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# A UNIQUE MATHEMATICAL QUEUING MODEL FOR WIRED AND WIRELESS NETWORKS

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## ABSTRACT

*The de-facto protocol for transmitting data in wired and wireless networks is the Transmission Control Protocol/Internet Protocol (TCP/IP). While a lot of modifications have been done to adapt the TCP/IP protocol for wireless networks, a lot remains to be done about the bandwidth underutilization caused by network traffic control actions taken by active queue management controllers currently being implemented on modern routers. The main cause of bandwidth underutilization is uncertainties in network parameters. This is especially true for wireless networks. In this study, two unique mathematical models for queue management in wired and wireless networks are proposed. The models were derived using a recursive, third-order, discrete-time structure. The models are; the Model Predictive Controller (MPC) and the Self-Tuning Regulator (STR). The MPC was modeled to bear uncertainties in gain, poles and delay time. The STR, with an assigned closed-loop pole, was modeled to be very robust to varying network parameters. Theoretically, the proposed models deliver a performance in network traffic control that optimizes the use of available bandwidth and minimizes queue length and packet loss in wired and wireless networks.*

**Keyword:** Active Queue Management, Bandwidth Utilization, Congestion, Internet Protocol, Model Predictive Controller, Self-Tuning Regulator, Transmission Control Protocol, Wired Networks, Wireless Networks.

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## 1. INTRODUCTION

Network traffic control [1] which includes prevention of congestion can be implemented using hardware and software solutions. In fact, a lot of theoretical attempts have been made to model Active Queue Management (AQM) controllers that have not been implemented practically [2]. A lot of variations in the communication network of today include Ad-hoc network, Self-organizing network and diverse forms of modified network topologies. They all make the need to develop more effective and better network traffic /congestion control tools more important than any other time in the history of network communication. Critical network node failures are sometimes the direct result of poor traffic management. This is true especially in wireless networks where there are frequent changes in network topology and very limited channels that must be shared among the teeming amount of wireless access points. The plain truth remains that wireless channels and the need to share them cause traffic control problems. Chief among these problems is reduced Quality of Service (QoS) [3].

The aim of modern AQM schemes tends towards preventing congestion by controlling network traffic to achieve high link utilization, prevent packet losses, minimize queue length and reduce queuing delays. Modeling networks theoretically, helps mainly with network parameter estimation. It is glaring that the way and manner, in which AQM controllers communicate with the TCP/IP, can be modeled using nonlinear differential equations. Congestions cannot be curbed completely since it depends on traffic patterns and not traffic-routing mechanisms. The internet of today, dominated by TCP flows, have AQM controllers that basically trigger the dropping/marketing of packets at the router level before congestion occurs [4].

Your Queue size management by AQM technique should aim to get the available bandwidth better utilized and reduce queuing delays for traffic flows in wired and wireless networks. The usual dynamic model for networks is first built and later modified to incorporate feedback control strategies. The challenge has always been, and still is, how to effectively manage network traffic [5] in a way that it minimizes queue delay time and makes optimal use of all available bandwidth. In short, the ultimate aim of any good AQM controller high link utilization whilst preventing congestion from ever happening in a network, which will help to improve the production output of the industry [6-8].

In this study, two mathematical models for the MPC and the STR are proposed to tackle the problem of bandwidth underutilization brought on by uncertainties in network parameters. A third-order, discrete-time system is assumed and used to develop the models after linearizing the usual non-linear differential equations that can be used to model existing AQM controllers like the Random Early Detector (RED) controller and the Proportional Integral (PI) controller.

## 2. RELATED WORKS

Wu *et al* [11] confirmed the surge in the use of wired and wireless networks and the antecedent rise in traffic that must be managed. The researchers highlighted some of the most recent efforts that have been made to control the voluminous traffic generated by all kinds of

devices trying to access network resources that are limited. The report the researchers gave was split into two parts, and they include, classical control methods and intelligent control theory. The researchers made some allusions to classical congestion control theories like the Reno, Tahoe and Vegas that made exclusive use of feedback control strategies in varying degrees. The main indicators of the control action to take using classical control strategies were the past signals of Round Transmission Time (RTT), Bit Acknowledgement Confirmation (ACK), and window size [9]. Three ways (tools) in which congestion control algorithms can be applied were identified by the write-up as slow start, congestion avoidance and fast retransmission. The researchers also noted that theoretical efforts that have been made to control network traffic stem from transfer functions and matrix control expressions. In the report, it was noted that more recent trends in network congestion control incorporate some form of intelligence that has made the art of controlling network traffic more dynamic. The paper made the following submissions:

The earliest models for congestion control were based on a set of nonlinear differential equations that had roots in hydromechanics. A typical example of this classic dynamic network traffic control model was given as:

$$\begin{cases} \frac{dw(t)}{dt} = \frac{1}{R(t)} - \frac{W(t)W[t-R(t)]}{2R(t)} - P[t-R(t)] \\ \frac{dq(t)}{dt} = \frac{N(t)}{R(t)}W(t) - C(t) \end{cases} \quad (1)$$

Where:

- $W$  – Desired window size
- $q$  – Queue length
- $C(t)$  – Bandwidth capacity of the link
- $P$  – Packet loss ratio
- $N$  – Number of TCP connections
- $R(t)$  – Round-transmission delay

The Proportional Integral (PI), Proportional Derivative (PD) and other classical network traffic control model were based on non-linear differential equations of first and second order.

The concept of network congestion control shifted from the use of non-linear differential equations to modern control theories that address the inability of the former to adequately describe the details of a more practical network congestion traffic controller. The example given was the state expression:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (2)$$

This enables the expression of a lot of independent variables as state vectors such as:

$$x^T(t) = [x_1(t), x_2(t), \dots, x_n(t)] \quad (3)$$

Where:

- $\dot{x}$  – Constitutes a first-order differential equation of the system
- $A$  – The n-order system matrix
- $U^T = [U_1, U_2, \dots, U_r]$  is an r-dimensional input vector
- $Y^T = [Y_1, Y_2, \dots, Y_m]$  is an m-dimensional output vector
- $B$  – The input matrix ( $n \times r$ )

$C$  – The output matrix ( $m \times n$ )

$D$  – The direct transmission matrix ( $m \times r$ )

A lot of modifications were done to the modern control theory based approach to network congestion control. The prominent of them was the inclusion of the probability ( $P$ ) of successful transmission event ( $S_1$ ) and the probability ( $1-P$ ) of a failed transmission event ( $S_2$ ).

