

Mathematical Methods and their Applications

**Ezekiel F. Adebisi
Segun A. Fatumo**

FIRST EDITION

**Ota
2005**

Acknowledgments

But there is a Spirit in man and the Inspiration of the Almighty gives them understanding (*Job 32,8*). To God be the glory for His divine Inspiration to accomplish this book project.

Our profound appreciation goes to the Chancellor, Covenant University, Dr. David Oyedepo for his prayers, warm encouragement all along, and for his earnest desire to ensure that staff and students of the University intensify efforts on research and book writing.

Special gratitude to Prof. (Mrs) Obayan, the Vice Chancellor, the Registrar, Dn. Yemi Nathaniel and Prof. T. S. Ibiyemi, Dean of the College of Science and Technology for their encouragement, support and commitment to excellence in all academic aspects.

We are grateful to all 200 level students 2003/2004 session of department of Computer and Information Sciences of Covenant University, who informed the need to write this book in the first place.

A big thank you to all our colleagues in the department of Computer and Informations Sciences, Covenant University, who supported us in one way or the other.

We owe our families special thanks, for allowing us for many days to work round the clock on the book. Special thanks to Mrs Oluwatoyin Adebisi for her moral support and “Martha” ministry in ensuring that we are focused to accomplish the task. Finally, we are particularly indebted to the following editorial and production staff of Covenant Univer-

sity Press, namely, Dr. M. O. Afolabi, Professor O. Taiwo and Ms Ofure Aito. Their editorial helps have brought this project to fruition.

Ezekiel F. Adebisi and Segun A. Fatumo
Ota, September, 2005

Preface

Here, we put together basic mathematics concepts and applied research contents, integrating them for proper understanding both to the undergraduate and the postgraduate students. In this book, there are three originalities, and they are related. First, the foundation theoretical concepts are presented in a format that makes application very easy to follow. Thereafter, the life application problems were presented to enable the readers see immediately, the application of the theories learned. Finally, we explain basic data structures that find application in our life application problems. No book has presented data structures this way.

We present the content of this book in three parts, namely, Part I: The Fundamentals, Part II: Expansion and Advanced Integration Methods, and Part III: Differential Equations and Numerical Analysis. Part I is all about the fundamental topics of mathematics. It provides the building bricks or languages with which all complex forms of mathematics can be built. For example, Gamma functions and related integrals of chapter 10 can not be taught without a minimum knowledge of Real Numbers and Functions of Real Variables, Limits, Continuity and Differentiability, Indefinite, Definite, and Improper Integrals and Infinite Series discussed in chapters 2, 4, 5, and 6 respectively. Or can Differentiation and Integration of Integrals in chapter 11 be taught without a minimum knowledge of Real Numbers and Functions of Real Variables, Limits, Continuity and Differentiability, and Indefinite, Definite and

Improper Integrals? Basically all topics discussed in part II and III required the understanding of all the topics discussed in part I.

In part II, we discussed techniques useful for differentiation and integration of (complex) integrals as introduced in Limits, Continuity and Differentiability and Indefinite, Definite and Improper Integrals of chapters 4 and 5 respectively. We also discussed here (that is part II) how expansions of functions that can not be done using Taylor and Maclaurin expansion techniques can be carried out. Lastly part III is all about differential equations, which basically consist of ordinary differential equations (ODEs) and the partial differential equations (PDEs). Since not all equations (algebraic or differential) can be solved analytically, we presented in this part, an introduction to numerical analysis, an alternate method for solving equations.

The unique additional context in this book to teaching mathematical methods to Computer Science students was discovered during the time Mr. Fatumo and myself were teaching the (2003/2004) 200 level class of Computer Science and Management Information Science (MIS) students. We realized that as we changed our approach from teaching only basic mathematics to a mixture of basic mathematics and their application, the students enjoyed our classes and found their final examination easier.

The book is basically useful for teaching mathematical methods and analysis of algorithms to computer science students, but note that it can also be generally used as a mathe-

matics methods textbook for the science and technology students, both at the undergraduate and the post-graduate levels.

