INTEGER PROGRAMMING ALGORITHM FOR PUBLIC TRANSPORT SYSTEM IN SUB-SAHARAN AFRICAN CITIES

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ABSTRACT

This paper addresses transportation problems caused by rapid urbanization of sub-Saharan Africa metropolitan areas combined with inadequate or poorly executed development plans of the cities. We work on minimizing the transportation problems by optimizing the factors responsible for the transport problems in Lagos Nigeria, being a typical sub-Saharan African city. The problem was formulated using an integer programming (IP) model and solved using a novel branch and bound algorithm. Sensitivity analysis was carried out to verify and demonstrate the significance of the parameters involved in the model. It was found that our proposed approach finds solutions that significantly reduce the transport problem in sub-Saharan African cities if properly applied.

Keywords: integer programming, sub-Saharan Africa, public transport system, branch and bound method.

1 INTRODUCTION

There has been a fast urbanization of sub-Saharan Africa metropolitan ranges as of late. Shockingly there has additionally been deficient or shoddy improvement arranges in these regions offering ascend to various transportation issues, including decayed physical attractions and solace of street based open transport [1]. Sub-Saharan African nations are regularly alluded to as the "Black" Africa and considered all the more legitimately "Africa". Lagos, Nigeria is a decent portrayal of sub-Saharan African city. Sub-Saharan nations incorporate nations that, topographically, lie south of the Saharan Desert. Politically, it comprises of all African nations that are completely or somewhat found south of the Sahara (barring Sudan, despite the fact that Sudan sits in the Eastern segment of the Sahara Desert). Somalia, Djibouti, Comoros and Mauritania are however geologically some portion of sub-Saharan Africa, additionally some portion of the Arab world [2]. Transportation is the life wire of any quickly developing urban city in sub-Saharan Africa. Lagos is one of the quickest developing urban areas in Africa. It has transportation issues, running from blockage to air contamination and car crashes [3], [4]. Considering the significance of compelling development of individuals, merchandise and data through spots in a nation, it is vital to discover answer for the issue of transportation in such territories as sub-Saharan African developing urban communities, as it influences the monetary, political, and social existence of this part the world. This cannot be accomplished without playing out the part of connecting supply and demand [5], [6]. This paper endeavours to upgrade the primary variables in charge of transportation in Lagos utilizing the novel Integer programming calculation. The issue related with people in general street transportation framework in Lagos, for example, ranges from clog to air contamination and auto collisions. The clog brings about commotion contamination. Additionally vitality utilization and land utilization are on the expansion because of the increment in the quantity of vehicles out and about at a specific time [6]–[8]. The reasons for these issues incorporate; basic example of the streets, the impromptu

WIT Transactions on The Built Environment, Vol 182, © 2019 WIT Press
www.witpress.com, ISSN 1743-3509 (on-line)
doi:10.2495/UT180311

development, and random land-utilize circulation. In the event that these three noteworthy reasons for the transportation issues can be comprehended then there would be a colossal change in the vehicle arrangement of Lagos and by expansion all sub-Saharan African urban areas which Lagos speaks to. Integer programming is an assortment of Mathematical programming models and strategies. In Linear programming, the choice factors are permitted to have fragmentary qualities while in Integer programming they are entirely permitted to have whole number qualities [8]. Issues possess large amounts of which the elements included are just in countable structures, for instance, number of individuals, number of machines, number of courses, number of houses, to specify yet a couple. In such cases fragmentary qualities do not bode well. Plan of both linear programming and Integer programming are the same. The main distinction is that the choice factors are restricted to whole number values in Integer programs. We have two essential sorts of Integer programming issues: unadulterated Integer programming and blended Integer programming, otherwise referred to as pure integer programs and mixed Integer programs respectively. If there should arise an occurrence of unadulterated Integer programming, all the choice factors are whole numbers, while in mixed Integer programs, the choice factors are blend of whole numbers and non-whole numbers. One of the techniques utilized as a part of taking care of whole number programming issues is branch and bound strategy. Tree graphs or diagrams are regularly utilized as a part of branch and bound strategy for arrangement. A tree diagram or chart is essentially a method for speaking to a succession of occasions. Tree graphs are especially valuable in likelihood since they record every single conceivable result in a reasonable and uncomplicated way [9], [10]. In this paper we considered immaculate Integer programming issue. So we displayed the issue of transportation issue in Lagos as an immaculate Integer programming issue [11]–[13]. The three noteworthy reasons for the issue speak to the choice factors. These factors are: structural pattern of the road, the planned growth and the land-use distribution.