--The model for network congestion control eventually graduated to incorporating some elements of fuzzy logic and neural networks. This is the most recent development in the field of network congestion control. Fuzzy logic control (FLC) boasts of being able to make good control decisions even when in the absence of a tangible number of network parameters. Mathematical network congestion control models that use fuzzy logic have strong adaptability. The researchers acknowledged the use of Artificial Neural Networks (ANN) to control network traffic especially in dynamic, unpredictable and superfast networks.

A group of researchers from Sharda University, India, conducted a comparative analysis between clustering, cross layer protocols and clustering for congestion control. The researchers proposed a model for controlling traffic in Mobile Ad hoc Networks (MANETs). The model proposed was a hybrid that was designed for the cluster gateways/heads. Network control decisions are based mainly on the queue size at the head of each MANET cluster [10].

Some group of researchers proposed a modified version of the Random Early Detection (RED) congestion control scheme called the three-section Random Early Detection (RED) based on nonlinear RED. The proposal involved a split of the packet dropping probability into three sections that enables the control scheme to make a better distinction between light, moderate and heavy load network condition[11]. Thus the modified RED Model, called, the Three-section Random Early Detection (TRED) becomes:

$$P_b = \begin{cases} 0, & ave \in [0, min_{th}] \\ 9Max_p \left( \frac{ave - min_{th}}{max_{th} - min_{th}} \right)^3, & ave \in [min_{th}, min_{th} + \Delta] \\ Max_p \left( \frac{ave - min_{th}}{max_{th} - min_{th}} \right), & ave \in [min_{th} + \Delta, min_{th} + 2\Delta] \\ 9Max_p \left( \frac{ave - max_{th}}{max_{th} - min_{th}} \right)^3, & ave \in [min_{th} + 2\Delta, max_{th}] \\ 1, & ave \in [max_{th}, +\infty] \end{cases} \quad (4)$$

The researchers ultimately presented the modified Random Early Detection model (TRED) to address the problem of link underutilization and large delay time. When using the conventional Random Early Detection under low and high traffic conditions.

Kahe *et al* [13] proposed a dynamic queue management model that also has the ability to automatically drop arriving packets when it becomes necessary to do so. The improvement involved the use of a Compensated Proportional Integral and Derivative (PID) controller. The model was designed exclusively for the Transmission Control Protocol/Internet Protocol Networks. A dynamic compensator that can take into account internal dynamics in a network was used to realize the PID active queue management controller. The authors declared that tests carried out on networks using the dynamic model yielded a better result in terms of queuing delay stability and resource utilization. The researchers set out to address the effect of packet drop rate on the flexibility of the queue length[14]. The queue length (dynamic) is given by:

$$\dot{q}(t) = X(t) - C \quad (5)$$

Where;

$X(t)$  – Aggregated traffic rate (packet/second)

$C$  – Link capacity (packets/second)

$q(t)$  – Router’s queue length (packets)

The above equation is the dynamic queue length of a conventional router. The researchers proposed a modified form of the dynamic queue length and gave it as:

$$\dot{q}(t) = X(t)(1 - P(t)) - C \quad (6)$$

Where:

**$P(t)$  – Packet drop probability**

The overall dynamic model proposed by the researchers addressed the issue of dropped/marked packets on core and edge routers that are continuously under high traffic load. The model switched between Explicit Congestion Notification (ECN) mode and packet dropping mode. The model used the former to inform sources of the onset of congestion when traffic is low and the latter when traffic is high. The dynamic model proposed by the researchers was based on the fluid-flow model and was given as:

$$\begin{cases} \dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)}{2} \cdot \frac{W(t-r(t))}{R(t-R(t))} \cdot P(t-R(t)) \\ \dot{q}(t) = N \frac{W(t)}{R(t)} (1 - P(t)) - C \end{cases} \quad (7)$$

Where:

$N$  – Traffic load (number of TCP sessions)

$\dot{W}(t)$  – TCP congestion window size (packets)

$R(t)$  – Round trip time (seconds) =  $q(t)/C + \tau_p$

$\tau_p$  – Propagation delay (seconds)

The active controller in the dynamic model was designed using Pade approximation and expression in terms of its transfer function was given as:

$$G_A(s) = \frac{b_o + b_1 S}{\left(\frac{2N(1-P_o)}{CR_o^2} + S\right) \left(\frac{1}{R_o} + S\right)} \quad (8)$$

Where;

$$b_o = -C^2 / (2N(1 - P_o)) - 4N/R_o^2$$

$$b_1 = C^2 R_o / (4N(1 - P_o)) - 2C / (1 - P_o)$$

$G_A(s)$  – Transfer function

The researchers used this model to create a PID controller that is robust especially in capturing unstable internal dynamics. The proposed model was absolutely self-tuning.

which proposed a robust controller that had two models (options) for traffic control in networks with very large delays. The researchers discovered that a lot of packets are lost due to the huge time delay between acknowledgements received from sent packets and when the onset of congestion is registered [13]. The model proposed, controlled queue length using data got from the bottleneck capacity. The researchers came up with an AQM controller that

was robust to the network parameter variations. The researchers called the model, a Two Degree of Freedom Internal Model Control (TDF-IMC). The transfer functions for the model's interval operation  $\hat{G}_p(s)$ , queue size regulation  $G_c(s)$ , Packet Error Rate (PER) and disturbance rejection rate  $G_d(s)$  were given as:

$$U(s) = \frac{G_c(s)}{1 + G_d(s) \times (G_p(s) - G_p(s)e^{-\theta s})} \times r(s) + \frac{-G_d(s)}{1 + G_d(s) \times (G_p(s) - G_p(s)e^{-\theta s})} \times (G_p(s) \times d_1(s) + d_2(s)) \tag{9}$$

$$Y(s) = \frac{G_p(s) \times G_c(s)}{1 + G_d(s) \times (G_p(s) - G_p^*(s)e^{-\theta s})} \times r(s) + \frac{1 - G_p^*(s)e^{-\theta s}G_d(s)}{1 + G_d(s) \times (G_p(s) - G_p^*(s)e^{-\theta s})} \times (G_p(s) \times d_1(s) + d_2(s)) \tag{10}$$

Where:

$d_1$  – Input disturbance

$d_2$  – Output disturbance

$U(s)$  – First internal model control

$Y(s)$  – Second internal model control

IEEE researchers proposed the use of a Learning-Automata-Like scheme at network gateways to help early detection decisions that would minimize packet loss and average queue-size[14]. The researchers used a Variable-Structure Stochastic Automata that acted on a vector called the Action Probability Vector  $P(t)$ . The Action Probability Vector was given as:

$$P(t) = [P_1(t), \dots, P_r(t)]^T \tag{11}$$

Where:

$P_i(t) (i = 1, \dots, r)$  – The probability that the automation will select an action at time  $t$ .

