International Journal of Civil Engineering and Technology (IJCIET)
Volume 9, Issue 8, August 2018, pp. 347–357, Article ID: IJCIET_09_08_035
Available online at http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=9&IType=8
ISSN Print: 0976-6308 and ISSN Online: 0976-6316

PREDICTION OF COLLAPSED LOAD OF STEEL COLUMNS USING FINITE STRIP METHOD

Olatokunbo Ofuyatan*
Department of Civil Engineering, College of Engineering,
Covenant University Ota, Lagos. Nigeria.

Simon Olawale
Department of Civil Engineering, Osun State University,
Osogbo, Osun State. Nigeria

Anthony Ede
Department of Civil Engineering, College of Engineering,
Covenant University Ota, Lagos. Nigeria.

Solomon Oyebisi
Department of Civil Engineering, College of Engineering,
Covenant University Ota, Lagos. Nigeria.

John Oluwafemi
Department of Civil Engineering, College of Engineering,
Covenant University Ota, Lagos. Nigeria.

Tolulope Alayande
Department of Civil Engineering, College of Engineering,
Covenant University Ota, Lagos. Nigeria.

*Corresponding Author : olatokunbo.ofuyatan@covenantuniversity.edu.ng

ABSTRACT

The solution of inelastic bifurcation of column is the eigenvalue and eigenvectors which can be solved by many methods such as Power Method and William-Wittrick algorithm which has been proven to be efficient. This study utilised a new method of solution algorithm that utilizes the decomposition of the net assembled structural global matrix into the lower and upper triangles leading to the calculation of the determinant of the decomposed matrix. The product of the squared leading diagonals is the determinant which is the error of the present calculation. This method has been demonstrated to be about 0.02% error. The conventional Finite Strip Method is programmed with the new solution algorithm using an object oriented programming concept (Java). The method is compared with the published work and the agreement *between the current work and the reported experimental and numerical work is excellent. Residual stress is successfully implemented with the new solution algorithm.*

Key words: Finite strip method, column, numerical, collapse, residual stress.

Cite this Article: Olatokunbo Ofuyatan, Simon Olawale, Anthony Ede, Solomon Oyebisi, John Oluwafemi, Tolulope Alayande, Prediction of Collapsed Load of Steel Columns using Finite Strip Method. *International Journal of Civil Engineering and Technology*, 9(8), 2018, pp. 347-357.
http://www.iaeme.com/IJCIET/issues.asp?JType=IJCIET&VType=9&IType=8

1. INTRODUCTION

Predicting the collapsed load of steel columns requires numerical computation because of material and geometric nonlinearities. The ability to reliably predict the collapsed load is fundamental to assessing the effects of local buckling, initial geometric imperfections in the section, residual stresses produced by manufacturing processes and material yielding and many other phenomena as noted by Ofuyatan *et al*. One of such methods is Finite Strip Method which is a versatile tool for the prediction of local, distortional and overall buckling of rolled and cold formed sections. The extent of the application of the method to the current work is limited to the material non-linearity. Although, the initial imperfection could be easily incorporated as demonstrated by Olawale, the emphasis now is the demonstration of the new solution algorithm. However, residual stress synonymous to rolled section is investigated. Li and Schafer have explored the solutions and provided design recommendations for two practical issues that develop when integrating computational member analysis with the conventional finite strip method (FSM) into cold-formed steel member design utilizing the direct strength method (DSM). First, FSM often fails to uniquely identify the relevant local and distortional member buckling modes. However, advancement in the field of FSM such as constrained finite strip method (cFMS) resolves some issues such as distortional buckling. cFSM also has some limitations such as inability to account for rounded corners in the model of cross-section. G. Eccher, K.J.R. Rasmussen, R. Zandonini presented the application of the isoparametric spline finite strip method to the elastic buckling analysis of perforated folded plate structures. The general theory of the isoparametric spline finite strip method was introduced. The kinematics assumptions, strain– displacement and constitutive relations of the Mindlin plate theory described and applied to the spline finite strip method. The corresponding matrix formulation was utilized in the equilibrium and stability equations to derive the stiffness and stability matrices. A number of numerical examples of flat and folded perforated plate structures illustrate the applicability and accuracy of the proposed.