2 MODEL FORMULATION

2.1 Decision variables representation

Let X_1 represent weight of structural pattern of the roads.
Let X_2 represent weight of the planned growth.
Let X_3 represent weight of land-use distribution.

2.2 Objective function

The objective here is to minimize the three decision variables subject to some constraints. The objective function can then be written as follows:

$$\text{Maximize } Z = X_1 + X_2 + X_3.$$

2.3 Constraints

From the data gathered we realised that the resources needed to improve on the structural pattern of the roads and planning for the growth in percentages of the total available resources is at most 45%, while the resources needed for structural pattern of the roads and proper land use distribution is at most 55%. Also, the non-negativity integer requirement is satisfied. Assuming the average budget for road transport is one billion naira, we have as constraints:

$$X_1 + X_2 \leq 45\% \text{ of } 10,000,000,000 \text{ naira}$$
$$X_1 + X_3 \leq 55\% \text{ of } 10,000,000,000 \text{ naira}$$
$$X_1, X_2, X_3 \geq 0, \text{ integers}$$

3 THE MODEL

The model can now be stated as

$$\text{Max } Z = X_1 + X_2 + X_3$$

Subject to

$$X_1 + X_2 \leq 4.5$$
$$X_1 + X_2 \leq 5.5$$
$$X_1, X2, X_3 \geq 0, \text{ integers}$$

4 MODEL SOLUTION

4.1 Solution method

The branch-and-bound method of solving integer programming problems uses a tree diagram framework [7].

Stage 1: Preparing the Tree Diagram:

The possible integer value sets of the variables X1, X2, X3 in the above integer program are:

$$X1 = 0,1,2,3,$$
$$X2 = 0,1,2,3,4$$
$$X3 = 0,1,2,3,4,5$$

The tree diagram representing the problem considered in this paper is shown Fig. 1. Some branches are infeasible with respect to one or both constraints. So, Fig. 1 represents the framework of tree diagram representing the integer model formed. Figs 2–6 represent the branches or components of the tree diagram.

The component of the resulting tree diagram is shown in Figs 2–6.

Stage 2: First stage branch and bound:

Solving linear programs involving integer values of X1 = 0,1,2,3,4, but disregarding the integer constraints imposed on X2 and X3:

With $X_1 = 0$:

$$\text{Max } Z = x1 + x2 + x3 = 0 + x2 + x3$$

Subject to

$$0 + X2 \leq 4.5$$
$$0 + X3 \leq 5.5$$
$$X1, X2, X3 \geq 0$$

This implies

with X1 = 0, gives X1=0, X2 = 4.5, X3 = 5.5. Which implies Z = 10

Similarly,

with X1 = 1, gives X1=1, X2=3.5, X3=4.5. Which implies Z = 9

with X1 =2, gives X1=2, X2=2.5, X3=3.5. Which implies Z = 8

with X1 =3, gives X1=3, X2=1.5, X3=2.5. Which implies Z = 7

with X1 =4, gives X1=4, X2=0.5, X3=1.5. Which implies Z = 6

It can be seen that the branch corresponding to X1 = 0 has the highest upper bound value of 10. The branch is then scheduled for further analysis.