The German word 'Rigorisum' employed in this book to describe Abstract questions after each chapter comes from the English word rigorous. Furthermore, C programming language is the pseudo-code language employed in this book. This language, just like English, has become a universal programming language for scientists across the globe.

The book can be read as follows:

1. An advanced Computer Science (CS) person familiar with mathematical methods may go straight to study the life application questions and get back to the basic mathematical methods discussed in this book, if the need arises.
2. The chapters has been arranged to allow systematic understanding of all the topics considered. Therefore, the book content may be divided into three. Each of these is ideal for usage as course content per semester.
3. Those who are interested in knowing further details about the methods discussed in this book should consider the publications listed under the bibliography section.

Ezekiel F. Adebisi and Segun A. Fatumo
Ota, September, 2005

Contents

Acknowledgments	i
Preface	iii

I The Fundamentals **xxvii**

1 Basic Data Structures	1
1.1 Index	2
1.2 Trie	5
1.3 Linked list	7
1.4 Hash table	17
1.5 Suffix trees	19
1.6 Suffix array	36
1.7 Directed Acyclic Word Graph	37
 2 Real Numbers and Functions of a Real Vari-	
ables	39
2.1 Real Numbers	39
2.2 Fundamental Laws governing Operations with	
Real Numbers	41

2.3	Inequalities	43
2.4	Functions of Real Variables	57
2.5	Rigorisum Questions	70
2.6	Life Application Questions	71
3	Vector Algebra	77
3.1	Basic Vectors and Components	79
3.2	Addition and Subtraction of Vectors	80
3.3	The Vector Product	84
3.4	Scalar Product	90
3.5	Triple Products of Vectors	92
3.6	Vector equation of lines and planes	94
3.7	Cylindrical and Spherical Coordinate Systems	103
3.8	Rigorisum Questions	106
4	Limits, Continuity and Differentiability	109
4.1	Limits	109
4.2	Continuity of Functions	113
4.3	Differentiability	118
4.4	Derivative	121
4.5	Derivatives of Trigonometric Functions	123
4.6	Rule of Differentiation	127
4.7	Rolle's theorem and finding Maxima and Minima	131
4.8	The First Mean-Value Theorem	134
4.9	Higher Order Derivatives and Leibnitz's Formula	137
4.10	Rigorisum Questions	141

5	Indefinite, Definite and Improper Integrals	145
5.1	The Definite Riemann Integral	148
5.2	Some Properties of Definite Integral	156
5.3	Evaluation of Integrals	157
5.4	The Mean Value Theorem	178
5.5	Improper Integrals	182
5.6	Rigorisum Questions	191
6	Infinite Series	195
6.1	Sequences and Series of Constants	195
6.2	Fundamental Theorems of Series	199
6.3	Convergence of Series	202
6.4	Absolute Convergence of Series	206
6.5	Sequence and Series of functions	216
6.6	Power Series	218
6.7	Theorem on Power Series	220
6.8	Taylor and Maclaurin Series	225
6.9	Expansion of certain Complex Functions . . .	235
6.10	Application of Taylor's expansion	236
6.11	Rigorisum Questions	238
6.12	Life Application Questions	241
7	Complex Numbers	249
7.1	The Complex Number System	249
7.2	The Representation of Complex Numbers . . .	250
7.3	Fundamental Operation with complex Numbers	258
7.4	De Moivre's Theorem and its Application . . .	261
7.5	Roots of complex numbers	265

7.6	The nth Root of unity	266
7.7	Dots and cross Product	267
7.8	Rigorisum Questions	268
8	Partial Differentiation	271
8.1	Function of a function	274
8.2	Higher Partial Derivatives	275
8.3	Total Derivative	278
8.4	Total differential Coefficient	280
8.5	Implicit Function	281
8.6	Higher Total Derivatives	282
8.7	Homogeneous Functions	283
8.8	Euler's Theorem on homogeneous function . .	284
8.9	Change of Variables	286
8.10	Taylor Theorem for function of two indepen- dent variables	287
8.11	Maxima and Minima	289
8.12	Rigorisum Questions	296
9	Matrix Operation	299
9.1	Basic Definition and operation	299
9.2	Elementary Row and Column Operations . . .	300
9.3	Determinant of a square Matrix	307
9.4	Solution of a set of linear equations (The direct approach)	310
9.5	Gaussian Elimination Method for solving a set of Linear equation	312
9.6	Eigenvalues and Eigenvectors	314

