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## Lightning Threat Forecast Simulation Using the Schrodinger-Electrostatic Algorithm

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### Abstract

Many models have been propounded for forecasting lightning. Though majority of the model had shown accuracy, the response time in detecting natural phenomenon is quite low. In this model, we used the mathematical experimentation of the micro scale plasmas to develop the macro scale atmospheric plasma which we believe is a major influence of lightning. The Schrödinger-electrostatic algorithm was propounded to further increase both the accuracy and alacrity of detecting natural phenomena. According to our theoretical experimentation, the air density plays a major role in lightning forecast. Our guess was verified using the Davis Weather Station to track the air density both in the upper and lower atmosphere. The air density in the upper atmosphere showed prospect as a vital factor for lightning forecast.

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### 1. Introduction

Lightning is a large-scale electrostatic concept which emanates from the global atmospheric electrical circuit (GAPC). The GAPC sets-up an electric field of opposite polarity between the earth and the atmosphere (air) with the intent of creating an electric energy. The electric field gradient generated by the GAPC continues up into the atmosphere to a point where the voltage reaches its maximum ( $V_{\max}$ ). Most time at  $V_{\max}$ ,

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thunderstorm via a release of energy stored in the storm cloud in form of lightning bolt. Modeling the dynamics of lightning became paramount to further understand salient phenomenon of the GAPC and accurately forecast likely occurrences. The most researched phenomenon is the Laplace growth phenomenon (Takashi *et al.*, 1989; Boris *et al.*, 1989; Takashi *et al.*, 1990; Duran *et al.*, 2010). The Laplace growth phenomenon results from physical mechanism that drives pattern formation in many disparate natural phenomena. Other models i.e. Dielectric Breakdown Model (Kim *et al.*, 2007) and Diffusion Limited Aggregation model (Fotheringham *et al.*, 1989) have also been used to detect natural phenomenon. Models on the plasma effects on lightning forecast have not been discussed at length in recent times (Holzworth *et al.*, 2011). Basically, plasmas can be initiated from the sun or from the atmosphere when gases are heated-up at a very high temperature. In a simple term, the thermally excited gas molecules are ionized into another state of matter known as plasma. The plasma can either be positively or negatively charged particle whose movement through the lightning channel constitutes the electrical current.

In this paper, we compared a micro-scale plasma generation (plasma in a cylindrical tube discharge) to a macro-scale plasma generation (global atmospheric electrical circuit). The focus of the research was the cloud to ground lightning with specific emphasis on the plasma spin and collision within particulates within the atmosphere. We introduce a new simulation model which is based on charged particle spin model. We introduce the Schrödinger-Electrostatic algorithm which is faster in detecting natural phenomenon.

## 2. The Schrödinger-Electrostatic Algorithm

In the micro-scale (plasma in a cylindrical tube discharge), the influence of pressure, density and temperature on plasma characteristics was investigated via the Langmuir probe (LP) technique (Araghi *et al.*, 2013). We propose that the same effect (as shown in Figure 1) could be seen in a macro-scale (global atmospheric electrical circuit), therefore we accounted for the atmospheric charges mainly from the hot plasma gas in the ionosphere. The charged particles spin at specific precessions. We propose that the nature of excited charge spin initiates the preliminary of lightning. Also, we propose that the types of plasma formation, plasma-spins and directional atmospheric-molecular strikes are responsible for different types of lightning witnessed in the world

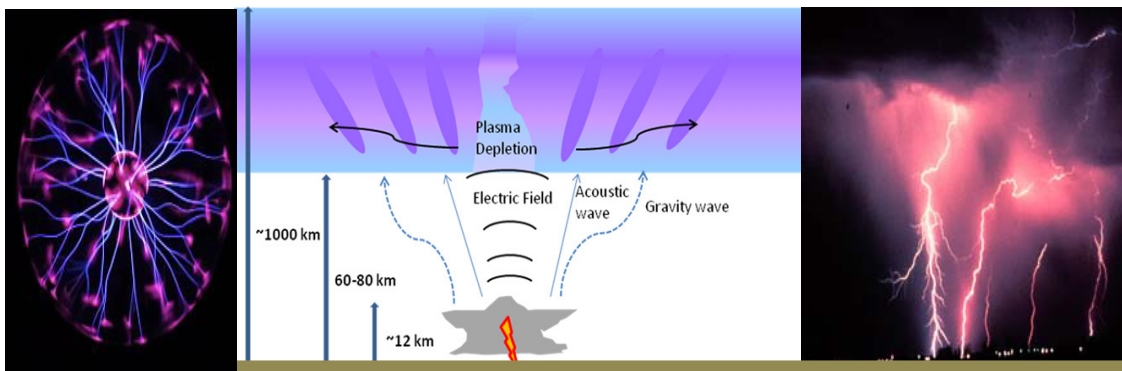


Fig 1. Prototype of micro-scale plasma interaction- expressed in a macro-scale global atmospheric electrical circuit. Adapted from <http://www.technology.org>