A group of researchers from IEEE checked and analyzed how a scheduler could be used to allocate bandwidth for the transmission of packets to and from multiple users that are connected to one access point [15]. The researchers discovered that the scheduler did its job of allocating channels to multiple users but based on fairness and latency requirements. The researchers developed a receiver side algorithm to alleviate the perceived unfairness in allocating bandwidth in the transport layer of the Transmission Control Protocol (TCP). The model the researchers proposed was a non-linear differential equation of the form:

$$\frac{dw}{dt} = \frac{1}{d} (\tau - b_q(t)) \tag{12}$$

$$q(t) = \begin{cases} 0, & \text{if } w(t) < \mu_c d \\ w(t) - \mu_c d, & \text{Otherwise} \end{cases} \tag{13}$$

Where:

$\mu_c$  – Service rate (byte/s)

$d$  – Propagation delay

$\tau$  – Traffic intensity (bytes/flow)

$b_q(t)$  – Dimensionless constant.

A Novel autonomous Proportional and Differential RED model (NPD-RED) for network congestion control was proposed by a group of scientist [16]. The proposed model uses a self-

tuning feedback controller that is both proportional and differential. The model considered the instantaneous queue length and the ratio of the most recent differential error signal to the buffer size. The proposed model was expressed by the differential equations:

$$\dot{W}(t) = \frac{2N}{R^2C} W(t) - \frac{RC^2}{2N^2} R(t - R) \quad (14)$$

$$\dot{q}(t) = \frac{NW(t)}{R} - \frac{q(t)}{R} \quad (15)$$

Where

- $W(t)$  – Time-derivative of  $W$
- $q$  – Current queue length (packets)
- $R$  – The round-trip time
- $N$  – Load factor (number of TCP connections)
- $C$  – Link capacity.

Some authors has proposed a novel AQM model based on fuzzy logic to address the issue of transmission control protocol's streaming services [17]. The mathematical analysis for the stability of the proposed fuzzy-logic-based AQM controller yielded the following equations:

$$\emptyset(x_1, x_2) > \frac{\frac{\beta}{a_2 x_2} + \frac{a_1}{a_2} x_1 + \frac{M}{\tau^2}}{\frac{M}{\tau^2} + \frac{(x_2 + C_0)^2}{2M}} \text{ if } x_2 > 0 \quad (16)$$

$$\emptyset(x_1, x_2) < \frac{\frac{\beta}{a_2 x_2} + \frac{a_1}{a_2} x_1 + \frac{M}{\tau^2}}{\frac{M}{\tau^2} + \frac{(x_2 + C_0)^2}{2M}} \text{ if } x_2 < 0 \quad (17)$$

Where

- $\emptyset(x_1, x_2)$  – Fuzzy controller
- $x_1$  – Tracking error
- $x_2$  – Tracking error change
- $\emptyset$  - Dropping probability

The stability analysis was made by replacing the dropping probability,  $P(t)$ , with the fuzzy control variable,  $\emptyset(x_1, x_2)$  to obtain the state space:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{M}{\tau^2} - \left( \frac{M}{\tau^2} + \frac{(x_2 + C_0)^2}{2M} \right) \emptyset(x_1, x_2) \end{cases} \quad (18)$$

Alavi *et al* developed a model that made use of frequency response manipulations via an interactive loop-shaping process to control congestion in networks with unpredictable parameters. The designed model addressed the issue of uncertain round trip delays and uncertain phase information [18]. The stability criterion for the implemented system as:

$$T_L(jw) \leq \left| \frac{G(s)P(s)}{1 + G(s)P(s)e^{-sR_0}} \right|_{s=jw} \leq T_u(jw) \quad (19)$$

Where:

- $T_L$  – Complementary sensitivity
- $T_u$  – Complementary sensitivity

$G(s)$  – Loop function

$P(s)$  – Loop function

$R_o$  – Round trip delay

Feng *et al* proposed an  $H_\infty$  approach to designing AQM based congestion controllers[16]. The model is based on modern control theory and the  $H_\infty$  parameter is used to design the time-delay aspect of the controller. The  $H_\infty$  model was derived from the model system depicted thus:

$$\begin{cases} x(t) = \sum_{i=0}^M A_i x(t - \tau_i(t)) + \sum_{i=0}^M B_i u(t - \tau_i(t)) + DV(t) \\ Z(t) = Hx(t) \\ x(t) = \emptyset \in C^n[-\tau_m, 0], \text{ when } t \in [-\tau_m, 0] \end{cases} \quad (20)$$

Where;

$x(t) \in R^n$  – System rate

$u(t) \in R^n$  – Control input

$V \in R^n$  – Exogeneous disturbance

$Z \in R^n$  – Controlled output

$\tau_i$  - Time delay

$\tau_m$  – The upper-band of the time delay

$R^n$  – N-dimensional Euclidean space

$C^n[a, b]$  – The space of continuous functions over  $[a, b]$  taking values in  $R^n$

### 3. METHODOLOGY

#### 3.1. The Typical TCP/AQM Model Formulation

In [19] the nonlinear differential equation that characterizes congestion in a router that is characterized by:

$$\frac{dW_i(t)}{dt} = \frac{1}{R_i(t)} - \frac{W_i(t)W(t - R_i(t))}{2R_i(t - R_i(t))} p(t - R_i(t)), w \geq 0 \quad (21)$$

$$\frac{dq(t)}{dt} = -C(t) + \sum_{i=1}^N \frac{W_i(t)}{R_i(t)}, q \geq 0 \quad (22)$$

$$R_i(t) = T_{p,i} + \frac{q(t)}{C(t)} \quad (23)$$

For N TCP flows noted as  $i = 1, 2, \dots, N$

$W_i$  – Average window size of each flow (packets)