9.7	Canonical Bases	325
9.8	Functions Of Matrices	334
9.9	Differentiation and Integration of Matrices . .	340
9.10	The Matrix Equation $AX + XB = C$	341
9.11	Rigorisum Question	342
9.12	Life Application Questions	348

II Expansion and Advanced Integration Methods 355

10 Gamma Functions and Related Integrals 357

10.1	The Gamma Function	357
10.2	The Beta Function	367
10.3	Relation between the Gamma and Beta Functions	369
10.4	The Psi(Digamma) Function	371
10.5	Rigorisum Questions	374
10.6	Life Application Questions	378

11 Differentiation and Integration of Integrals 381

11.1	Differentiation of Indefinite Integrals	381
11.2	Differentiation of Definite Integrals	384
11.3	Integration of a Definite Integral	388
11.4	Rigorisum Questions	390

12 Complex Variables 393

12.1	Functions, Limits and Continuity	393
12.2	Differentiation and the Analytic Functions . .	416
12.3	Integration and Analytic Functions	429

12.4 Taylor and Laurent Series	442
12.5 The Residue Theorem and its Applications . .	455
12.6 Rigorisum Questions	482
12.7 Life Application Questions	484
13 Fourier and Mellin Transforms	489
13.1 Introduction	489
13.2 Fourier Series	490
13.3 Fourier series under a different Interval	502
13.4 Sum of functions series	506
13.5 Cosine and Sine Series.	512
13.6 Integration and Differentiation of a Fourier series.	515
13.7 Complex Fourier series	520
13.8 The Parseval Theorem	522
13.9 Fourier's Integral Theorem	523
13.10 Properties of the Fourier Integral	528
13.11 Some special functions and their transforms	531
13.12 Fourier cosine and sine transforms	539
13.13 Convolution and the Convolution theorem .	541
13.14 Table of Fourier integrals for some standard functions	546
13.15 Mellin Transform	548
13.16 Fast Fourier Transform and Its Applications	548
13.17 Rigorisum Questions	551
13.18 Life Application Questions	555
14 Numerical Integration	559
14.1 Rectangular Rule	560

14.2 MidPoint Rule	561
14.3 Trapezoidal Rule	565
14.4 Simpson's Rule	567
14.5 Application of Simpson's Rule	568
14.6 Series Expansion Method	570
14.7 A little Numerical Analysis	571
14.8 Rigorisum Question	572

III Differential Equations and Numerical Analysis 575

15 Ordinary Differential Equations (ODE)	577
15.1 Introduction	577
15.2 Formation of ODEs	580
15.3 First Order Equations	582
15.4 Linear Equations	602
15.5 Linear homogeneous equations with constant coefficients	604
15.6 Linear inhomogeneous constant coefficient equa- tions	609
15.7 Nonlinear Second Order Differential Equations	621
15.8 Simultaneous Equations	624
15.9 The Laplace Transformation	626
15.9.2 Laplace transformation of functions . .	627
15.9.6 Inverse Transforms.	633
15.9.8 Transforms of differential coefficient . .	637

15.9.9 Solution of Ordinary Differential Equations	638
15.10 Rigorisum Questions	642
16 Partial Differential Equations (PDE)	645
16.1 Introduction	645
16.2 The General Form of a PDE Solution	648
16.3 General Solution of Second Order Constant Coefficient Equation	650
16.4 Solution by Direct Integration and Separation of Variables	659
16.5 Other Methods for Solving Partial Differential Equations	671
16.6 Rigorisum Questions	672
17 Introduction to Numerical Analysis	675
17.1 Computer Arithmetic and Errors	675
17.2 Sources of error in a computation	676
17.3 Solution of Equation	679
17.4 Interpolation	685
17.5 Solving differential Equations	701
17.6 Rigorisum Questions	722
Bibliography	725
Index	733