2. FINITE STRIP METHOD

The general concept of Finite Strip Method has been well established as an efficient means of predicting buckling of plate and plated structures. In most cases, elastic approach has been very popular even in the age of powerful computational power. The extension of the elastic finite strip method to the inelastic range is accomplished by using the deformation theory of plasticity applied to thin plates. Based on this theory of plasticity, the nonlinear material properties are given by the following equations:

$$\{\sigma\} = E_{20}[F]\{\varepsilon\} \qquad (1)$$

where the elasto-plastic modular matrix [F] is given by

$$[F] = \begin{bmatrix} f_{11} & f_{12} & 0 \\ f_{12} & f_{22} & 0 \\ 0 & 0 & f_{33} \end{bmatrix} \quad (2)$$

and the coefficients of [F] are given by

$$f_{11} = \frac{(1+3\tau)}{\mu}$$

$$f_{12} = \frac{2(1-(1-2\nu)\tau)}{\mu}$$

$$f_{22} = \frac{4}{\mu}$$

$$f_{33} = \frac{1}{2+2\nu+3e}$$

$$e = E_{20}/E_S$$

$$\tau = E_T/E_{20}$$

$$\mu = 5 - 4\nu + 3e - (1-2\nu)^2\tau \quad (3)$$

where

E_t is the tangent modulus

E_s is the secant modulus

E_{20} is Young's modulus of elasticity at ambient temperature.

These moduli are obtained from the uniaxial stress-strain relation.

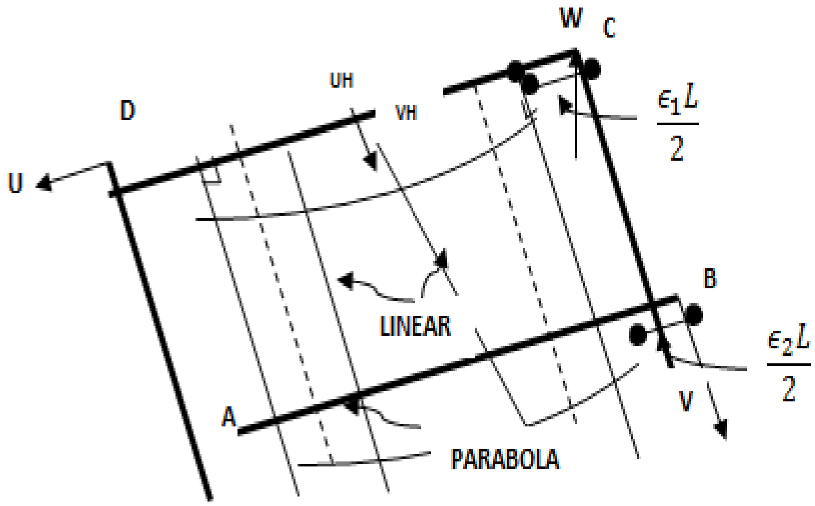

Figure 1a. A basic plate showing the Hookean and membrane displacements of a strip

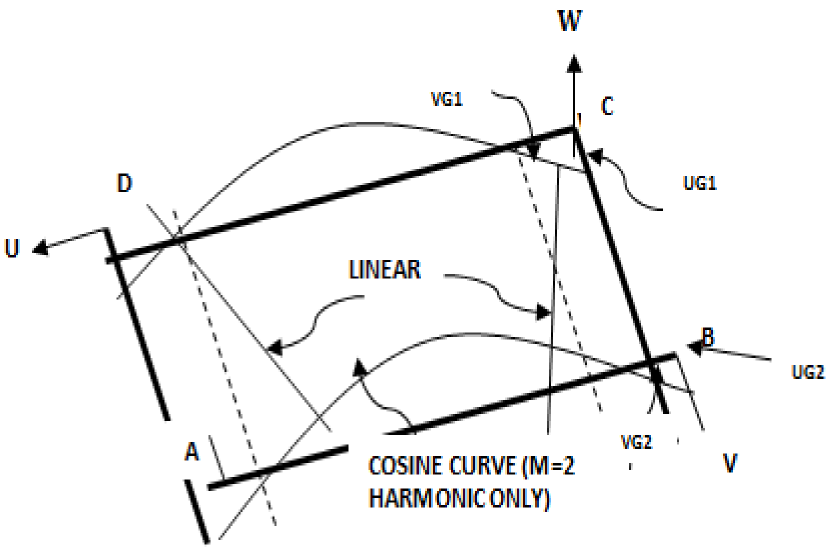

Figure 1b A basic plate showing the Hookean and membrane displacements of a strip

