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ON STABILITY OF QUANTUM STOCHASTIC DIFFERENTIAL EQUATION

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ABSTRACT

The long run behaviour of solutions of Lipschitzian quantum stochastic differential equation (QSDE) with non-instantaneous impulse is studied. This is achieved by imposing some conditions on the coefficients associated with the map P. Using the fixed point approach, we show that a solution exists under the given conditions and subsequently establish Ulam's type stability. We present some examples to further justify its application.

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Keywords: Ulams-Hyers stability, QSDE, Non-instantaneous impulse functions, Fixed point method.

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1. INTRODUCTION

Stability of solutions of impulsive ordinary differential equations (ODEs), partial differential equations (PDEs), Functional differential equations (FDEs), etc. have been of interest to many authors [1, 4-7, 9-13]. Wang and Feckan (2013) [13], established stability results for stochastic differential equations. [4, 5, 9] established similar results when the impulse conditions are combinations of the traditional initial value problems and the short term perturbations. However, the perturbation terms in these classes of equations cannot show the dynamic change of evolution processes as it should in some applications. To address some of these limitations, Liao and Wang (2014) in [7], studied generalized Ulam-Hyers-Rassias (U-H-R) stability of solutions for a class of equations with non-instantaneous impulses and provided some examples to show their applications.

Some results on existence of solution of impulsive quantum stochastic differential equations (IQSDEs) and quantum stochastic differential inclusions (QSDIs) have been established in [2, 3, 8]. So far, results on stability of these equations have not been investigated. Considering the importance of the long run behaviour of systems in real life applications, is a motivation for this study.

This paper is concerned with the study of U-H-R stability of the following QSDE (Also known as nonclassical ordinary differential equation (NODE)) with non-instantaneous impulse functions:

$$\frac{d}{dt}\langle \eta, \phi(t)\xi \rangle = P(t, \phi(t))(\eta, \xi), t \in (s_k, t_k], k = 1, ..., m$$
$$\langle \eta, \phi(t)\xi \rangle = \langle \eta, q_k(\phi(t))\xi \rangle, t \in (t_{k,} s_k]$$
$$\langle \eta, \phi(0)\xi \rangle = \langle \eta, \phi_0 \xi \rangle, t \in I := [0, T] \qquad (1.1)$$

Where $(t, \phi) \to P(t, \phi)(\eta, \xi)$ is well defined in [2, 3], $\eta, \xi \in \mathbb{D} \underline{\otimes} \mathbb{E}$ is arbitrary,

$0 = t_0 = s_0 < t_1 \le s_1 \le t_2 < ... < s_{m-1} \le t_m \le s_m \le t_{m+1} = T, P : I \times \tilde{\mathrm{B}} \to sesq\,(\mathbb{D} \underline{\otimes} \mathbb{E})$ continuous, and

$$q_k : [t_k, s_k] \to \mathbb{R}.$$

Note that the sesquilinear form valued map P is assumed to be real valued since $C \approx \mathbb{R}^2$, hence, the methods of [7, 12] are applicable to this setting.

2. PRELIMINARIES

1. $\tilde{\mathrm{B}}$ is a topological vector space.
2. $(\mathbb{D} \underline{\otimes} \mathbb{E})$ is a complex space.
3. $C(I, \tilde{\mathrm{B}}), PC(I, \tilde{\mathrm{B}})$ are spaces of continuous and piecewise continuous functions.
4. Define $PC^1(I, \tilde{\mathrm{B}}) := \{\phi \in PC(I, \tilde{\mathrm{B}}) : \phi' \in PC(I, \tilde{\mathrm{B}})\}$
5. The sesquilinear equivalent forms $PC(I, sesq(D \underline{\otimes} E))$ and $PC^{'}(I, sesq\,(\mathbb{D} \underline{\otimes} \mathbb{E}))$

Of the above spaces are defined in a similar manner with the usual supremum norm defined in [2].

Definition 2.1. A stochastic process is called a solution of Eq. (1.1) if, it satisfies the following:

$$\langle \eta, \phi(0), \xi \rangle = \langle \eta, \phi_0 \xi \rangle$$
$$\langle \eta, \phi(t), \xi \rangle = \langle \eta, q_k(t, \phi(t))\xi \rangle, t \in (t_k, s_k]$$
$$= \langle \eta, \phi_0 \xi \rangle + \int_0^t P(s, \phi(s))(\eta, \xi) ds, t \in [0, t_1];$$
$$= \langle \eta, q_k(t, \phi(t)))\xi \rangle + \int_{s_k}^t P(s, \phi(s))(\eta, \xi) ds, t \in [t_k, s_k], k = 1, ..., m$$

Subsequently, $t \in I, \eta, \xi \in (D \otimes E))$ and $k = 1, ...m$ except otherwise stated.