$q$  – Average queue length (packets)

$R_i$  – Round trip delay (seconds)

$C$  – Link capacity (packets/sec)

$T_{p,i}$  – Propagation delay

$P$  – Packet marking probability



According to [20] the equation derived from [19] can be simplified according to TCP flows with similar round trip time and route as:

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t-R(t))} p(t-R(t)), w \geq 0 \quad (24)$$

$$\dot{q}(t) = -C(t) + \frac{N(t)}{R(t)} W(t), q \geq 0 \quad (25)$$

Where:

$$\dot{q} = q \in [0, \bar{q}] \quad (26)$$

$$W = W \in [0, \bar{W}] \quad (27)$$

The quantities  $q$  (queue length) and  $W$  (window size) are positive and bounded but;

$\bar{q}$  – Buffer size

$\bar{W}$  – Maximum Window Size

$P$  – Marking probability with values: (0, 1) [21]

But equation (21) is not easy to solve because of its nonlinearity. To linearize equation (21) the following was done:

1.  $[W, q]$  were made the states.
2. The input was taken as  $P$ .
3.  $N, C$  and  $R$  were made constants.

Thus,

$$N(t) \equiv N, C(t) \equiv C, R(t) \equiv R_o.$$

The state  $(W, q)$  at equilibrium  $(W_o, q_o, P_o)$  defined as  $[W, q] = (0,0)$

Equation (21) can now be further simplified as:

$$\dot{W} = 0 \rightarrow W_o^2 P_o = 2 \quad (28)$$

$$q = 0 \rightarrow W_o = \frac{R_o C}{N} \quad (29)$$

$$R_o = \frac{q_o}{C} + T_p \quad (30)$$

So that;

$$\delta \dot{W}(t) = -\frac{N}{R_o^2 C} (\delta w(t) + \delta w(t - R_o)) - \frac{R_o C^2}{2N^2} \delta p(t - R_o) \quad (31)$$

$$\delta \dot{q}(t) = \frac{N}{R_o} \delta W(t) - \frac{1}{R_o} \delta q(t) \quad (32)$$

With;

$$\delta W = W - W_o; \delta q = q - q_o \text{ and } \delta P = P - P_o \quad (33)$$

In the frequency domain, equation (9) can be expressed as:

$$\delta W(s) = \frac{1}{s} \left[ -\frac{N}{R_o^2 C} (1 + e^{-sR_o}) \delta W(s) - \frac{R_o C^2}{2N^2} \delta P(s) e^{-sR_o} \right] \quad (34)$$

$$\delta q(s) = \frac{1}{S} \left[ \frac{N}{R_o} \delta W(t) - \frac{1}{R_o} \delta q(t) \right] \quad (35)$$

[19] showed that the delay term in the dynamic window control equation becomes insignificant if:

$$\frac{N}{R_o^2 C} \ll \frac{1}{R_o} \quad (36)$$

But

$$\frac{N}{R_o^2 C} = \frac{1}{W_o R_o} \quad (37)$$

A reasonable assumption according to [19] is to make  $W_o \gg 1$  so that the delay term can be ignored. Thus a simpler way of expressing equation (21) would be:

$$\delta \dot{W}(t) = -\frac{2N}{R_o^2 C} \delta W(t) - \frac{R_o C^2}{2N^2} \delta P(t - R_o) \quad (38)$$

$$\delta q(t) = \frac{N}{R_o} \delta W(t) - \frac{1}{R_o} \delta q(t) \quad (39)$$

The transfer function derived from the ratio of the delayed marking probability,  $(t - R_o)$ , to the window size,  $\delta W$ , is given by;

$$P_{tcp}(S) = \frac{\frac{R_o C^2}{2N^2}}{S + \frac{2N}{R_o^2 C}} \quad (40)$$

The transfer function derived from the ratio of the window size,  $\delta W$  to the queue length,  $\delta q$  is given by:

$$P_{queue}(S) = \frac{\frac{N}{R_o}}{S + \frac{1}{R_o}} \quad (41)$$

The time delay in the delayed marking probability expression can be written in the frequency domain as  $e^{-sR_o}$ . Thus, the combined (nominal) transfer function for the nonlinear differential equation shown in equation (1) can be written as:

$$P(S) = \frac{\frac{C^2}{2N} e^{-sR_o}}{\left(S + \frac{1}{R_o}\right) \left(S + \frac{2N}{R_o^2 C}\right)} \quad (42)$$

Where:

$P(s)$  – Nominal Transfer Function.

From [18] and with the block diagram manipulation discussed therein, the uncertainties in equation (42) can be expressed as:

$$\Delta(S) = \frac{2N^2 S}{R_o^2 C^3} (1 - e^{-sR_o}) \quad (43)$$

But in wireless networks, packet losses can arise from link failures and according to [16] congestion control models can be modified to take this peculiarity into account.

Normally the window size dynamic for wired networks is given by the equation:

$$\frac{dW_i(t)}{dt} = \frac{1}{R_i(t)} - \frac{W_i(t)W_i(t - R_i(t))}{2R_i(t - R_i(t))} \cdot P(t - R_i(t)) \quad (44)$$

In equation (44) above, the marking probability is fed back to the source (downlink communication) [16]. Thus, the chance of the marking probability getting lost during downlink communication is given as;

$$P_{dl,i}, \text{ where } i = 1, \dots, N$$

When a loss does occur, the window size is reduced by one (1). This is possible because the convention is; the source should use the most recent past packet marking probability to reduce the window size by one (1). Equation (21) now changes to:

$$\begin{aligned} \frac{dW_i(t)}{dt} = & \frac{1}{R_i(t)} - (1 - P_{dl,i}(t)) \frac{W_i(t)W_i(t - R_i(t))}{2R_i(t - R_i(t))} P(t - R_i(t)) \\ & - P_{dl,i}(t) \frac{(W_i(t) - 1)W_i(t - R_i(t))}{R_i(t - R_i(t))} \cdot P(t - R_{ah,i}(t)) \end{aligned} \quad (45)$$

Where;

$R_{ah,i}$  – The difference between the current time and the last marking probability received time.