This idea was mathematically experimented by adopting the time- independent Schrödinger equation used to account for charged particle spins originates from

$$i\hbar \frac{\partial}{\partial t} \psi - \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = 0 \tag{1}$$

The mathematical experimentation (as shown in the appendix) yielded the governing equation shown below

$$2 \left| E_o e \left( \frac{a^2}{x} - x \right) \right| E_r E_o e \left( \frac{a^2}{x^2} - 1 \right) = \frac{j\beta}{8\pi} \left[ \frac{E_r e_r}{x} (\sin\theta + \cos\theta) \right] \beta e^{-j\beta x} \tag{2}$$

### 3. Results and Discussion

Equation (2) is vital to the forecast,  $\Gamma = E_r E_o$  is the lightning flash. This is the region where the positive and negative electric fields creates the lightning flash.  $\left( \frac{a^2}{r^2} - 1 \right)$  is the plasma potential of the 'second kind' which shows the work done by the two charged region to shift in space.  $e$  is the collective plasma spin which initializes lightning.  $j$  is the plasma current,  $\theta = \omega t$  is the phase angle which describes the directional strikes of the plasma on molecules in the atmosphere.  $E_o e \left( \frac{a^2}{x} - x \right)$  is the Electric potential( $\phi_o$ ) of the collective plasma in the atmosphere.  $\frac{E_r e_r}{x}$  is the magnitude electric current transmitted per distance( $\chi_o$ ).  $j\beta$  is equivalent to air density in the atmosphere( $\mu$ ).  $E_r E_o e \left( \frac{a^2}{x^2} - 1 \right)$  describes the lightning path but in reality  $E_r E_o e \left( \frac{a^2}{x^2} - 1 \right) \equiv \Gamma$ . Therefore, equation (2) in practical term is written as

$$2|\phi_o|\Gamma = \frac{\mu}{8\pi} [\chi_o (\sin\theta + \cos\theta)] \beta e^{-\mu x} \tag{3}$$

From equation (2), the lightning flash pattern from cloud-to-ground is dependent on air density which we propose to be the major indicator of the atmospheric charge dynamics. The atmospheric charge dynamics was traced theoretically to the directional collision of spinning plasma which determines the various bright sides as shown in figure2.

