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## On a third-order boundary value problem at resonance on the half-line

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#### Abstract

In this paper, we establish existence of solutions for the following boundary value problem on the half-line: $\left(q(t) u^{\prime \prime}(t)\right)^{\prime}=g\left(t, u(t), u^{\prime}(t), u^{\prime \prime}(t)\right), \quad t \in(0, \infty)$ subject to the boundary conditions $u^{\prime}(0)=$ $\sum_{i=1}^{m} \alpha_{i} \int_{0}^{\xi_{i}} u(t) \mathrm{d} t, u(0)=0, \lim _{t \rightarrow \infty} q(t) u^{\prime \prime}(t)=0$. We establish sufficient conditions for the existence of at least one solution using coincidence degree arguments. An example is provided to validate our result.


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$$
\begin{aligned}
& \text { في هذا البحث نثبت وجود حلول للمسـألة الحدية التالية في نصف مستقيم: } \\
& \left(q(t) u^{\prime \prime}(t)\right)^{\prime}=g\left(t, u(t), u^{\prime}(t), u^{\prime \prime}(t)\right), \quad t \in(0, \infty) \\
& \text { تحت الشـروط الحدية: } \\
& u^{\prime}(0)=\sum_{i=1}^{m} \alpha_{i} \int_{0}^{\xi_{i}} u(t) d t, u(0)=0, \lim _{t \rightarrow \infty} q(t) u^{\prime \prime}(t)=0 \text {. } \\
& \text { نقدم بعض شروط الكفاية لوجود حل واحد على الأقل مستخدمين حجّة تطابق الدرجات. }
\end{aligned}
$$

## 1 Introduction

In this paper, we derive sufficient conditions for the existence of solutions for the following boundary value problem on the half-line

$$
\begin{align*}
\left(q(t) u^{\prime \prime}(t)\right)^{\prime} & =g\left(t, u(t), u^{\prime}(t), u^{\prime \prime}(t)\right), \quad t \in[0, \infty)  \tag{1.1}\\
u^{\prime}(0) & =\sum_{i=1}^{m} \alpha_{i} \int_{0}^{\xi_{i}} u(t) \mathrm{d} t, u(0)=0, \quad \lim _{t \rightarrow \infty} q(t) u^{\prime \prime}(t)=0 \tag{1.2}
\end{align*}
$$

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