List of Tables

1.1	The T_{leaf} and T_{branch} tables of fig. 1.15 representing the suffix trees (fig. 1.14) for $S = abab$.	26
3.1	Rectangular, Cylindrical and Spherical Conversion table	105
5.1	Short list of Indefinite Integrals of elementary functions	147
9.1	An efficient manner for storing a sparse matrix.	349
10.1	Expected values of $\frac{F(n)}{n}$ defined in lemma 53 for different values of a 's and n 's.	378
10.2	Observed values of $F(n)/n$	379
13.1	Fourier integrals for some standard functions .	547
13.2	Mellin integral for some standard functions. .	549
15.1	A short list of Laplace transforms of common functions	632

16.1 Eigenvalues w and its corresponding eigenfunc-	
tions.	665
17.1 The forward difference table for $f(x) = e^x$ with	
$h = 0.2$	689
17.2 The difference table for the function $f(x)$ de-	
finned in the table above.	692
17.3 The trace out table to find $f(1.6)$	694
17.4 The trace out table to find $f(-3)$	695
17.5 The trace out table to find $f(0.2)$	696
17.6 The trace out table to find $f(3.1)$	697
17.7 The trace table required using the Gregory-	
Newton backward difference formula of (17.34)	
to find $f(4.4)$	699
17.8 The trace tables required using the Gregory-	
Newton backward difference formula of (17.34)	
to $f(7)$	700
17.9 Complete iterations of the method applied in	
example 17.5.1	703
17.10 Exact solution table for $y = 2x + e^{2-x}$	704
17.11 Complete iterations of the iteration that began	
in equation (17.57).	705
17.12 Supplementary table for the iteration that be-	
gan in equation (17.57).	706
17.13 Iterative display of the exact solution of (17.58)	
using the integrating factor (I.F)	707
17.14 The Euler's solution for equation (17.58).	708

17.15	Comparing the Exact and the Euler-Cauchy solutions.	711
17.16	Suppl. table for comparing the Exact and the Euler-Cauchy solutions.	712
17.17	Comparing the Exact and the Euler solutions.	713
17.18	Suppl. table for comparing the Exact and the Euler solutions.	714
17.19	Comparing the Euler and the Euler-Cauchy solutions.	715
17.20	Suppl. table for comparing the Euler and the Euler-Cauchy solutions.	716
17.21	An auxiliary table for solving the problem of example 17.5.5	719
17.22	Comparing the Runge-Kunta, the Euler-Cauchy, and the Euler methods error % s.	720
17.23	A finite difference table.	723

List of Figures

1.1	A Sample trie	6
1.2	A singly linked list.	8
1.3	A doubly linked list.	8
1.4	A circular linked list.	8
1.5	A binary tree and its representation.	9
1.6	Linked list insertion.	11
1.7	Linked list deletion.	12
1.8	Adding to a singly linked list with head ‘h’. . .	13
1.9	Data struture definition for ELEMENT that define the singly linked list used in fig. 1.8 above.	14
1.10	Dynamic creation of a binary tree node. . . .	15
1.11	Add a node to the binary tree.	16
1.12	Data struture definition for the binary tree node.	17
1.13	A hashing scheme.	20
1.14	a) The suffix trees for $S = abab$, b) another one for $S = GTATCTAGG$	21

1.15	The suffix trees for $S = abab$ following the T_{branch} and T_{leaf} representation above. The first child references are represented by the vertical arcs, and the branchbrother and the leaf references are represented by the horizontal arcs.	26
1.16	The sequence of Σ^+ -Trees for $S = abab$. ST_5 is the suffix trees required.	32
1.17	Kurtz[32] modified version of McCreight Algorithm for constructing a suffix trees ST	33
2.1	A simple graph relating some circular functions.	50
2.2	One dimensional display of Linear inequalities of one variable.	51
2.3	Two dimensional display of Linear inequalities in (a) one variable and (b) two variables . . .	52
2.4	Two dimensional display of two linear inequalities in two variables.	53
2.5	Two dimensional display of four linear inequalities in two variables.	54
2.6	The feasible region of solution.	56
2.7	Graphs of $y = x^3$ and $y = x^2$ in the $x - y$ plane.	63
2.8	a) The function $y = e^{ax} \cos bx$ in the $x - y$ plane, b) the function $y = \sin x$ in the $x - y$ plane.	66
2.9	Graph of $x - [x]$ for $-3 \leq x \leq 3$	67
3.1	a) \vec{AB} b) $\vec{BA} = -\vec{AB}$	78
3.2	The vector $\vec{U} + \vec{V}$ is defined to be the vector \vec{OP} .	81

