit is mPa.s (cp)

$\dot{\gamma}$ = The shear rate (sec⁻¹)

The two parameters J_o and μ_p can be determined from Eq. 4 and 5, respectively. Fluids that exhibit Bingham Plastic behaviour are characterized by a yield point (J_o) and plastic viscosity (μ_p) that is independent of the shear rate.

Some water-based slurries and sewage sludge are examples of Bingham plastic fluid. Most of the water-based cement slurries and water-based drilling fluids exhibit Bingham plastic behaviour. Drilling muds are often characterized with YP and PV values. However, the Bingham plastic model does not accurately represent the behaviour of drilling fluid at very low shear rate in the annulus and at very high shear rate at the bit (Lauzon and Reid, 1979).

The proposed modified Bingham plastic model: The modified Bingham plastic model was proposed in order to eliminate and/or reduce the error associated with the Bingham plastic model at bot low and high shear rate conditions.

This is done by introducing a dimensionless stress correction factor which helps to combat the effect of under estimation of stresses at high shear rate conditions by Bingham plastic model and over estimation of stresses at lower shear rate conditions by the model. Recall, that the Bingham plastic rheological model is represented as: Hence, taking the square root of each stress term the RHS of Eq. 6 gives:

$$\tau = \left[\tau_o^{0.5} + (\mu_p \gamma)^{0.5} \right] \tag{7}$$

Introducing a dimensionless stress correction factor K to Eq. 7, gives:

$$\tau = \left[\tau_o^{0.5} + (\mu_p \gamma)^{0.5} \right]^k \tag{8}$$

Linearizing Eq. 8 by taking its logarithmic value gives:

$$\log \tau = k \log \left[\tau_o^{0.5} + (\mu_p \gamma)^{0.5} \right] \tag{9}$$

Hence, the dimensionless stress correction factor K can be expressed as:

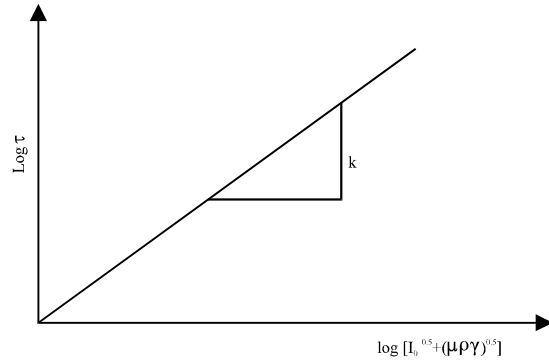


Fig. 5: Modified bingham plastic model rheogram

$$k = \frac{\log \tau}{\log \left[\tau_o^{0.5} + (\mu_p \gamma)^{0.5} \right]} \tag{10}$$

A plot of $\log \tau$ versus $\log \left[J_o^{0.5} + (\mu_p K)^{0.5} \right]$ and passing through the origin will result in straight line with slope k as shown in the Fig. 5 The power law model is expressed as:

$$\tau = k \gamma^n \tag{11}$$

where, n is the fluid flow behaviour index which indicates the tendency of a fluid to shear thin and it is dimensionless and k is the consistency coefficient which serves as the viscosity index of the system and the unit is lb/100ft².sⁿ which can be converted to Pa.sⁿ by multiplying by a factor of 0.51. When n<1, the fluid is shear thinning and when n>1, the fluid is shear thickening (Reiner, 1926). The parameters k and n can be determined from a plot of $\log \tau$ versus $\log \dot{\gamma}$ and the resulting straight line's intercept is $\log k$ and the slope is n.

The Hershel-Buckley Model: An extension of the Bingham plastic model to include shear rate dependence. Mathematically, it is expressed as:

$$\tau = \tau_{oH} + k_H \gamma^{nH} \tag{12}$$

Where:

$\dot{\gamma}$ = The shear rate (sec⁻¹)

τ = The shear stress (Pa)

n_H = The flow behaviour index (dimensionless)

k_H = The HRBM consistency index in (Pa.sⁿ)

J_{oH} = The HBRM yield stress (Pa)

If the yield stress of a fluid sample is known from an independent experiment, the parameters k_H and n_H can be determined by linearizing Eq. 12 as follows:

$$\log(\tau - \tau_{oh}) = \log k + n \log(\gamma) \quad (13)$$

And a plot of $\log(J - J_{oh})$ versus $\log(\dot{\gamma})$ will result in a straight line with intercept $\log kh$ and slope nH , respectively.