But

$$R_{ah,i}(t) = VR_i(t)$$

Where;

$V$  – An integer that is greater than or equal to two ( $V \geq 2$ ).

The modified queue dynamics for wireless networks can be expressed as:

$$\frac{dq(t)}{dt} = -C(t) + \sum_{i=1}^N \frac{W_i(t)}{R_i(t)} (1 - P_{ul,i}(t)) \quad (46)$$

Where;

$P_{ul,i}$  – The uplink channel loss probability.

Equations 45 and 46 completely describe the dynamics of traffic flow in wireless networks. The equations are sometimes called  $N+1$  dynamic equation. The statistical property of fading on all wireless channels is assumed to be the same. Thus equations 45 and 46 can be written as:

$$\begin{aligned} \dot{W}(t) = & \frac{1}{R(t)} - (1 - P_{dl}(t)) \frac{W(t)W(t - R(t))}{2R(t - R(t))} P(t - R(t)) \\ & - P_{dl}(t) \frac{(W(t) - 1)W(t - R(t))}{R(t - R(t))} \cdot P(t - R_{ah}(t)); w \geq 0 \end{aligned} \quad (47)$$

$$\dot{q}(t) = -C(t) + (1 - P_{ul}(t)) \frac{N(t)}{R(t)} W(t), q \geq 0 \quad (48)$$

To linearize the wireless TCP Model, the variable link bandwidth that is associated with wireless networks is taken as a disturbance [20].

When:

1.  $R$ ,  $N$ ,  $P_{dl}$  and  $P_{ul}$  are made constants.
2. The nominal value,  $C_o$  of  $C(t)$  is known.
3. The variable link bandwidth represented by  $\delta(t) = C(t) - C_o$  is taken into account; equations 47 and 48 become linearized and can be expressed as:

$$dW(t) = -\frac{(1 + P_{dl})W_o P_o}{2R_o} \delta W(t) - \frac{(W_o + P_{dl}W_o - 2P_{dl})P_o}{2R_o} \delta W(t - R_o) - \frac{1}{R_o C_o} [\delta q(t) - \delta q(t - R_o)] - \frac{(1 - P_{dl})W_o^2}{2R_o} \delta P(t - R_o) - \frac{P_{dl}(W_o - 1)W_o}{R_o} \delta P(t - R_{aho}) + \frac{q_o}{R_o C_o^2} [\delta C(t) - \delta C(t - R_o)] \quad (49)$$

$$\delta \dot{q}(t) = \frac{(1 - P_{ul})N}{R_o} \delta W(t) - \frac{1}{R_o} \delta q(t) - \frac{T_p}{R_o} \delta C(t) \quad (50)$$

$$R_o = \frac{q_o}{C_o} + T_p \quad (51)$$

$$W_o = \frac{R_o C_o}{N(1 - P_{ul})} \quad (52)$$

$$P_o = \frac{2}{(1 + P_{dl})W_o^2 - 2P_{dl}W_o} \quad (53)$$

### 3.2. Mathematical Analysis of Proposed Model

To be able to cope with the time-varying nature of network parameters, this study proposes the use of adaptive control strategies. Two adaptive controllers are modeled:

1. A Model Predictive Controller (MPC) with online model estimation.
2. A Self-Tuning Regulator (STR) based on closed-loop pole assignment.

### 3.3. The Recursive Least Squares (RLS) Network Parameter Estimator

The Recursive Least Squares (RLS) technique is used in both models for parameter estimation. The parameter estimation technique is used recursively. Thus, the result obtained at time instant  $(K-1)$  is used to get the result at the current time,  $K$  [26-31].

If  $\hat{\theta}(k-1)$  represents the least-squares estimates got from  $k-1$  measurements, then the Hessian matrix  $\Phi^T \Phi$  will be non-singular for all values of  $K$ .

If  $P(K)$  is used to represent the inverse Hessian at time  $K$ , its mathematical expression is written as:

$$P^{-1}(k) = \Phi^T(k)\Phi(k) \quad (54)$$

But

$$\Phi(k) = \begin{bmatrix} \Phi(K-1) \\ \Phi(k) \end{bmatrix} \quad (55)$$

$$\Phi^T[k]\Phi[k] = \Phi^T(k-1)\Phi(K-1) + \Phi^T \Phi(k) \quad (56)$$

This makes  $P^{-1}(k)$  to become:

$$P^{-1}(k) = P^{-1}(k-1) + \Phi^T(k)\Phi(k) \quad (57)$$

The least-squares estimate can be written mathematically as:

$$\hat{\theta}(k) = [\Phi^T(k)\Phi(K)^{-1}\Phi^T(k)y(k)] = P(k)\Phi^T(k)y(k) \quad (58)$$

In another form,

$$\hat{\theta}(K) = P(k)[\phi(k-1)y(k-1) + \phi(k)y(k)] \quad (59)$$

When equation (22) and (23) are combined;

$$\phi^T(k)y(k) = P^{-1}(k-1)\hat{\theta}(k-1) = P^{-1}(k)\hat{\theta}(k-1) - \phi(k)\phi^T(k)\hat{\theta}(k-1) \quad (60)$$

The recursive least-squares can then be expressed as:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\phi(k)[y(k) - \phi^T(k)\hat{\theta}(k-1)] = \hat{\theta}(k-1) + K(k) \in(k) \quad (61)$$

But;

$$K(k) = P(k)\phi(k) \quad (62)$$

$$\in(k) = y(k) - \phi(k)\hat{\theta}(k-1) \quad (63)$$

Where;

$y(k)$  – Signal

$\in(k)$  – The residual (error in predicting the signal  $y(k)$ )

$\hat{\theta}(k-1)$  – Recursive least-squares estimate

Using the small matrix inversion lemma, the Hessian function  $P(k)$  can be derived recursively as:

$$P(k) = P(k-1) \left[ 1 - \frac{1}{1 + \phi^T(k)P(k-1)\phi(k)} \phi(k)\phi^T(k)P(k-1) \right] \quad (64)$$

To get and take into cognizance the time-varying nature of the network parameters, the least-squares estimate can be rewritten as:

$$V(\theta, k) = \frac{1}{2} \sum_{i=1}^k \lambda^{k-i} (y(i) - \phi^T(i)\theta)^2 \quad (65)$$

Where:

$\lambda$  – The forgetting factor with values between  $0 < \lambda < 1$

The recursive least square algorithm is therefore written mathematically as:

$$\theta(k) = \theta(k-1) + k(k) \in(k) \quad (66)$$

$$K(k) = \frac{P(k-1)\phi(k)}{\lambda + \phi^T P(k-1)\phi_k} \quad (67)$$

$$P(k) = \frac{1}{\lambda} (1 - K(k)\phi^T)P(k-1) \quad (68)$$

### 3.4. The Model Predictive Controller with Online Model Estimator

The MPC makes explicit use of system models to decide on what action to take at any given time to control network traffic. It has a very wide range of control actions it can make. All control actions taken by the MPC always comes at a cost function although the controller is designed to take the control decision that best minimizes the cost function [32-34].