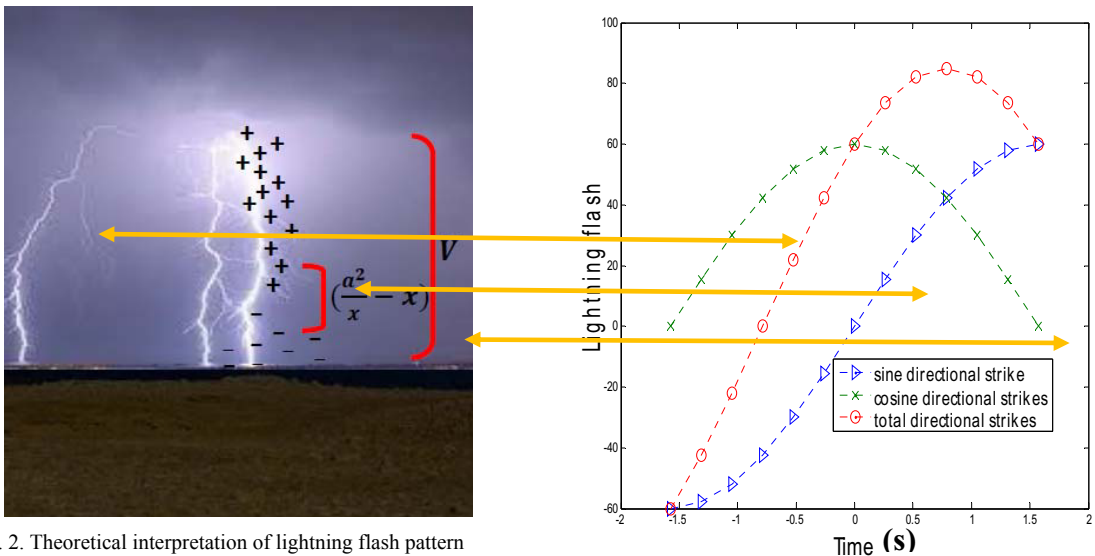


Fig. 2. Theoretical interpretation of lightning flash pattern

The dependence of air density on lightning as proven – theoretically was experimentally monitored (on a rainy day i.e. characterized by a thunderstorm and series of lightning) using the Davis weather Station. The Davis weather station automatically logs weather parameter with time. In the upper atmosphere (figure 3), the wavelike-undulations showed a detailed instability in the atmosphere due to charge dynamics. The tendencies of having cloud -to -ground lightning are prevalent at the trough e.g. the red ring signifies the lightning flashes while the yellow ring signifies the only thunderstorm. At the lower atmosphere (figure 4), the features are blurred i.e. without accuracy of activities as shown in figure (3). The physics of the air density concept is the evidence of the accumulation of wider areas of charged region in space which herald imminent presence of lightning or thunderstorms. As the plasma spins through the atmosphere, they strikes water molecules, create a kind of charge i.e. positive or negative and localize within specific charged region. As a positive or negative charged region drift in the atmosphere, voltage builds-up simultaneously. The result is an electrical conducting channel known as lightning. When the charge regions (as shown in figure 1) accumulates and expands two dimensionally within space, the potential increases. The increase of the charge potential can either be intra ( $(\frac{a^2}{x^2} - 1)$ ) or internally ( $(\frac{a^2}{x} - x)$ ) influenced by our model.

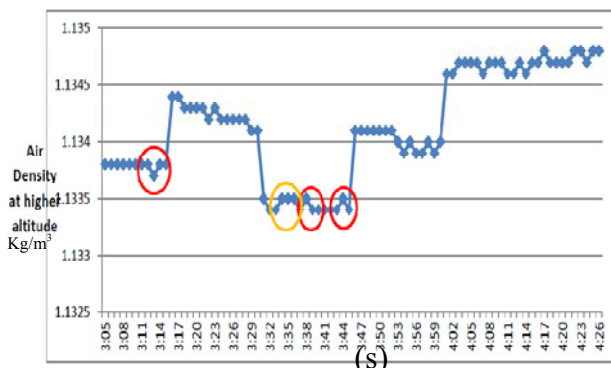


Fig 3. Experimental lightning flash pattern in upper atmosphere

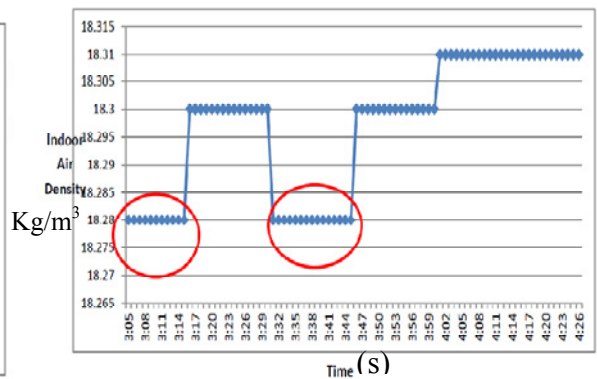


Fig 4. Experimental lightning flash pattern in lower atmosphere

#### 4. Conclusion

The lightning forecast within a region can be determined by close monitoring of the air density of the upper atmosphere. Extensively, other parameters which relates to the magnitude and types of the lightning can be determined by equation (3).

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### Appendix A. Micro and Macro Lightning Modeling using the Schrödinger-Electrostatic Algorithm

We propose that the nature of excited charge spin initiates the preliminary of lightning. We therefore introduce the time-independent Schrödinger equation to account for its spins as

$$i\hbar \frac{\partial}{\partial t} \psi - \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = 0 \quad (1)$$

The langrangian density related to equation [1] is given as

$$\mathcal{L}_1 = \frac{1}{2} \left[ \left| \frac{\partial \psi}{\partial t} \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi|^2 - V|\psi|^2 \right] \quad (2)$$

We apply the minimum coupling rule to describe the interaction of  $\psi$  with the electrostatic field i.e.

$$\frac{\partial}{\partial t} \mapsto \frac{\partial}{\partial t} + ieV, \nabla \mapsto \nabla - ieA \quad \text{where } V = V_o + E_o \left( \frac{a^2}{x} - x \right)$$

where  $V$  is the total potential across atmospheric surfaces,  $V_o$  is a constant on the surface of the charged air,  $E_o$  is the electric field and  $x$  is the width of the lightning strokes,  $\left( \frac{a^2}{x} - x \right)$  is the plasma potential,  $x$  is the Dybe length. Equation [2] transforms into

$$\mathcal{L}_1 = \frac{1}{2} \left[ \left| \frac{\partial \psi}{\partial t} + ie\psi\phi \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi - ieA\psi|^2 - V|\psi|^2 \right] \quad (3)$$

$$\mathcal{L}_1 = \frac{1}{2} \left[ \left| \frac{\partial \psi}{\partial t} + ie\psi V_o + iE_o e\psi \left( \frac{a^2}{x} - x \right) \cos \omega t \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi - ieA\psi|^2 - V|\psi|^2 \right] \quad (4)$$