The moment-curvature relationship is given by

$$\{M\} = \frac{E_{20}t^3}{12}[F]\{\chi\} \tag{4}$$

and the curvature is given by

$$\{\chi\} = \left\{-\frac{\partial^2 W}{\partial x^2} - \frac{\partial^2 W}{\partial y^2} 2\frac{\partial^2 W}{\partial x \partial y}\right\}$$

and **t** is the plate thickness

To obtain the stiffness and stability matrices, the out-of-plane and in-plane effects are considered individually.

a) Out-of-Plane: The combination of the resulting equations enables all buckling modes to be considered. The out-of-plane displacement function is given by

$$f_o = \{W\} \tag{5}$$

$$f_o = \{Z\}^T \sin \pi\, \xi \{\delta_o\} \tag{6}$$

where

$$\{Z\}^T = \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \\ C_6 \end{Bmatrix} \tag{7}$$

And $C_3 = b(\eta - 2\eta^2 + \eta^3)$

$$C_4 = 1 - 3\eta^2 + \eta^3$$

$C_5 = b(-\eta^2 + \eta^3)$

$C_6 = 3\eta^2 - 2\eta^3$

$$\xi = \frac{x}{\lambda}$$

$$\eta = \frac{y}{b} \tag{8}$$

$$\{\delta_o\} = \begin{Bmatrix} \theta_1 \\ w_1 \\ \theta_2 \\ w_2 \end{Bmatrix}$$

(10)The out-of-plane strain vector is related to the curvature as given below

$$\{\varepsilon_o\} = z\{\chi\} \tag{11}$$

$$\{\varepsilon_o\} = \{B_o\}\{\delta_o\} \tag{12}$$

and $[B_o]$ is given as

$$[B_o] = \begin{bmatrix} \frac{z\pi^2}{\lambda^2}\{Z\}^T sin\pi\xi \\ -\frac{z}{b^2}\{Z'_{\eta\eta}\}^T sin\pi\xi \\ \frac{2\pi z}{b\lambda}\{Z'_{\eta}\}^T cos\pi\xi \end{bmatrix} \tag{13}$$

Applying the principle of virtual work, the change in the internal virtual work for the out-of-plane behaviour can easily be established leading to the out-of-plane stiffness matrix as documented in many literature.

$$[K_o] = \int_{vol}[B_o]^T[F][B_o]dvol \tag{14}$$

Similarly, the out-of-plane stability matrix is obtained considering the out-of-plane displacement functions shown in eqn. 15 and membrane bending strain vector in eqn. 16 respectively.

$$\{f_o\} = [N_o]\{\delta_o\} \tag{15}$$

$$\{\varepsilon_{bo}\} = \frac{1}{2}\begin{Bmatrix} (\frac{\partial W}{\partial x})^2 \\ (\frac{\partial W}{\partial x})^2 \\ \frac{\partial W}{\partial x}\frac{\partial W}{\partial y} \end{Bmatrix} \tag{16}$$

$$\{\varepsilon_{bo}\} = \frac{1}{2}\begin{bmatrix} \frac{\partial W}{\partial x} & 0 \\ 0 & \frac{\partial W}{\partial y} \\ \frac{\partial W}{\partial y} & \frac{\partial W}{\partial x} \end{bmatrix}\begin{Bmatrix} \frac{\partial W}{dx} \\ \frac{\partial W}{dy} \end{Bmatrix} \tag{17}$$

$$\{\varepsilon_{bo}\} = \frac{1}{2}\{\delta_o\}^T\begin{bmatrix} \frac{1}{\lambda^2}[N'_{o,\xi}]^T[N'_{o,\xi}] \\ \frac{1}{b^2}[N'_{o,\eta}]^T[N'_{o,\eta}] \\ \frac{1}{b\lambda}\{[N'_{o,\eta}]^T[N'_{o,\xi}] + [N'_{o,\xi}]^T[N'_{o,\eta}]\} \end{bmatrix}\{\delta_o\} \tag{18}$$

The out-of-plane stability matrix is obtained as shown in eqn. 19. Detail of its development is documented in Olawale [2].

$$[S_o] = \int_{vol} [B_{so}]\{\sigma\}^T dvol \tag{19}$$

b) In-Plane:

The in-plane displacement function is given by

The in-plane displacement function is given by

$$\{f_i\} = \{U \;\; V\} = [N_i]\{\delta_i\} \tag{20}$$

$$f_i = \begin{bmatrix} \{X\}^T cos\pi\xi \\ \{Y\}^T sin\pi\xi \end{bmatrix} \{\delta_i\} \tag{21}$$

where

$$\{X\}^T = \{0 \quad C_1 \quad 0 \quad C_2\}$$
$$\{Y\}^T = \{C_1 \quad 0 \quad C_2 \quad 0\} \tag{22}$$

and

$$C_1 = 1 - \eta$$
$$C_2 = \eta \tag{23}$$

$$\{\delta_i\} = \{v_1 \quad u_1 \quad v_2 \quad u_2\} \tag{24}$$

The in-plane strain vector is given by

$$\{\varepsilon_i\} = \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \end{Bmatrix} \tag{25}$$

$$\{\varepsilon_i\} = [B_i]\{\delta_i\} \tag{26}$$

and

$$\{B_i\} = \begin{bmatrix} -\frac{\pi}{\lambda}\{X\}^T sin\pi\xi \\ \frac{1}{b}\{Y'_\eta\}^T sin\pi\xi \\ (\frac{1}{b}\{X'_\eta\}^T + \frac{\pi}{\lambda}\{Y\}^T)\; cos\pi\xi \end{bmatrix} \tag{27}$$

The in-plane stiffness matrix is obtained following the same principle of increment in virtual work and is given by eqn. 28

$$[K_i] = \int_{vol} [B_i]^T [F][B_i] dvol \tag{28}$$

In a similar vein as the out-of-plane stability matrix, the in-plane stability matrix $[S_i]$ is obtained using in-plane bending strain given in eqn. 29.

$$\{\varepsilon_{bi}\} = \frac{1}{2}\begin{Bmatrix} (\frac{\partial U}{\partial x})^2 + (\frac{\partial V}{\partial x})^2 \\ (\frac{\partial U}{\partial y})^2 + (\frac{\partial V}{\partial y})^2 \\ 2(\frac{\partial U}{\partial x}\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y}) \end{Bmatrix} \qquad (29)$$

This can be expressed as

$$\{\varepsilon_{bi}\} = \frac{1}{2}\begin{bmatrix} \frac{\partial U}{\partial x} & \frac{\partial V}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial U}{\partial y} & \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial y} & \frac{\partial V}{\partial y} & \frac{\partial U}{\partial x} & \frac{\partial V}{\partial x} \end{bmatrix} \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \end{Bmatrix} \qquad (30)$$

The in-plane stability matrix is thus obtained following the same principle as out-plane stability matrix as shown in eqn. 31

$$[S_i] = \frac{bt\pi^2}{\lambda}\int_0^1 \sigma_x[\{X\}\,\{X\}^T + \{Y\}\,\{Y\}^T]d\eta \qquad (31)$$

The resulting out-of-plane and in-plane eigenvalue problems are solved using the new algorithm reported by Olawale and Ogunbiyi [1].

$$[[K_o] - [S_o]]\{\sigma_o\} = 0 \qquad (32)$$

Also, in-plane equilibrium is given as

$$[[K_i] - [S_i]]\{\sigma_i\} = \qquad (33)$$

The new solution algorithm used for the computation of the buckling load of column has been reported in Olawale and Ogunbiyi[1] and briefly described here for completeness.

The assembled global matrix is given as [A_G]. This can be decomposed into lower and upper triangular matrices such that [L] $[L]^T$ = [A_G], where

$$[L] = \begin{bmatrix} L_{11} & 0 & 0 \dots\dots\dots\dots\dots 0 \\ L_{21} & L_{22} & 0 \dots\dots\dots\dots\dots 0 \\ \vdots & \vdots & \vdots \qquad\qquad \vdots \\ \vdots & \vdots & \vdots \qquad\qquad \vdots \\ L_{n1} & L_{n1} & \dots\dots\dots\dots\dots\dots L_{nn} \end{bmatrix} \qquad (34)$$

and $[L]^T$ is the transpose of the [L] matrix. The multiplication of the lower and upper triangular matrices [L] and $[L]^T$ yields the global matrix which can be assembled in the form below

$$[A_G] = \begin{bmatrix} a_{11} \dots\dots\dots\dots\dots\dots\dots a_{1n} \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ a_{n1} \dots\dots\dots\dots\dots\dots\dots a_{nn} \end{bmatrix}$$