Next we re-frame the concept of Ulam's type stability for the purpose of this paper.

Let $PC(I, \tilde{\mathrm{B}}) := \{\phi \in \tilde{\mathrm{B}} : \phi(t) \ge 0\}, \Phi_{\eta\xi}, \ge 0$ and $\Phi_{\eta\xi}(t) \in PC(I, sesq\,(\mathbb{D} \underline{\otimes} \mathbb{E}))$

The following inequality will be useful:

$$\left|\frac{d}{dt}\langle\eta,\phi(t)\xi\rangle - P(t,\phi(t)(\eta,\xi)\right| \le \Psi_{\eta\xi}, t\in(s_k,t_{k+1}]$$
$$\left|\langle\eta,\phi(t)\xi\rangle - \langle\eta,q_k(t,\phi(t))\xi\rangle\right| \le \Phi_{\eta\xi}(t), t\in(t_k,s_k] \qquad (2.1)$$

Definition 2.2. Equation (1.1) is U-H-R stable with respect to $\Phi_{\eta\xi},\Psi_{\eta\xi}(t))$ if we can find $M_{\eta\xi}>0$ such that for each solution $\phi\in PC'(I,\tilde{B})$ of (2.1), there exists a solution $\Phi\in PC'(I,\tilde{B})$ of Eq.(1.1) with

$$\|y(t)-\phi(t)\|_{\eta\xi} \le M_{\eta\xi}(\Phi_{\eta\xi},\Psi_{\eta\xi})), t\in I \qquad (2.2)$$

Eq. (1.1) has found applications in quantum stochastic control theory and quantum dynamical systems, see [2, 8].It is worth mentioning that this method will be more useful in many applications such as numerical analysis, Physics, especially when exact solutions are difficult to come by.

Definition 2.3. A stochastic process $\phi\in PC^1(I,\tilde{B})$ is a solution of (2.1) if and only if there exists a function $F_{\eta\xi}\in PC^1(I,sesq(D\underline{\otimes}E))$ and $F_{\eta\xi}\in PC^1(I,sesq(\mathbb{D}\underline{\otimes}\mathbb{E}))$ such that

i $|F_{\eta\kappa}(t)| \le \Phi_{\eta\kappa}(t), t\in I,$ and $|F_{\eta\kappa}| \le \Psi_{\eta\kappa}$

ii $\langle\eta,\phi(t)\xi\rangle = P(t,\phi(t))(\eta,\xi), t\in(s_k,t_{k+1}]$

iii $\langle\eta,\phi(t)\xi\rangle = (\eta,q_k(t,\phi(t))\xi)+F_{\eta\xi}, t\in(s_k,t_k]$

Definition 2.4. Also, $\phi\in PC(I,\tilde{B})$ if is a solution of the (2.1), then it is also a solution of the following integral inequality:

$$\|\phi(t)-q_k(t,\phi(t))\|_{\eta\xi} \le \Phi_{\eta\xi}, t\in(s_k,t_{k+1}]$$
$$\left\|\phi(t)-\phi(0)-\int_0^t P(s,\phi(s))ds\right\|_{\eta\xi} \le \int_0^t \Psi_{\eta\xi}(s)ds, t\in[0,t_1];$$
$$\left\|\phi(t)-q_k(t_k,\phi(t_k))\int_{s_\kappa}^t P(s,\phi(s))ds\right\| \le \Phi_{\eta\kappa}+\int_0^t \Psi_{\eta\kappa}(s)ds, t\in[s_k,t_{k+1}] \qquad (2.3)$$

We state the following established result and refer the reader to [7] and the references therein:

Lemma 2.1. Let v,a,b be real valued piecewise continuous functions, where a is nondecreasing. Assume the following inequality holds:

$v(t)\le a(t)+\int_0^t b(s)v(s)ds+\sum_{0<t_i<t}\gamma_i v(t_i^-), t\ge 0,$ where $b(t)>0,\gamma_i>0, i=1,\dots,m.$ Then the following inequality also holds:

$$v(t)\le a(t)(1+\gamma)^i + e^{\int_0^t b(s)ds}, t\in(t_i,t_{i+1}],$$

where $\gamma=\max\{\gamma_i, i=1,\dots,m\}$.