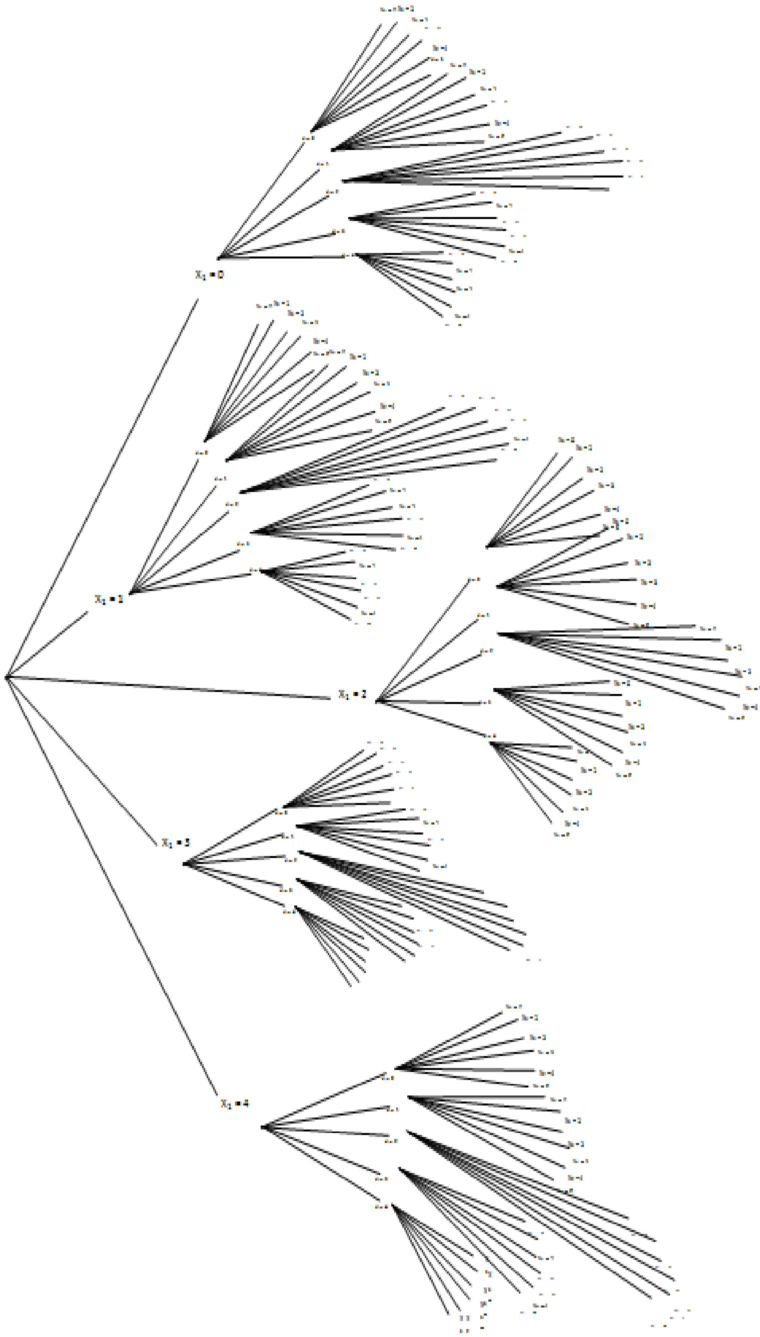

Figure 1: Tree diagram framework of the integer program model under consideration.