where

$$L_{11} = (a_{11})^{1/2}$$

$$L_{1j} = a_{1j} \,/\, L_{11}$$

$$L_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} L^2{}_{ki}\right)^{1/2}$$

$$L_{ij} = \left(a_{ij} - \sum L_{ki} L_{kj}\right) / L_{ii}$$

$$i = 2, 3, \dots \dots . . , n$$

$$j = i + 1, i + 2, \dots \dots . . , n$$

$$i > j$$

The determinant of the matrix [A_G] is given by

$$\left|[A_G]\right| = \left|[L][L]^T\right| = \left|[L]\right|\left|[L]^T\right|$$

$$= L_{11}^2 L_{22}^2 L_{33}^3 \dots\dots L_{nn}^2$$

$$= \prod_{L=1}^{n} L_{11}^2 = \prod_{L=1}^{n}\left(a_{ii} - \sum_{k=1}^{i-1} L_{ki}^2\right) i = 2,3,\dots n$$

$$\lambda = \frac{\sigma}{L^*}$$

$$\sigma_{building} = \lambda L^*$$

One of the advantages of the new solution method is that the need for the introduction of sub strips with a strip to achieve accuracy is totally eliminated. This, in effect reduces the bandwidth requirement and the computational effort is greatly reduced.

3. RESULTS AND DISCUSSION

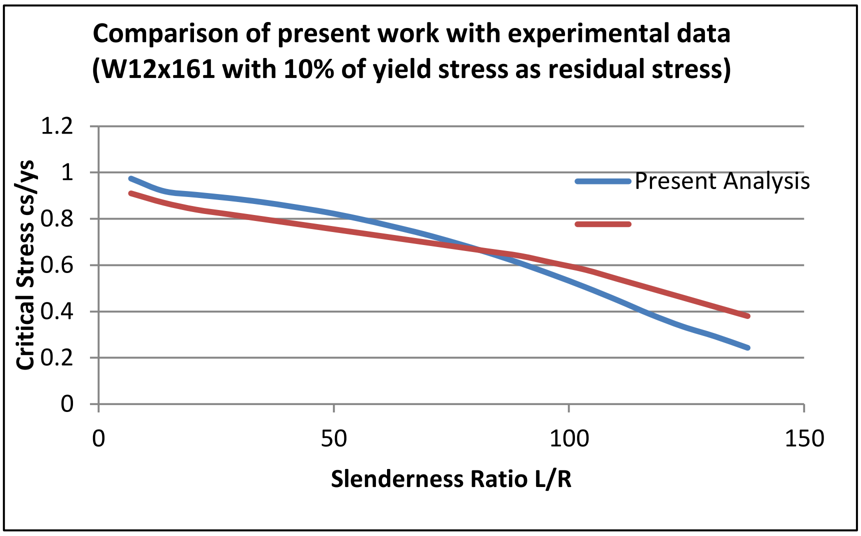

Figure 2 Comparison of the current analysis to European Convection Curve with Residual Stress of 10% Yield Stress.

The development is programmed in Java which is an object oriented programming language and the result of the current work is compared with European Convention curve B3-24 with 10% of yield stress for the residual stress level that is multi-linear distribution. The two computational results compare very closely and it can be said that current work is accurate to an appreciable extent for further parametric study. The plot of this comparison is shown in figure 2.

The behaviour of various rolled steel shapes is studied at various residual stress levels ranging from 10% to 50% yield stress. The residual stress distribution is a multi-linear relationship as shown in figure 3. The variation of the critical stress with the slenderness ratio for different residual stress levels indicates that the critical stress is adversely affected by the presence of residual stress that can be either locked in during rolling process or welding procedure. This effect is clearly shown in figure 4 & 5 and it can be seen that the bigger the shape the more adversely the critical stress is affected. The depreciation in the critical stress for small shape can be seen to be gradual throughout the range of slenderness ratios considered. Another interesting observation is that residual does not affect very stocky columns with slenderness ratio of about 20. It can be seen that only the intermediate and slender columns are adversely affected by the presence of residual stress.