Fluids that exhibit a yield point and viscosity that is stress or strain dependent cannot be adequately described by the Bingham plastic model. The Herschel Buckley Model corrects this deficiency by replacing the plastic viscosity term in the Bingham plastic model with a power law expression.

RESULTS AND DISCUSSION

Analysis of trans PKO mud sample: The mud Viscometer readings for TRANSPKO mud samples is presented in Table 1. Based on this readings, the predicted shear stress by each model is then analyzed.

Determination of model parameters for TRANSPKO
The Bingham plastic model: The plastic viscosity for the Bingham plastic model is obtained by using Eq. 5 as 0.01734 mPa.s while the yield stress is 5.61 Pas. Hence, the Bingham plastic equation for TRANSPKO is:

$$\tau_o = 5.61 + 0.01734\gamma \quad (14)$$

Hence, the BPRM shear stress for TRANSPKO is given by Eq. 14:

The modified Bingham plastic model: The modified Bingham plastic is obtained from the plot of $\log J$ versus $\log [J_o^{0.5} + (\mu DK)^{0.5}]$ which results in straight line passing through the origin with slope 2.278 as shown in Fig. 6:

$$\log \tau = 2.278 \log [\tau_o^{0.5} + (\mu \rho \gamma)^{0.5}] \quad (15)$$

Hence, the stress correction factor is given as 2.278. Hence, the modified Bingham plastic equation for Trans PKO is given as:

$$\tau = [\tau_o^{0.5} + (\mu \rho \gamma)^{0.5}]^{2.278} \quad (16)$$

Converting Eq. 16 to Pascal units gives Eq. 17:

$$\tau = [1.2915 + 0.09404\gamma^{0.5}]^{2.278} \quad (17)$$

Model parameters for power law and Hershel-Buckley Models: The n and k parameters for power law were obtained by a plot of $\log J$ versus $\log \dot{\gamma}$ as shown in Fig. 7 to give a straight line with Eq. 18:

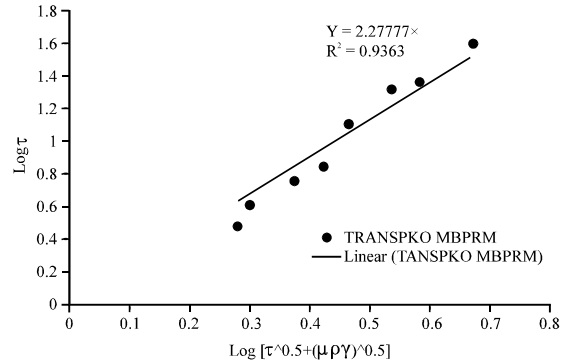


Fig. 6: Modified bingham plastic rheogram for TRANSPKO

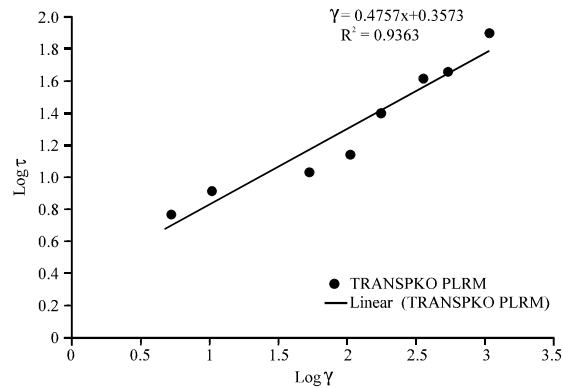


Fig. 7: Power law rheogram for TRANSPKO

Table 1: Viscometer readings for TRANSPKO mud

Speed (rpm)	Dial reading (lb/100ft ²)	Shear rate (sec ⁶¹)
600	79	1022.00
300	45	511.00
200	41	340.06
100	25	170.30
60	14	102.18
30	11	51.09
6	8	10.22
3	6	5.11