3.3	The vector $\vec{U} + \vec{V}$ is defined to be the vector \vec{OP} .	82
3.4	Associative law	83
3.5	$(\vec{U} \times \vec{V})$	85
3.6	$(\vec{V} \times \vec{U})$	86
3.7	the projection of \vec{V} onto \vec{U}	91
3.8	The area of the parallelepiped defined by \vec{U} , \vec{V} and \vec{W}	94
3.9	The graph of $O\vec{W} = O\vec{U} + U\vec{W}$	95
3.10	Graphical representation of the perpendicular- ity of \hat{n} and $U\vec{V}$	99
3.11	Rectangular, cylindrical and spherical coordi- nate systems.	103
3.12	a) Converting Rectangular to Cylindrical Coor- dinate, b) Converting Spherical to Cylindrical coordinate.	104
4.1	The graph showing $f(x)$ for $x \leq 1$ and $f(x) =$ $\frac{1}{2}$ for $x > 1$	111
4.2	a) The function $y = x^2$ in the $x - y$ plane, b) The function $f(x) = \frac{1}{x}$ also in the $x - y$ plane.	115
4.3	Graphical definition of differential coefficient of $f(x)$ with respect to x	119
4.4	a) A maximum function, b) A minimum function.	132
4.5	a) The function $y = x $ in the interval $-a \leq$ $x \leq a$, b) Parallel tangents at maximum and minimum points.	133
4.6	Continuous and differentiable function at inter- val $a \leq x \leq b$	135

5.1	The area bounded by $y = 3x^2 + 14x + 15$, the x -axis and the ordinates at $x = -1$ and $x = 2$.	148
5.2	The graph relating Riemann integral to the area under a continuous curve.	150
5.3	Another graph relating Riemann integral to the area under a continuous curve.	152
5.4	An odd function, such that $\int_{-a}^a f(x) dx = 0$. .	154
5.5	a) $F(k, \phi)$ at $k = \sin \alpha$, $\alpha = 0, \pi/4, \pi/2$ and $0 \leq \phi \leq \pi/2$, $E(k, \phi)$ at $k = \sin \alpha$, $\alpha = 0, \pi/4, \pi/2$ and $0 \leq \phi \leq \pi/2$	174
7.1	Argand diagram for $P(z)$ and its conjugate $P(\bar{z})$	252
7.2	a) Equal vectors \vec{OP} and \vec{AB} , b) Parallelogram law for vector addition.	257
7.3	Stereographic projection: The mapping of the plane to the sphere.	258
10.1	Gamma function and its reciprocal for $-4 \leq x \leq 4$	358
10.2	The behaviour of Psi (digamma) function for $-4 \leq x \leq 4$	372
12.1	a) A curve between points P and Q in the z -plane, b) The same curve between P and Q but in the ω -plane (another plane)	400
12.2	a) The function $y = mx$ in the z -plane, b) The transformed version of $y = mx$ in the w -plane, where $w = \frac{1}{z}$	402