3. MAIN RESULTS

We state the following useful hypotheses:

S_1 Let $K^p_{\eta\kappa} > 0$ be a constant such that

$$\|P(t,\phi_1) - P(t,\phi_2)\|_{\eta_\kappa} \le K^p_{\eta\xi}\|\phi_1 - \phi_2\|_{\eta\kappa},$$

For each $t \in I, \phi_1, \phi_2 \in \tilde{B}$.

S_2 For $q\kappa \in C([t_k, s_k] \times \tilde{B}, \tilde{B})$, let there be constants $L_k > 0$ such that

$$\|q_\kappa(t,\phi_1) - q_\kappa(t,\phi_2)\|_{\eta\xi} \le L_\kappa\|\phi_1 - \phi_2\|_{\eta\xi}, \quad \text{For each } t \in I, \phi_1, \phi_2 \in \tilde{B}.$$

S_3 *Let* $l_\Psi > 0$ a constant and let $\Psi \in PC(I, \tilde{B})$ be a nondecreasing function such that

$$\int_0^t \Psi(s)ds \le l_\Psi \Psi(t), \text{ for each } t \in I.$$

The following result is a consequence of definition 2.1.

Theorem 3.1. Let the map P in Eq. (1.1) be continuous for each and let the hypotheses $S_1 - S_3$ hold. Then equation (1.1) has a unique solution $\phi \in PC'(I, \tilde{B})$ provided

$$\{L_k + K^p_{\eta\xi}T, k = 1,...m\} < 1 \tag{3.1}$$

Proof: The proof is an adaptation of the method employed in [2].

We give a sketch as follows and refer the reader to the reference [2] for details.

Transform the Eq. (1.1) to a fixed point problem by defining the map Γ as follows:

Let $\Gamma : PC(I, sesq\,(\mathbb{D} \underline{\otimes} \mathbb{E})) \to PC(I, sesq\,(\mathbb{D} \underline{\otimes} \mathbb{E}))$

$$\Gamma(\phi)(t)(\eta,\xi) = \langle \eta, \phi(0)\xi \rangle + \int_0^t P(s,\phi(s))(\eta,\xi)ds$$
$$+ q_\kappa(t,\phi(t))(\eta,\xi) \tag{3.2}$$

and by the assumption $(S_1 - S_2)$, we have

$$\begin{aligned}|\Gamma(\phi)(t)(\eta,\xi) - \Gamma(y)(t)(\eta,\xi)| &\le \int_0^t |P(s,\phi(s))(\eta,\xi) - P(s,y(s))(\eta,\xi)|ds \\ &\quad + |q_\kappa(t,\phi(t)(\eta,\xi) - q_k(t,y(t)(\eta,\xi)| \\ &\le K^p_{\eta\kappa}\int_0^t \|\phi - y\|_{\eta\kappa} ds + L_\kappa\|\phi - y\|_{\kappa\xi} \\ &\le \left(L_\kappa + K^p_{\eta\kappa}T, k = 1,...,m\right)\|\phi - y\|_{\kappa\xi} \\ &\le \|\phi - y\|_{\kappa\xi}\end{aligned}$$

Where $\phi(0) = y(0)$. Showing that (3.1) is satisfied and hence, Γ is a con- traction operator on $PC(I, sesq\,(\mathbb{D} \otimes \mathbb{E}))$ and a fixed point exists, which is a unique solution of (1.1).

Next, is the main result on stability.

Theorem 3.2: Let the conditions $S_1 - S_2$ and (3.1) hold. Then Eq. (1.1) is

U-H-R stable.

Proof: Let

$\phi \in PC'(I,\tilde{\mathrm{B}})$ be a solution of Eq. (1.1). Then

$$\langle \eta, \phi(t)\xi \rangle = \langle \eta, q_\kappa(t,(\phi(t))\xi \rangle, t \in (t_\kappa, s_k];$$
$$= \langle \eta, \phi_0 \xi \rangle + \int_0^t P(s,\phi(s))(\eta,\xi)ds, t \in [0,t_1];$$
$$= \mathrm{q}_k(s_k,\phi(s_k)) + \int_{s_\kappa}^t (P(s,\phi(s))(\eta,\xi) + F_{\eta\xi}(s)ds, \quad t \in (s_k, t_{k+1}].$$

From (2.3) and S_3 we get

$$\left\|\phi(t) - q_\kappa(t_k,\phi(t_k))\right\|_{\eta\xi} - \int_{s_\kappa}^t \left\|P(s,\phi(s))\right\|_{\eta\xi} ds$$
$$\le \Phi_{\eta\xi} + \int_0^t \Psi_{\eta\xi}(s)ds,$$
$$\le \Phi_{\eta\xi} + \mathrm{l}_\Psi \Psi_{\eta\xi}\ (\mathrm{t}), t \in [s_k, t_{k+1}]$$