$$\log \tau = 0.4757 \log \gamma + 0.3573 \quad (18)$$

Hence, the PLRM for (TRANSPKO) is given as:

$$\tau = 1.1611\gamma^{0.4757} \quad (19)$$

Equation 19 is used to generate the power law stresses in Table 2. Similarly, the resulting straight line equation from the plot of $\log(J - J_{oh})$ against $\log \dot{\gamma}$ for Hershel-Bulkley equation for (TRANSPKO) as shown in Fig. 8 is:

$$\log(\tau - \tau_{oh}) = 0.8175 \log \gamma + (-0.6075) \quad (20)$$

Table 2: Stress values of different models for TRANSPKO mud

Dial speed	Dial reading (lb/100ft ²)	Shear rate (sec ⁻¹)	Measured (Pa)	PLRM (Pa)	BPRM (Pa)	HBRM (Pa)	MBPRM/RPMRPM (Pa)
600	79	1022	40.29	31.3655	23.3315	39.3943	33.9136
300	45	511	22.95	22.5534	14.4707	23.6769	21.1381
200	41	340.6	20.91	18.5971	11.5160	17.8579	16.5338
100	25	170.3	12.75	13.3735	8.5630	11.4567	11.4700
60	14	102.18	7.14	10.4885	7.3818	8.5903	9.1427
30	11	51.09	5.61	7.5424	6.4959	6.1980	7.0940
6	8	10.22	4.08	3.5076	5.7872	3.9019	4.8055
3	6	5.11	3.06	2.5224	5.6986	3.5377	4.3351

Table 3: Viscometer readings for INTERPKO mud

Speed (rpm)	Dial readings (lb/100ft ²)	Shear rate (sec ⁻¹)
600	57	1022.00
300	34	511.00
200	26	340.60
100	21	170.30
60	17	102.18
30	11	51.09
6	8	10.22
3	6	5.11

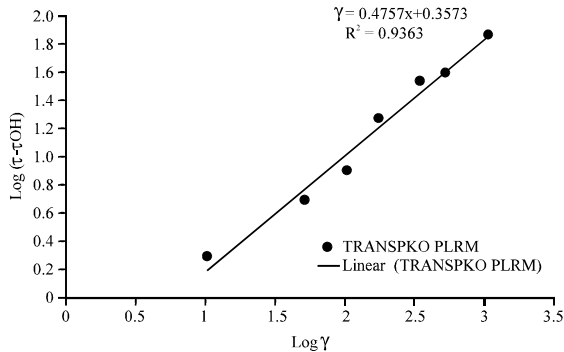


Fig. 8: Hershel Buckley rheogram for TRANSPKO

Hence, the Hershel Buckley equation for (TRANSPKO) is given as:

$$\tau = 3.06 + 0.1259(\gamma^{0.8173}) \quad (21)$$

Equation 21 is used to generate the HBRM stresses in Table 2:

Analysis of ITER PKO mud sample: The mud viscometer readings for INTERPKO mud samples is presented in Table 3. Based on this readings, the predicted shear stress by each model is then analyzed.

Determination of model parameters for INTERPKO mud sample

The Bingham plastic model: The plastic viscosity is obtained by using equation 5 as 0.01173 mPa.s while the yield stress is 5.61 Pa from Eq. 4. Hence, the Bingham plastic rheology model for INTER PKO is given as (Fig. 9):

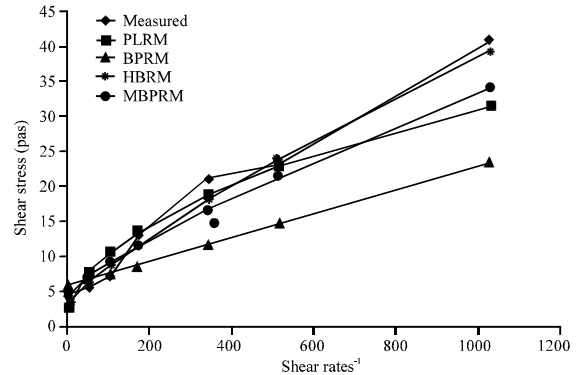


Fig. 9: Shear stress versus shear rate graph for TRANSPKO

$$\tau = 5.61 + 0.01173\gamma \quad (22)$$

Equation 22 is used to generate the power law stresses in Table 4.