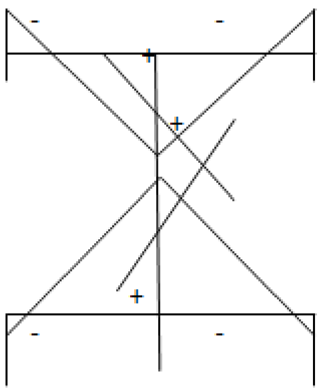

Figure 3 Residual stress distribution

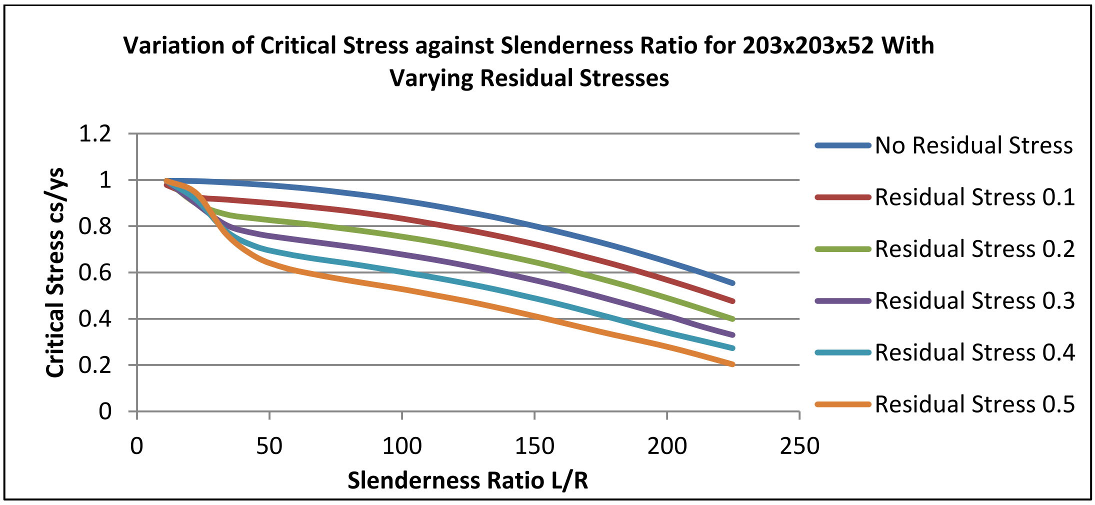

Figure 4 Critical Stress variation with slenderness ratio at various level of residual stress for normal rolled section.

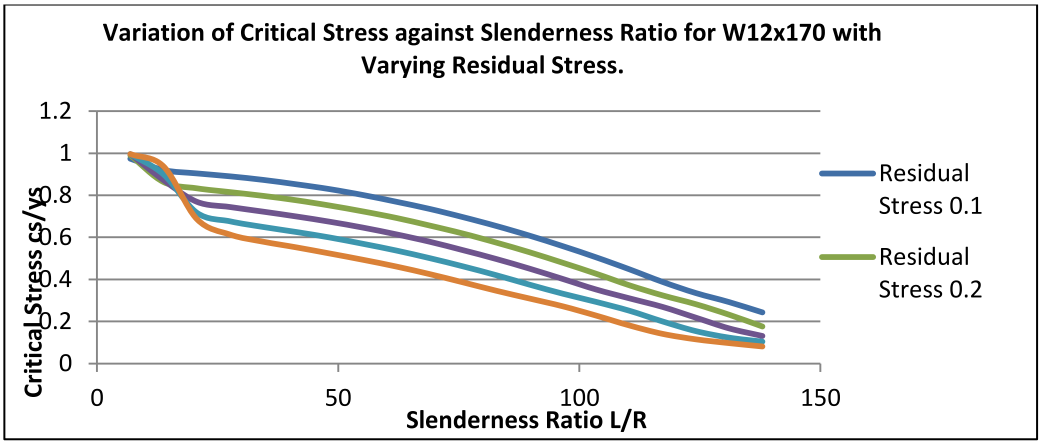

Figure 5 Critical Stress variation with slenderness ratio at various level of residual stress for wide flange rolled section