We Apply the solution of the standing wave  $\psi(x, t) = e^{iS(x,t)} T(x, t)$  in equation [4] where  $E, B : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$ , the lagrangian density takes the form

$$\mathcal{L}_1 = \frac{1}{2} \left\{ E_{rt}^2 - |E_z|^2 + \left[ \frac{\hbar^2}{2m} |eA|^2 + |V_o e|^2 - \left( |E_o e \left( \frac{a^2}{x} - x \right) \right)^2 \right] + 2E_o V_o e^2 \right\} E_r^2 \quad (5)$$

Considering the lagrangian density of the particle in an electrostatic fields  $E_1$ - $E_2$  field of the atmospheric plasma

$$\mathcal{L}_o = \frac{1}{8\pi} (|E_1|^2 - |E_2|^2) \quad (6)$$

Where the values of electric and magnetic was adapted from Glenn (2003) and restructured into the circular monopole plasma tube (2008). We applied the micro scale (plasmas in antenna tube) to the macro scale (atmospheric lightning flash)

$$E_1(a, z) = (\beta E_r(a, z)e_r + E_z(a, z)e_z)e^{-j\beta r} \sin \theta \quad (7)$$

$$E_2(a, z) = (\beta E_r(a, z)e_{r1} + E_z(a, z)e_{z1})e^{-j\beta r} \cos \theta \quad (8)$$

where  $e_r = e_{r1} = \frac{\xi m}{4\pi r}$  and  $e_z = e_{z1} = \frac{\xi m j}{4\pi z^2}$  are the parameters which describes the nature and dynamics of the spins inside the tropopause.

The boundary conditions for equation [7] are

$$\begin{cases} E_1(a, 0) = E_\alpha(z) \\ E_1(\infty, z) = 0 \\ E_1(a, x) = E_\alpha(z) \cdot \alpha \\ E_1(a, \infty) = 0 \end{cases} \quad (9)$$

The boundary conditions for equation [8] are

$$\begin{cases} E_2(a, 0) = E_\gamma(z) \\ E_2(\infty, z) = 0 \\ E_2(a, x) = E_\gamma(z) \cdot \gamma \\ E_2(a, \infty) = 0 \end{cases} \quad (10)$$

where  $\alpha$  and  $\gamma$  are the attenuation factors of the electrical fields;  $E_\gamma(z)$  and  $E_\alpha(z)$  are the electric fields generated by the polar difference;  $x$  is the length of plasma antenna;  $\beta$  is the frequency of excited power;  $j$  is the plasma current;  $r$  represents the radius or horizontal component of the antenna;  $z$  represents the vertical component of the antenna;  $m$  represents the magnitude of the electrons in the atmosphere;  $\xi$  represents the electrical permeability;  $\mu_o$  represents the magnetic permeability;  $e_r$  is the spin factor which determines the electron spin along the horizontal component of the plasma;  $e_z$  is the spin factor which determines the electron spin along the vertical component of the plasma;  $e_{r1}$  is the spin factor which determines the electron spin along the horizontal component within the electric field of the tropopause;  $e_{z1}$  is the spin factor which determines the electron spin along the vertical component within the electric field of the tropopause;

Therefore the total action of lagrangian density is given by

$$D = \iint \mathcal{L}_1 + \mathcal{L}_o \quad (11)$$

Then the Euler-Lagrange equation associated to the function  $S = S(E_r, E_z = E_o, z)$  gives rise to the following systems of equation

$$E_r + \left[ \frac{\hbar^2}{2m} |eA|^2 + |V_o e|^2 - \left( |E_o e \left( \frac{a^2}{r} - r \right) |^2 \right) + 2E_o V_o e^2 + \beta e_r \right] E_r = \beta E_r e_r e^{-j\beta r} (\sin\theta + \cos\theta) \quad (12)$$

$$\frac{\partial}{\partial t} [(V_o e) E_r^2] - \frac{\partial}{\partial t} \left[ \left( E_o e \left( \frac{a^2}{x} - x \right) \right) E_r^2 \right] = 0 \quad (13)$$

$$2 \left| E_o e \left( \frac{a^2}{x} - x \right) \right| E_r E_o e \left( \frac{a^2}{x^2} - 1 \right) = \frac{j\beta}{8\pi} \left[ \frac{E_r e_r}{x} (\sin\theta + \cos\theta) \right] \beta e^{-j\beta x} \quad (14)$$