12.3 a) The function $x^2 + y^2 = a^2$ in the z -plane,	
b) The function $x^2 + y^2 = a^2$ in the w -plane,	
where $w = \frac{1}{z}$	403
12.4 a) Lines $u = c_1$ and $v = c_2$ in the w plane,	
b) Corresponding curves in the z -plane, where	
$w = z^2$	404
12.5 z complete circuit around the origin.	405
12.6 A branch point at $z = i$	407
12.7 Branch points at $z = \pm i$	408
12.8 Branch lines for $w = f(z) = (z^2 + 1)^{\frac{1}{2}}$	409
12.9 Real axis at $z = 0, \pm 2\pi, \pm(2/3)\pi, \pm(2/5)\pi, \dots$	422
12.10 a) Region $ z < 2$, b) Region $1 < z < 2$. . .	435
12.11 a) Non-overlapping simple closed curves C_1 ,	
C_2, C_3, \dots, C_n , b) Region form by the functions	
$y = x^2$ and $y^2 = x$	438
12.12 a) Overlapping simple closed curves C_1 and	
C_2 , b) Simple closed curve C , point $z = a$ in-	
side and outside C	441
12.13 The annulus or annular region between con-	
centric circles C_1 and C_2 of radius R_1 and R_2	444
12.14 Analytic continuation of an analytical func-	
tion $f(z)$ inside some circles of convergence C_1 ,	
C_2, C_3, \dots, C_n	453
12.15 A simple closed contour, small enough to	
avoid any other poles of $f(z)$	457
12.16 A simple closed curve C with singularities	
a, b, c, \dots	462
12.17 Contour C with multiple poles.	464

12.18	Residues at some given poles for function defined at (12.188).	466
12.19	a) Contour C with radius R formed by the line from $-R$ to $+R$ and the semicircle Γ above the x -axis, b) A unit circle C with center at the origin.	468
12.20	A rectangle region C having vertices at $-R$, R , $R + \pi i$, and $-R + \pi i$	473
12.21	A square C_N region.	478
13.1	$f(x) = 0$ for $-\pi < x < 0$ and $f(x) = 1$ for $0 < x < \pi$	493
13.2	a) $f(x) = x$ in the range $-\pi < x < \pi$ but extended to satisfy the periodic relation $f(x + 2\pi k) = f(x)$, b) $f(x)$ defined in (13.21) and also extended to satisfy the periodic relation $f(x + 2\pi k) = f(x)$	497
13.3	a) $f(x) = e^x$ in the range $-\pi < x < \pi$ but extended to satisfy the periodic relation $f(x + 2\pi k) = f(x)$, b) $f(x) = x$ in the range $-1 < x < 1$, but also extended to satisfy the periodic relation $f(x + 2) = f(x)$	504
13.4	The Sawtooth wave function $f(x) = x + \pi$	507
13.5	The Partial sums $S_n(x)$ of (13.59).	508
13.6	The rectangular or periodic square wave function	509
13.7	The first three partial sums of the $f(x)$ given in (13.65).	511

13.8 a) $f(x) = x^2$ in the range $0 < x < 2$ but extended to satisfy the periodic relation $f(x + 4k) = f(x)$, b) $f(x) = x^2$ in the range $0 \leq x \leq \pi$ but also extended to satisfy the periodic relation $f(x + 2\pi k) = f(x)$	514
13.9 a) $f(t) = 1$ for $-a/2 < t < a/2$, $f(t) = 0$ otherwise, b) An amplitude spectrum of $f(t)$ of (13.108)	526
13.10 $f(t) = 1$ for $0 < t < a$, and zero otherwise.	527
13.11 a) Continuous amplitude $ F(\omega) $ for $f(t)$ of (13.113), b) Continuous phase spectrum $\phi(\omega)$ for $f(t)$ of (13.113).	528
13.12 The function $f_k(t - a)$ in (13.135)	534
13.13 The triangle function	539
14.1 a) An area defining the values $[x, f(x)]$, b) Approximating an area by a set of rectangles.	560
14.2 Midpoints of subintervals in interval $[a, b]$	561
14.3 Another approximation of an area by a set of rectangles.	562
14.4 Mid-points of subintervals in $[1, 2]$	565
14.5 Sub-interval of interval $[a, b]$ for Trapezoidal rule.	565
14.6 a) Trapezoidal rule: Interval with $h = 3/2$, b) Trapezoidal rule: Interval with $h = 3/4$, c) Trapezoidal rule: Interval with $h = 3/8$, and d) Trapezoidal rule: Interval with $h = 1/2$	568
14.7 Partition of interval $[a, b]$ into an even number N of equal subintervals.	568