The method adopted in modeling the MPC involves:

1. The possible outcomes for any control decision that is made (prediction horizon). The future outputs,  $y(k+i/k)$  for  $i = 1, \dots, N_p$  are predicted for each sample,  $k$  taken and it depends on past values of the input and output signals and also on future signals,  $U(k+i/k)$ .

In which case,  $i = 0, \dots, N_p - 1$ . All these parameters are computed when a control decision has to be made.

2. All control decisions are kept as close as possible to the reference trajectory  $W(k + i)$ .
3. Only the first control move  $U(k/K)$  is used. All other possible control moves are rejected at the instant the next sample  $y(k + 1)$  is taken, the entire process from step one to three is repeated.

### 3.4.1. Basic Elements of the MPC

a. *The Process Model:* all MPCs are designed to be able to predict the best possible control action whilst capturing all the dynamics of typical high-speed network traffic.

b. *Objective Function:* all MPCs follow a certain control law to determine the future output,  $y$ . The control decision made follows a reference signal ( $W$ ) over  $N_p$  – the Prediction Horizon. All control decisions or effort,  $\Delta U$  is always penalized. Generally, the objective function is expressed as:

$$J(N_1, N_p, N_u) = \sum_{i=N_1}^{N_p} \delta(i)[y(k + i/k) - W(K + i)]^2 + \sum_{i=1}^{N_u} \lambda(i)[\Delta u(k + i - 1)]^2 \quad (69)$$

Where;

$N_1$  – The Minimum horizon

$N_p$  – The endpoint of the prediction horizon

$N_u$  – The control horizon

$\delta(i)$  – The weighting factor that penalizes tracking errors

$\lambda(i)$  – The weighting factor that penalizes control moves

A lot of constraints can be included in the objective function to make it more complex. Some of these constraints are slew rate limits, amplitude limits and output signal limits. These limits can be expressed mathematically as:

$$\begin{aligned} U_{min} &\leq U(k) \leq U_{max} \forall t \\ \delta U_{min} &\leq U(k) - U(k - 1) \leq \delta U_{max} \forall t \\ y_{min} &\leq y(k) \leq y_{max} \forall t \end{aligned}$$

$U_{min}$  – Least control effort

$U(K)$  – Control effort when sample  $k$  is taken

$U_{max}$  – Maximum control effort

$y(k)$  – Output when sample  $k$  is taken

$t$  – Time

$y_{min}$  – Minimum output

$y_{max}$  – Maximum output

$U(k - 1)$  – Previous control effort

$k$  – Sample instant

c. *Control Law Computation:* The aim of computing the control law is to minimize the objective function. This is achieved by predicting the output  $\hat{y}(k + i/k)$  based on past input and output values along with future control actions. This is done with the goal of minimizing the cost function since there will be a penalty for any control action eventually taken. Linear

models without constraints derive their control law analytically but with constraints, the control law is derived using iterative techniques.

### 3.5. The Model Predictive Controller Design

1) A recursive algorithm is used along with a third order discrete-time structure to design the unique MPC. The uniqueness of the proposed mathematical model makes the controller robust to uncertainties in network parameters. The model represented by the state-space equations:

$$\begin{aligned} x_{k+1} &= Ax_k + BU_k + W_k \\ y_k &= Cx_k + DU_k + V_k \end{aligned} \quad (70)$$

Where;

$$y = \delta q; U = \delta P$$

Through recursive least squares, the state matrices  $A, B, C$  and  $D$  are computed thus:

$$A = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \hat{a}_{13} \\ \hat{a}_{21} & \hat{a}_{22} & \hat{a}_{23} \\ \hat{a}_{31} & \hat{a}_{32} & \hat{a}_{33} \end{bmatrix}, \quad B = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix}, \quad C = [\hat{c}_1 \quad \hat{c}_2 \quad \hat{c}_3], \quad D = \hat{d}$$

Assuming the State noise,  $W_k$  and Measurement noise,  $V_k$  to be Gaussian with covariance  $V$ , and zero mean,  $W$ .

$$\begin{bmatrix} W_k \\ V_k \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} W & Z \\ Z^T & V \end{bmatrix} \right)$$

Where;

$Z$  – Cross-covariance

The observer represented as [23-25];

$$\begin{aligned} \hat{x}_k + 1/k &= A\hat{x}_{k/k-1} + BU_k + k(y_k - \hat{y}_{k/k-1}) \\ \hat{y}_{k/k-1} &= C\hat{x}_{k/k-1} + DU_k \\ k &= (APC^T + Z)(CPC^T + V)^{-1} \\ P &= W + APA^T - (APC^T + Z)(CPC^T + V)^{-1}(Z^T + CPA^T) \end{aligned}$$

The estimates from  $k + 2$  to  $k + N_p$  are:

$$\begin{aligned} \hat{x}_{k+i+1/k} &= A\hat{x}_{k+1/k} + BU_k + U_k \\ \hat{y}_{k+i/k} &= C\hat{x}_{k+i/k} + DU_{k+i/k} \end{aligned}$$

Where;