For $t \in (s_k, t_k],$ we obtain

$$\left\|\phi(t) - q_k(t,\phi(t)\right\|_{\eta\xi} \le \Phi_{\eta\xi},$$

$$\left\|\phi(t) - \phi(0) - \int_0^t P(s,\phi(s)ds\right\|_{\eta\xi} \le l_\psi \Psi_{\eta\xi}(t).$$

Therefore, for each $t \in [s_k, t_{k+1}]$, we get

$$\left\|\phi(t) - y(t)\right\|_{\eta\xi} \le \left\|\phi(t) - q_\kappa(t_k,\phi(t_k)) - \int_{s_\kappa}^t P(s,\phi(s))ds\right\|_{\eta\xi}$$

And when $t \in [0, t_k],$ yields

$$+ \left\|q_\kappa(t_s,\phi(s_k)) - q_\kappa(s_k, y(s_k))\right\|_{\eta\xi}$$
$$+ \int_{s_\kappa}^t \left\|P(s,\phi(s)) - P(s,y(s))\right\|_{\eta\xi} ds$$
$$\le (1 + l_\psi)(\Phi_{\eta\xi} + \Psi_{\eta\xi}(t)) + L_k \sum_{0<t_k<t} \left\|\phi(s_\kappa) - y(s_\kappa)\right\|_{\eta\xi}$$
$$+ \mathrm{K}_{\eta\xi} \int_{s_\kappa}^t \left\|\phi(s_\kappa) - y(s_\kappa)\right\|_{\eta\xi} ds$$

Applying Lemma 2.1, yields

$$\left\|\phi(t) - y(t)\right\|_{\eta\xi} \le (1 + l_\psi)(\Phi_{\eta\kappa} + \Psi_{\eta\kappa}(t))(1 + L_\kappa)^m \exp(K_{\eta\xi} t_{t+1}), t \in (s_k, t_{k+1}]. \tag{3.3}$$

Moreover, for $t \in (s_k, t_k],$ we obtain

$$\left\|\phi(t) - y(t)\right\|_{\eta\xi} \le \left\|\phi(t) - q_\kappa(t,\phi(t))\right\|_{\eta\xi}$$
$$+ \left\|q_\kappa(t,\phi(t)) - q_\kappa(t, y(t))\right\|_{\eta\xi}$$
$$\le \Phi_{\eta\xi} + L_\kappa \left\|\phi(t) - y(t)\right\|_{\eta\xi}$$
$$\le \frac{1}{1 - L_q} \Phi_{\eta\xi}, L_q = \max\{L_\kappa, k = 1,...,m\} < 1 \tag{3.4}$$

Again, for each $t \in [0,t_1]$, we obtain

$$\|\phi(t)-y(t)\|_{\eta\xi} \le l_\psi \Psi_{\eta\xi}(t) + K_{\eta\xi}\int_0^t \|\phi(s)-y(s)\|_{\eta\xi} ds$$

By Gronwall's Inequality, we get

$$\|\phi(t)-y(t)\|_{\eta\xi} \le l_\psi \Psi_{\eta\xi}(t) e^{K_{\eta\xi t_1}} \tag{3.5}$$

Hence, by putting (3.3),(3.4) and (3.5) together, we obtain

$$\|\phi(t)-y(t)\|_{\eta\kappa} \le \left((1+l_\psi)(1+L_\kappa)^m e^{K_{\eta\xi} t_{k+1}} + \frac{1}{1-L_q} + l_\psi e^{K_{\eta\xi t_1}} \right)(\Phi_{\eta\xi} + \Psi_{\eta\kappa}(t))$$

$$:= M_{\eta\kappa}(\Phi_{\eta\xi} + \Psi_{n\kappa}(t))$$

where $M_{\eta\xi}$ is as defined in Definition 2.2 This implies that equation (1.1) is generalized U-H-R stable with respect to $(\Phi_{\eta\xi}, \Psi_{\eta\xi}(t))$.