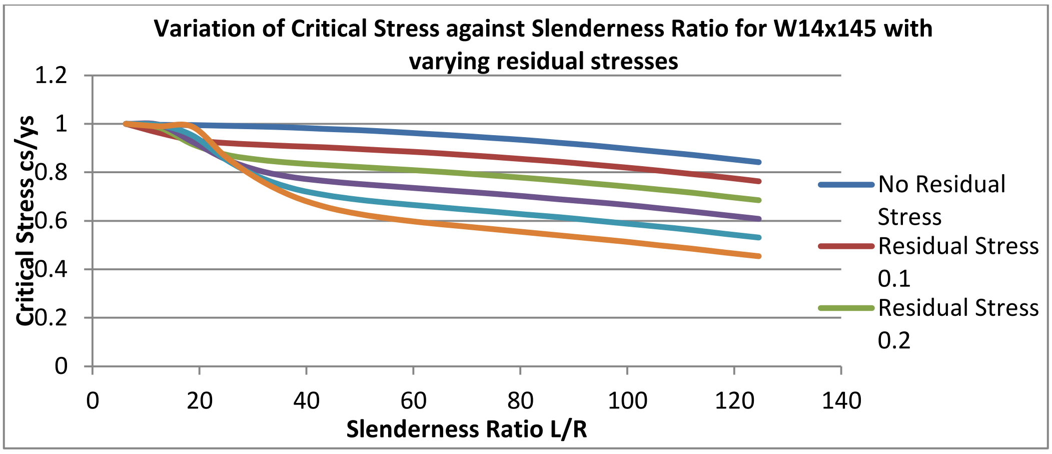

Figure 6 Critical Stress variation with slenderness ratio at various level of residual stress for wide flange rolled section

4. CONCLUSIONS

From this study, the power of new solution algorithm has been demonstrated in predicting the collapsed load of steel columns with or without residual stresses. It is also observed that residual stresses do not affect short columns but the intermediate and slender columns are susceptible to the presence of residual stress. The residual stress can have the capacity of the column if the magnitude of the residual stress is so high if not controlled at the time of production of the steel shapes. Also, the bigger the shape, the more the effect of residual stress on the capacity of columns is pronounced.

REFERENCES

[1] Olawale, S.O., and Ogunbiyi, M.A., (2015). Buckling Analysis of Columns using Decomposed Matrix Approach, Nigerian Institute of Civil Engineers Proceedings, National Development Strategies toward Sustainable Civil Infrastructure, p50-59.

[2] Olawale, S.O., (1988). Collapse Behaviour of Steel Columns in Fire, Ph. D Thesis, University of Sheffield.

[3] Ofuyatan Olatokunbo, Adeola A. Adedeji , Olofinade Rotimi, Simon Olawale "Pseudo-dynamic Earthquake Response Model of Wood-frame With Plastered

typha((*minima*)Bale Masonry-infill" International Conference on Engineering for a Sustainable World" icesw 2017, Department of Mechanical Covenant University, Ota

[4] Ofuyatan Olatokunbo, Adedeji Adeola, Omeje Maxwell, Olawale Simon "Interaction Assessment and Optimal Design of Composite Action of Plastered Typha Strawbale," Advances in Materials Research, an International Journal, Vol. 6 No. 2. (2017) 221-231. DOI: http://doi.org/10.12989/amr.2017.6.2.221

[5] Plank, O., and Wittrick, A., (1974). Buckling under combined loading of thin flat-walled structures by complex finite strip method. International journal of Numerical Method in Engineering.

[6] Rafael, S., and Sridharan S. A., (1985). Interactive buckling analysis with finite strips. International Journal of Numerical Method in Engineering vol. 21, p145-161.

[7] Wittrick, A and Williams, O., (1971). A General Algorithm for Computing Natural Frequencies of Elastic Structures. Quarterly Jour. Mech. Appl. Math. 24.

[8] [8] Williams, O and Wittrick, A., (1969). Computational Procedure for a Matrix Analysis of the Stability and Vibration of Thin Flat-walled Structures in Compression", International Journal of Mechanics and Science vol. 11, p979-998.

[9] Wittrick, A. and Williams O., (1974). Buckling and Vibration of Anisotropic Plate Assemblies under Combined Loadings. Int. Jour. Mech. Sci. vol.16.

[10] Gunjavate P.V., M M Jadhav (2013) Application of finite strip method -A REVIEW International Journal of Advanced Engineering Technology E-ISSN 0976-3945 IJAET/Vol. IV/ Issue II/April-June, /96-97

[11] Schafer B.W., LI. Z. (2010) Application of the finite strip method in cold-formed steel member design. Journal of Constructional Steel Research 66 (2010) 971_980 5.

[12] Eccher,G., Rasmussen K.J.R, Zandonini R. (2008) Elastic buckling analysis of perforated thin-walled structures by the isoparametric spline finite strip method. Thin-Walled Structures 46 (2008) 165–191.