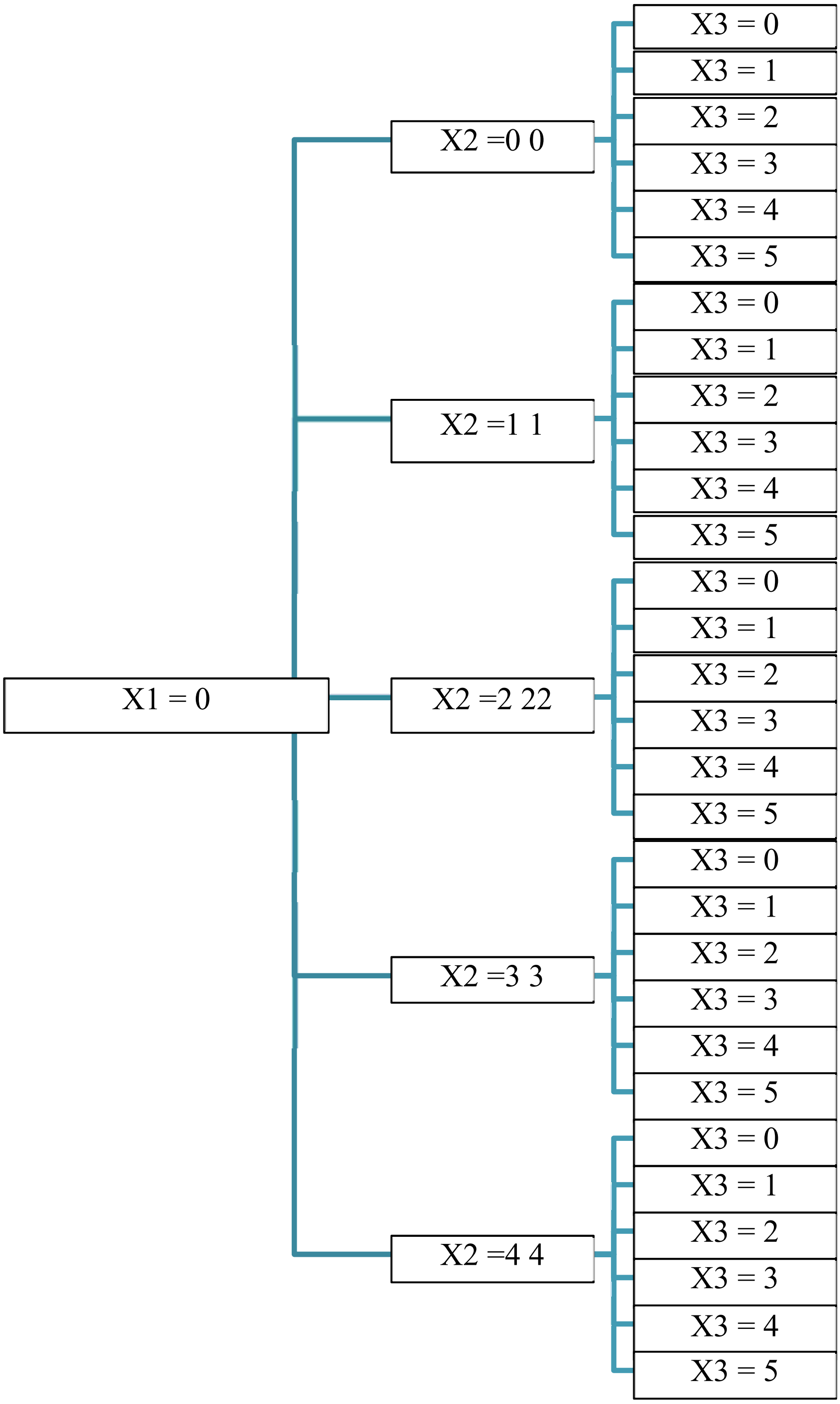

Figure 2: Component of the tree diagram for $X_1 = 0$.