$U_{k+i/k}$  – The input used for prediction purpose

$$\begin{aligned} \hat{x}_{k+2/k} &= A\hat{x}_{k+1/k} + BU_{k+1/k} \\ \hat{x}_{k+3/k} &= A\hat{x}_{k+2/k} + BU_{k+2/k} \\ &= A(A\hat{x}_{k+1/k} + BU_{k+1/k}) + BU_{k+2/k} \\ &= A^2\hat{x}_{k+1/k} + ABU_{k+1/k} + BU_{k+2/k} \\ &\vdots \\ &\vdots \end{aligned}$$

$$\begin{aligned} \hat{x}_{k+j/k} &= A^{j-1}\hat{x}_{k+1/k} + \sum_{i=1}^{j-1} A^{j-i-1}B_{k+1/k} \\ \hat{y}_{k+1/k} &= C\hat{x}_{k+1/k} + DU_{k+1/k} \\ \hat{y}_{k+2/k} &= C\hat{x}_{k+2/k} + DU_{k+2/k} \\ &= C(A\hat{x}_{k+1/k} + BU_{k+1/k}) + DU_{k+2/k} \\ &\vdots \\ &\vdots \\ \hat{y}_{k+j/k} &= CA^{j-1}\hat{x}_{k+1/k} + C\left(\sum_{i=1}^{j-1} A^{j-i-1}B_{k+1/k}\right) + DU_{k+j/k} \end{aligned}$$

Let

$$y_k = \begin{bmatrix} y_{k+1} \\ \vdots \\ \vdots \\ \vdots \\ y_{k+N} \end{bmatrix}, U_k = \begin{bmatrix} U_{k+1} \\ \vdots \\ \vdots \\ \vdots \\ U_{k+N} \end{bmatrix}$$

And

$$\Lambda = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{N_p-1} \end{bmatrix}, \Phi = \begin{bmatrix} D & 0 & 0 & \cdots & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 & \cdots & \vdots \\ \vdots & \vdots & D & \cdots & D & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ CA^{N_p-1} & BCA^{N_p-2}B & \cdots & \cdots & \cdots & \cdots & 0 \\ & & & & & CB & D \end{bmatrix}$$

Thus;

$$y_k = \Lambda \hat{x}_k + \Phi U_k$$

The cost function of the MPC model is:

$$J(\hat{x}_{k+1/k}, U_k) = \frac{1}{2} \sum_{i=1}^{N_p} [Q(\hat{y}_{k+i/k} - r_{k+i})^2 + S(U_{k+i/k} - U_{k+i-1/k})^2]$$

Where;

$Q, S$  – The MPC weights (scalars)

$$\begin{aligned} R_t &= \begin{bmatrix} r_{k+1} \\ \vdots \\ \vdots \\ \vdots \\ r_{k+N_p} \end{bmatrix} \\ &\therefore \frac{1}{2} \sum_{i=1}^{N_p} Q(\hat{y}_{k+i/k} - r_{k+i})^2 \\ &= \frac{1}{2} Q(y_t - R_t)^T (y_t - R_t) \\ &= \frac{1}{2} U_t^T \Phi^T \bar{Q} \Phi U_t + U_t^T [\Phi^T \bar{Q} \Lambda \hat{x}_{k+1/k} - \Phi^T \bar{Q} R_t] + C_1 \end{aligned}$$



Where;

$C_1$  – A constant

$$\bar{Q} = \begin{bmatrix} Q & & & \\ & Q & & \\ & & \ddots & \\ & & & Q \end{bmatrix}$$

$$\therefore \frac{1}{2} \sum_{i=1}^{N_p} \|U_{k+i/k} - U_{k+i-1/k}\|_S^2$$

$$= \frac{1}{2} U_t^T \bar{S} U_t - U_{k+\frac{1}{k}}^T S U_k + C_2$$

Where;

$C_2$  – A Constant

$$\bar{S} = \begin{bmatrix} 2S & -S & & & & \\ -S & 2S & -S & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & -S & 2S & -S \\ & & & & -S & S & \end{bmatrix}$$

Combining the scalar weights and cost function:

$$J(\hat{x}_{k+1/k}, U_k) = \frac{1}{2} U_t^T H U_t + U_t^T f + C_3$$

Where;

$C_3$  – A constant

$$H = \Phi^T \bar{Q} \Phi + \bar{S}$$

But

$$f = \Gamma \begin{bmatrix} \hat{x}_{k+1/k} \\ R_t \end{bmatrix} - \begin{bmatrix} S U_k \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

$$\Gamma = [\Phi^T \bar{Q} \Lambda - \Phi^T \bar{Q}] \quad (71)$$

## 2) Possible Input Constraints:

The packet marking probability defined in  $[1,0]$

$$\therefore 0 \leq P \leq 1$$

$$\rightarrow 0 \leq P_o + \delta P \leq 1$$

$$\rightarrow -P_o \leq \delta P \leq 1 - P_o$$

Implemented in the algorithm for the MPC it appears as:

$$\begin{bmatrix} I \\ -I \end{bmatrix} U_t \leq \begin{bmatrix} 1 - P_o \\ -P_o \end{bmatrix}$$

### 3) Robustness and Stability Issues

It is usually difficult to obtain a stable MPC design especially when the horizon is finite and there are control constraints [35]. The determinants of the stability of the MPC are the poles of the state observer and regulator. The poles of the state observers can be adjusted using network parameters while the regulator can be adjusted using horizons and weightings.

The determinants of robustness to network uncertainties are the time constant,  $(\tau)$ , the continuous-time process,  $a = e^{-\frac{T}{\tau}}$ , and the functional delay coefficient,  $m = \frac{b_0}{b_0+b_1}$ . The uncertainty limits obtained for a delay of,  $0 \leq d \leq 10$ , was,  $(0.5 < a < 0.98, 0 \leq m \leq 1)$ .

The proposed model allows for the following uncertainties:

- Uncertainties at the pole of more than  $\mp 20\%$  for a wide working zone ( $a < 75$ ) and normal delay values.
- Uncertainties in the gain ( $G$ ) and the gain of the process,  $(\gamma G)$ . The values of  $\gamma$  are allowed to fluctuate between 0.5 and 1.5.
- Uncertainties in the fractional delay coefficient,  $(m)$  of up to 300% .
- Uncertainties caused by dominant poles  $(k \times a)$  that have not been modeled with values of  $k$  close to one(1).
- Uncertainties caused by delay errors of up to two (2) units through the entire pole range.