The modified Bingham plastic model: The modified Bingham plastic is obtained from the plot of Log J versus log [J₀^{0.5} + (μD)^{0.5}] will result in straight line passing through the origin with slope 2.2848 as shown in Fig. 10:

$$\log \tau = 2.2848 \log [\tau_0^{0.50193} + (\mu\rho_\gamma)^{0.5}] \quad (23)$$

Hence, the stress correction factor is given as 2.2848. Hence, the modified Bingham plastic equation for INTERPKO is given as:

$$\tau = [\tau_0^{0.5} + (\mu\rho_\gamma)^{0.5}]^{2.2848} \quad (24)$$

Converting Eq. 24 to Pascal units gives Eq. 25:

$$\tau = [1.6915 + 0.07735\gamma^{0.5}]^{2.2848} \quad (25)$$

Equation 25 is used to generate the MBPRM stress shown in Table 4.

Table 4: Stress values of different models for INTER PKO mud sample

Dial speed (rpm)	Dial (lb/100ft)	Shear rate (sec ⁻¹)	Measured d(Pa)	PLRM (Pa)	BPRM (Pa)	HBRM (Pa)	MBPRM (Pa)
600	57	1022	29.07	22.9476	17.5981	27.5078	26.0300
300	34	511	17.34	17.4126	11.6040	18.1261	16.8228
200	26	340.6	13.26	14.8170	9.6052	14.4090	13.4491
100	21	170.3	10.71	11.2440	7.6076	10.0539	9.6710
60	17	102.18	8.67	9.1749	6.8086	7.9553	7.9170
30	11	51.09	5.61	6.9625	6.2093	6.0768	6.3409
6	8	10.22	4.08	3.6686	5.7298	4.0403	4.5384
3	6	5.109	3.06	2.7840	5.6699	3.6641	4.1604