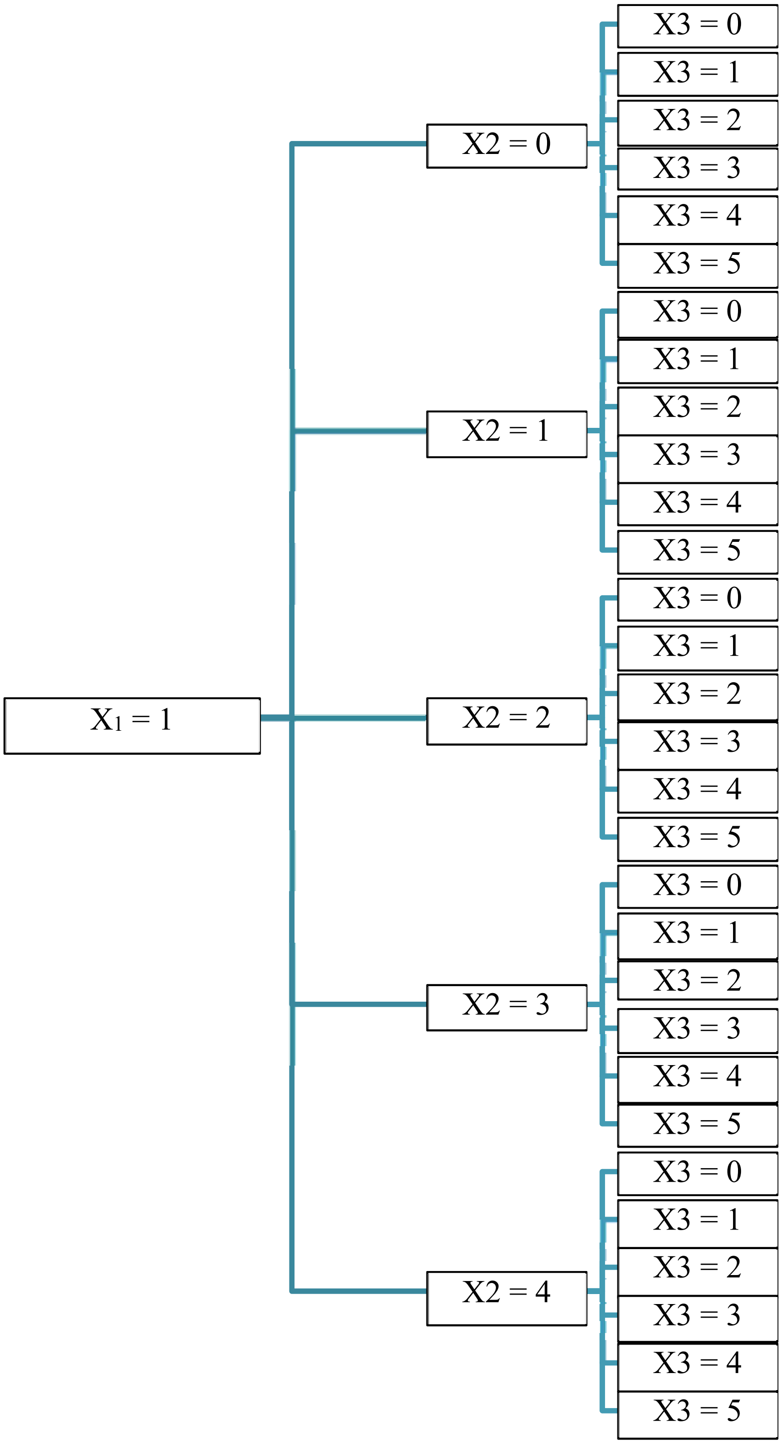

Figure 3: Component of the tree diagram for $X_1 = 1$.