4. EXAMPLE

Let $I=[0,2], P(t,\phi(t)(\eta,\xi) = (e^{2t}-1)(\langle \eta, \phi(t)\xi), K_{\eta\xi} = 1/6, \langle \eta, q_\kappa(t,\phi(t))\xi \rangle = \phi(t)\exp\left(\frac{e^{2t}}{2} - t - 1/2\right), L_k = 1/6$. Now let $\Psi_{\eta\xi}(t)$ and $\Phi_{\eta\xi}, I = 1/2$. Let

$$\langle \eta, \phi(0)\xi \rangle = \langle \eta, \xi \rangle = e^{\langle \eta,\xi \rangle} = 1.$$

Considering the following problems:

$$\begin{aligned} \frac{d}{dt}\langle \eta, \phi(t)\xi \rangle &= P(t,\phi(t)(\eta,\xi) \\ &= (e^{2t}-1)(\langle \eta, \phi(t)\xi \rangle, t \in (0,1] \\ \langle \eta, \phi(t)\xi \rangle &= \langle \eta, q_k(t,\phi(t))\xi \rangle \\ &= \phi(t)\exp\left(\frac{e^{2t}}{2} - t - 1/2\right), t \in (1,2] \end{aligned} \tag{4.1}$$

$$\begin{aligned} \frac{d}{dt}\langle \eta, y(t)\xi \rangle &= P(t,y(t)(\eta,\xi) \\ &= (e^{2t}-1)(\langle \eta, y(t)\xi \rangle, t \in (0,1] \\ \langle \eta, y(t)\xi \rangle &= \langle \eta, q_k(t,y(t))\xi \rangle \\ &= y(t)\exp\left(\frac{e^{2t}}{2} - t - 1/2\right), t \in (1,2] \end{aligned} \tag{4.2}$$

Where $\phi(0) = y(0)$. Let $y \in PC^1(I,\tilde{B})$ be a solution of (4.2). Then, we find $F_{\eta\xi}(.) \in PC^1(1,\tilde{B})$ and $F_1 \in \tilde{B}$ such that

$$\left|F_{\eta\xi}(t)\right| \le (e^{2t}), t \in (0,1], \left|F_{\eta\xi},1\right| \le 1\!\!\diagup_{\!2},$$

$$\frac{d}{dt}\langle \eta, y(t)\xi\rangle = P(t, y(t)(\eta,\xi) + F_{\eta\kappa}(t)$$
$$=(\mathrm{e}^{2t}\text{-}1)(\langle \eta, y(t)\xi\rangle, + F_{\eta\kappa}(t), \mathrm{t} \in (0,1]$$
$$\langle \eta, y(t)\xi\rangle = \langle \eta, q_k(t, y(t))\xi\rangle + F_{\eta\kappa}(t), 1$$
$$= \mathrm{y}(t)\exp\left(\frac{\mathrm{e}^{2t}}{2} - t - 1\!\!\diagup_{\!2}\right), t \in (0,2] \tag{4.3}$$

Integrating (4.3), yields

$$\langle \eta, y(t)\xi\rangle = \langle \eta, y(0)\xi\rangle + \int_0^t ((e^{2s}-1)(\langle \eta, y(s)\xi)ds, t \in (0,1],$$

And

$$\langle \eta, y(t)\xi\rangle = \langle \eta, q_k(t, y(t))\xi\rangle + F_1 = y(t)\exp\left(\frac{e^{2t}}{2} - t - 1\!\!\diagup_{\!2}\right) + F_{\eta\kappa}, 1.$$

By Theorem 3.1, (4.1) has a unique solution which we present by

$$\langle \eta, \phi(t)\xi\rangle = \langle \eta, \phi(0)\xi\rangle + \int_0^t ((e^{2t}-1)(\langle \eta, \phi(s)\xi\rangle)ds, t \in (0,1].$$

And

$$\langle \eta, \phi(t)\xi\rangle = \phi(t)\exp(e^{2t} - t - 1\!\!\diagup_{\!2}).$$

And for $t \in [0,1]$, we obtain

$$\left|\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle\right| \le \int_0^t e^{2s}ds \le \frac{e^{2t}}{2} - \frac{1}{2} \le e^{2t}.$$

Again we obtain

$$\left|\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle\right| \le \frac{1}{6}\left|\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle + F_{\eta\kappa}, 1\right|$$
$$\le \frac{1}{6}\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle + \frac{1}{2}$$

And this yields

$$\left|\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle\right| \le \frac{6}{5}$$

Which finally result to

$$\left|\langle \eta, \phi(t)\xi\rangle - \langle \eta, y(t)\xi\rangle\right| \le \frac{6}{5}\left(\frac{1}{2} + e^{2t}\right), t \in I.$$

5. CONCLUSION

This shows that the solution of Eq. (1.1) is generalized U-H-R stable with $\Phi_{\eta\xi} = \frac{1}{2}$ and $\Psi_{\eta\kappa}(t) = e^{2t}$.

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest.

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