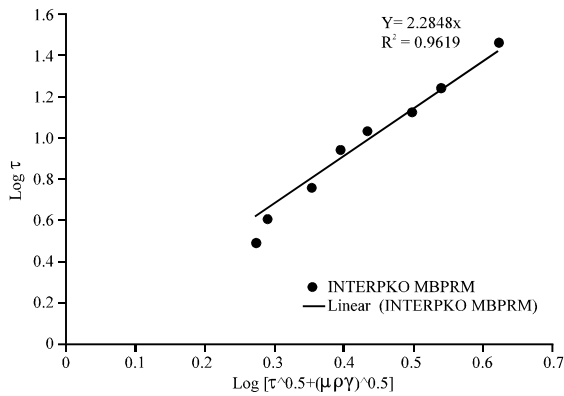


Fig. 10: Modified Bingham plastic model rheogram for INTERPKO

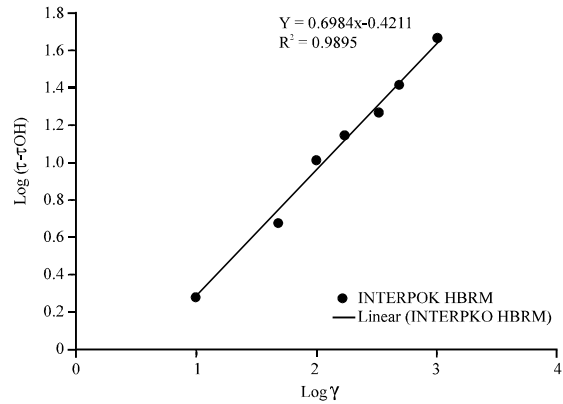


Fig. 12: Hershel-Buckley rheogram for INTERPKO mud sample

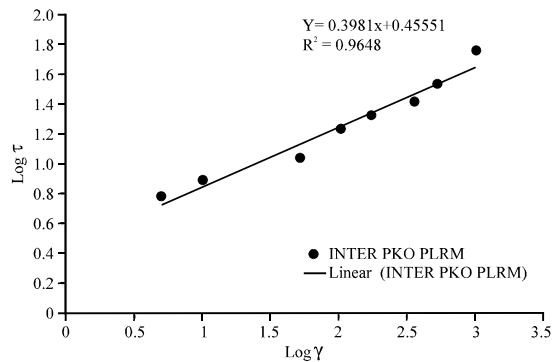


Fig. 11: Power law rheogram for INTERPKO

Model parameters for power law and Hershel-Buckley Models: The power law rheological model parameters were obtained from a plot of log J versus log (as shown in Fig. 11 which gives a straight line with Eq. 26:

$$\log \tau = 0.3981 \log \gamma + 0.4551 \quad (26)$$

Hence, the PLRM for INTER PKO is given as:

$$\tau = 1.4544 \gamma^{0.3981} \quad (27)$$

Equation 27 is used to generate the power law stresses in Table 4. A plot of log (J - J_{oh}) against log (as shown in Fig. 12 gives a straight line with equation:

$$\text{Log}(\tau - \tau_{oh}) = 0.6984 \log \gamma + (-0.4211) \quad (28)$$

Hence, the HRBM for INTERPKO is:

$$\tau = 3.06 + 0.1934(\gamma^{0.6984}) \quad (29)$$

Equation 29 is used to generate the power law stresses in Table 4.

Rheological model performance evaluation: The performance of the modified Bingham plastic model was compared with that of the Bingham plastic model and Power law model by quantifying the degree of deviation of their stresses from that of Hershel-Buckley rheological model. HBRM was chosen because according to the yield power law (Herschel-Buckley) rheological model accurately predicts mud rheology and offers many advantages over the Bingham plastic and power law rheological models because it more accurately characterizes mud behaviour across the entire shear rate. Hence, two statistical methods were used to measure this degree of deviation. These are:

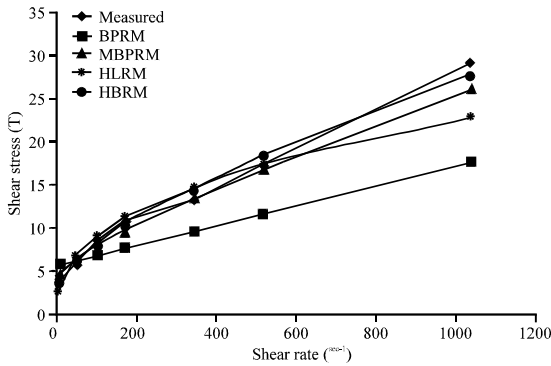


Fig. 13: Shear stress versus shear rate graph for INTERPKO

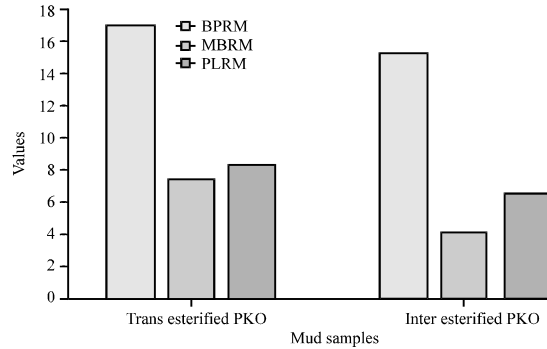


Fig. 15: Standard deviation of average percentage error of the rheological models

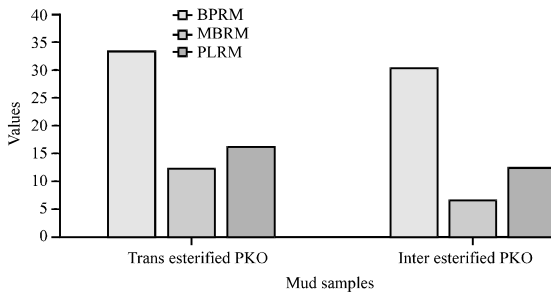


Fig. 14: Absolute Average Percentage Error (E_{AAP}) of the rheological model

Table 5: Absolute Average Percentage error (E_{AAP}) of the rheological models

Mud sample	BPRM	MBPRM	PLRM
Trans esterified PKO	33.5877	12.3438	16.0735
Inter esterified PKO	30.3538	6.7166	12.2885