### 3.6. The Self-Tuning Regulator (STR)

This controller uses specific data measurements to adapt the behaviour of itself per time. The proposed model for this controller is based on closed-loop pole assignment. The general self-tuning regulator control law is given by:

$$f(q)U(k) = -g(q)y(k) + h(q)w(k) \tag{72}$$

Where;

$y(k)$  – Measured output at time instant  $k$

$w$  – The reference trajectory

$f(q), g(q), h(q)$  – Controller polynomials

$$f(q) = 1 + F_1q^{-1} + F_2q^{-2} + \dots + F_{nf}q^{-nf}$$

$$g(q) = g_0 + g_1q^{-1} + g_2q^{-2} + \dots + F_{ng}q^{-ng}$$

$$h(q) = h_0$$

The discrete-time system can thus be represented as:

$$a(q)y(k) = q^{-d}b(q)u(k) + e(k) \tag{73}$$

Where;

$d$  – Delay

$a(q), b(q)$  – The system polynomials

$$a(q) = 1 + a_1q^{-1} + a_2q^{-2} + \dots + a_{na}q^{-na}$$

$$b(q) = b_0 + b_1q^{-1} + b_2q^{-2} + \dots + b_{nb}q^{-nb}$$

When the polynomial,  $t(q) = 1 - t_1q^{-1}$  is defined and the values of  $f(q), g(q)$  and  $h_o$  that satisfy the equations below is found:

$$f(q)g(q) + q^{-d}g(q)b(q) = t(q) \quad (74)$$

$$h_o = \left[ \frac{t(q)}{b(q)} \right]_{q=1}$$

$$ng = na - 1; nf = nb$$

Then the closed-loop system can be made a first order system with pole,  $t_1$ . The foundational model chosen for STR is similar to the one by the MPC. The STR is designed using a linearized third-order model. This is done to enable the design of a discrete-time system that is robust to changes in delay and network parameters.

If the Round trip time,  $R_o$ , is chosen as the sampling time and the discrete-time delay,  $d$  is made one (1) then;

$$a(q) = 1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3}$$

$$b(q) = b_o + b_1q^{-1} + b_2q^{-2} + b_3q^{-3}$$

Recall;

$$nf = nb = 3; ng = na - 1 = 3 - 1 = 2$$

So,

$$f(q) = 1 + F_1q^{-1} + F_2q^{-2} + F_3q^{-3}$$

$$g(q) = g_o + g_1q^{-1} + g_2q^{-2}$$

$$h(q) = h_o = \left[ \frac{t(q)}{b(q)} \right]_{q=1} = \frac{1 - t_1}{b_o + b_1 + b_2 + b_3}$$

The control parameters;  $[F_1, F_2, F_3, g_o, g_1, g_2]^T$ , are solved using the equation (74) in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & b_o & 0 & 0 \\ a_1 & 1 & 0 & b_1 & b_o & 0 \\ a_2 & a_1 & 1 & b_2 & b_1 & b_o \\ a_3 & a_2 & a_1 & b_3 & b_2 & b_1 \\ 0 & a_3 & a_2 & 0 & b_3 & b_2 \\ 0 & 0 & a_3 & 0 & 0 & b_3 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ g_o \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} -t_1 - a_1 \\ -a_2 \\ -a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expressed another way,

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ g_o \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & b_o & 0 & 0 \\ a_1 & 1 & 0 & b_1 & b_o & 0 \\ a_2 & a_1 & 1 & b_2 & b_1 & b_o \\ a_3 & a_2 & a_1 & b_3 & b_2 & b_1 \\ 0 & a_3 & a_2 & 0 & b_3 & b_2 \\ 0 & 0 & a_3 & 0 & 0 & b_3 \end{bmatrix}^{-1} \begin{bmatrix} -t_1 - a_1 \\ -a_2 \\ -a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The STR's control action is calculated as:

$$F(q)U(k) = -g(q)y(k) + h_o w(k)$$

When linearized:

$$y(k) = \delta q(k); U(k) = \delta P(k) \quad (75)$$

Where;

$q$  – The queue length

$P$  – The drop probability

When the reference queue length is set as  $q_o$  and the reference trajectory  $w$  is set as zero:

$$(1 + F_1q^{-1} + F_2q^{-2} + F_3q^{-3})U(t) = -(g_o + g_1q^{-1} + g_2q^{-2})y(t) \quad (76)$$

$$U(t) = -F_1U(t-1) - F_2U(t-2) - F_3U(t-3) - g_o y(t) - g_1 y(t-1) - g_2 y(t-3) \quad (77)$$

#### 4. CONCLUSION

Bandwidth underutilization occurs due to network traffic control actions taken by more conventional Active Queue Management (AQM) controllers. Underutilization of bandwidth in TCP/IP networks is also caused by the unpredictable nature of delay times and network parameters. It is made worse by penalties that AQM controllers must pay for every network traffic control action taken. The modeled MPC can cope very well with time-varying delays and network parameter uncertainties. It can also manage all types of constraints on buffer size and input saturation. The MPC model proposed in this study is inherently predictive and has been modeled with various constraints integrated into it for optimal performance. The problem of time-delay in computing and deciding on the best control action has been solved, by the recent crop of high speed and top performing hardware used in the communication industry. Thus, the MPC model will not have any issue with time-delays caused by the computation of the best or optimal control action. Closed-loop pole assignment was used in the proposed STR model. The closed-loop pole determines the stability of the controller, and, like all basic control system designs in the Z-plane, the proposed STR was designed to have its closed-loop pole within the unit circle. The MPC and STR models proposed in this study, have theoretically being able to take into account the problematic uncertainties in delay time and network parameters, that make conventional AQM controllers, underutilize network bandwidths in TCP/IP networks, especially in wireless scenarios. The goal of this study was to propose mathematical models for congestion control using AQM techniques. The proposals, while being theoretical, aim to deliver an AQM controller that would minimize queue delay time, optimize bandwidth utilization while being robust to variations in network parameters. In this study, mathematical models for two controllers, namely, the Model Predictive Controller (MPC) with online predictive estimation and the Self-Tuning Regulator with closed-loop pole assignment were proposed. The models were derived, to tackle the daunting problem of bandwidth underutilization.

#### FUTURE WORK

The proposed mathematical models would be simulated using NS3<sup>®</sup> and OPNET<sup>®</sup> to visually appraise their performance.

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