Table 6: Standard deviation of Average Percentage error of the rheological models

Mud sample	BPRM	MBPRM	PLRM
Trans esterified PKO	17.0846	7.40980	8.3112
Inter esterified PKO	15.3419	4.07899	6.5138

- C Absolute Average Percentage error (E_{AAP})
- C Standard deviation of Average Percentage error (E_{AAP})

The E_{AAP} is given by the Eq. 30 as:

$$E_{AAP} = \left[\frac{1}{N} \sum \left| \frac{(\tau_{HBRM} - \tau_{calculated})}{\tau_{HBRM}} \right| \right] * 100 \quad (30)$$

The Absolute Average Percentage error (E_{AAP}) of the rheological models is shown in Table 5 and Fig. 14. The INTERPKO mud sample showed the lowest average percentage error for all the rheological models as shown in Table 5 and Fig. 14, respectively. Similarly, the modified Bingham plastic model has the lowest average percentage error for the two mud samples. The standard deviation of average percentage error is obtained using Eq. 31:

$$SD_{E_{AAP}} = \sqrt{\frac{\sum f (\% \text{ error} - E_{AAP})^2}{\sum f}} \quad (31)$$

The standard deviation of average percentage error of the rheological models is shown in Table 6 and graphically represented in Fig. 15.

From Fig. 9, the shear stress values predicted by the proposed model shows a good identity with that of the Herschel Buckley model both at lower and higher shear rates. While the Bingham plastic model results showed a clear deviation from the measured and Herschel Buckley stress values both at low and at high shear rate conditions. Although, more stress values accuracy for the proposed model can be seen at the outset of higher shear rate conditions. Similarly, from Table 2, at dial speed of 100 rpm, the stress values predicted by the model shows a good similarity with that of the Herschel Buckley and the measured Values.

From Fig. 13, the modified Bingham plastic model stress values showed better accuracy than the Bingham plastic and the power law model results when compared with both the Herschel Buckley and the measured stress values at both low and high shear rate conditions. The stress values predicted by the Bingham plastic model is high at low shear rate conditions and low at high shear rate conditions as shown in Table 4.

The INTERPKO mud sample showed the lowest average percentage error for all the rheological models as shown in Table 5 and Fig. 14, respectively. Similarly, the modified Bingham plastic model has the lowest average percentage error for the two mud samples.

From Table 6 and Fig. 15, it is evident that the proposed modified Bingham plastic rheological model showed the least standard deviation of its error from the Hershel Buckley Model for the two mud samples while a very wide deviation was exhibited by Bingham plastic model. The INTERPKO mud sample has the least deviation for all the rheological models.

CONCLUSION

From the model performance evaluation of the rheological models, the following inferences can be drawn. The Bingham plastic rheological model cannot accurately represent the rheology of synthetic based drilling mud because it underestimates the stress values at high shear rate conditions and overestimate the stress values at low shear rate conditions. The proposed modified Bingham plastic model accurately characterized the mud systems at both low and high shear rate conditions. The performance of the proposed model also showed a higher accuracy than that of power law model at low shear rate conditions.

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NOMENCLATURE

TRANSPKO = Trans Esterified Palm Kernel Oil
INTERPKO = Inter Esterified Palm Kernel Oil
(SD_{r, AAP}) = Standard Deviation of Absolute Average Percentage error yield point
PV = Plastic Viscosity
n = Flow behaviour index
k = Consistency index
k_H = Hershel Buckley consistency index
n_H = Hershel Buckley flow index
J_{oH} = Hershel Buckley yield stress

Greek symbols:

μ = Viscosity
J = Shear stress
(= Shear rate
J_o = Yield stress

Abbreviations:

BPRM = Bingham Plastic Rheological Model
MBPRM = Modified Bingham Plastic Rheological Model
HBRM Hershel-Buckley Rheological Model
PLRM = Power Law Rheological Model

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