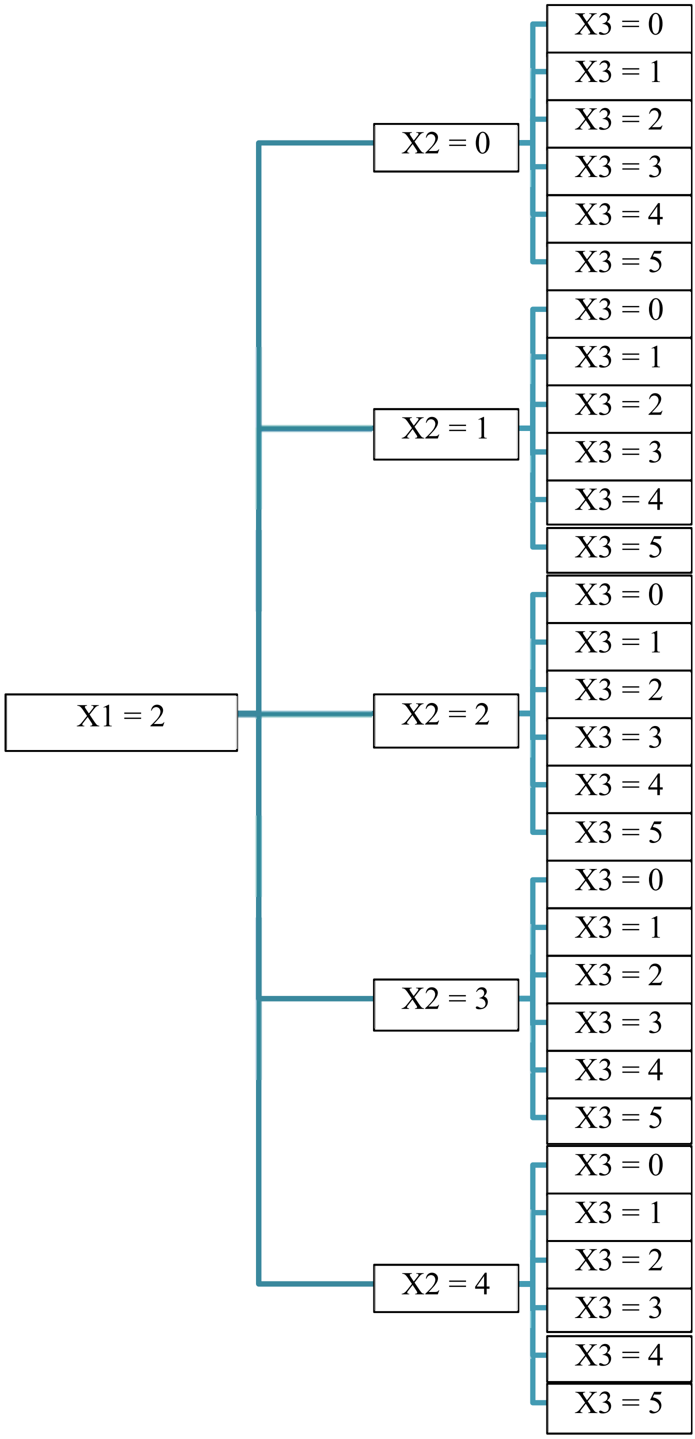

Figure 4: Component of the tree diagram for $X_2 = 2$.

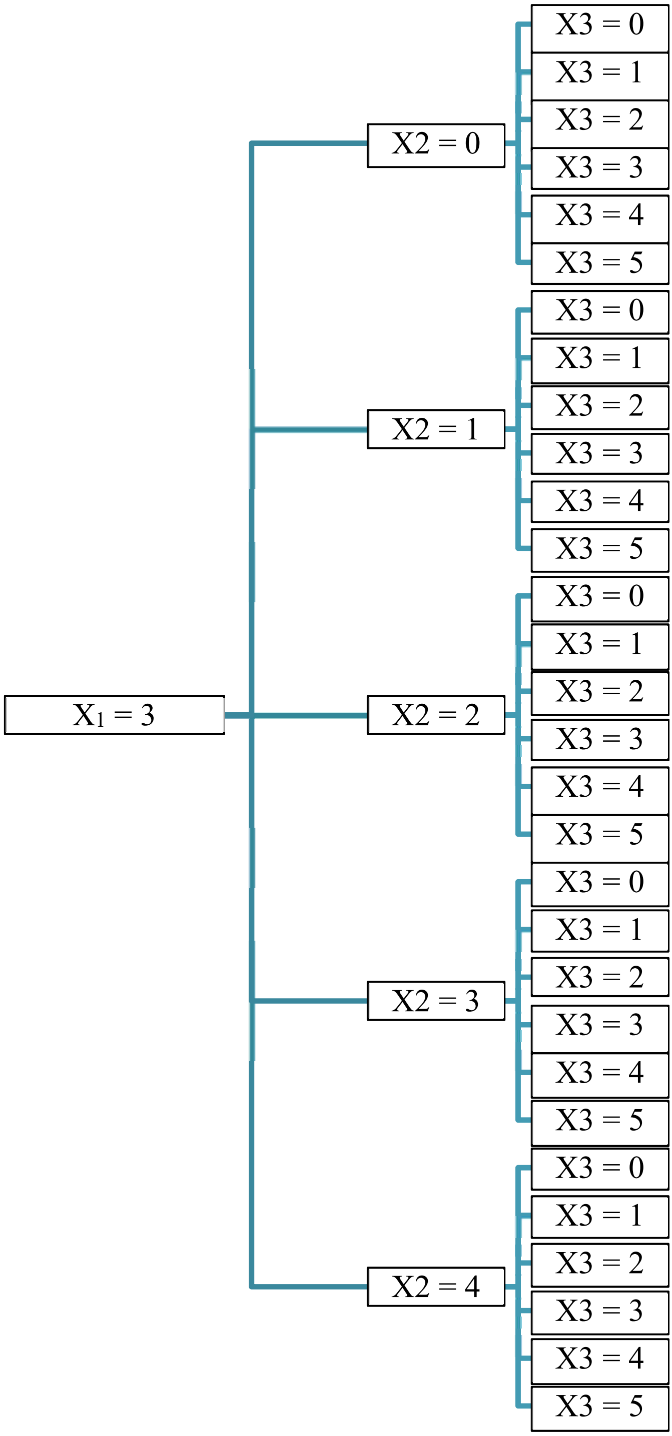

Figure 5: Component of the tree diagram for $X_1 = 3$.

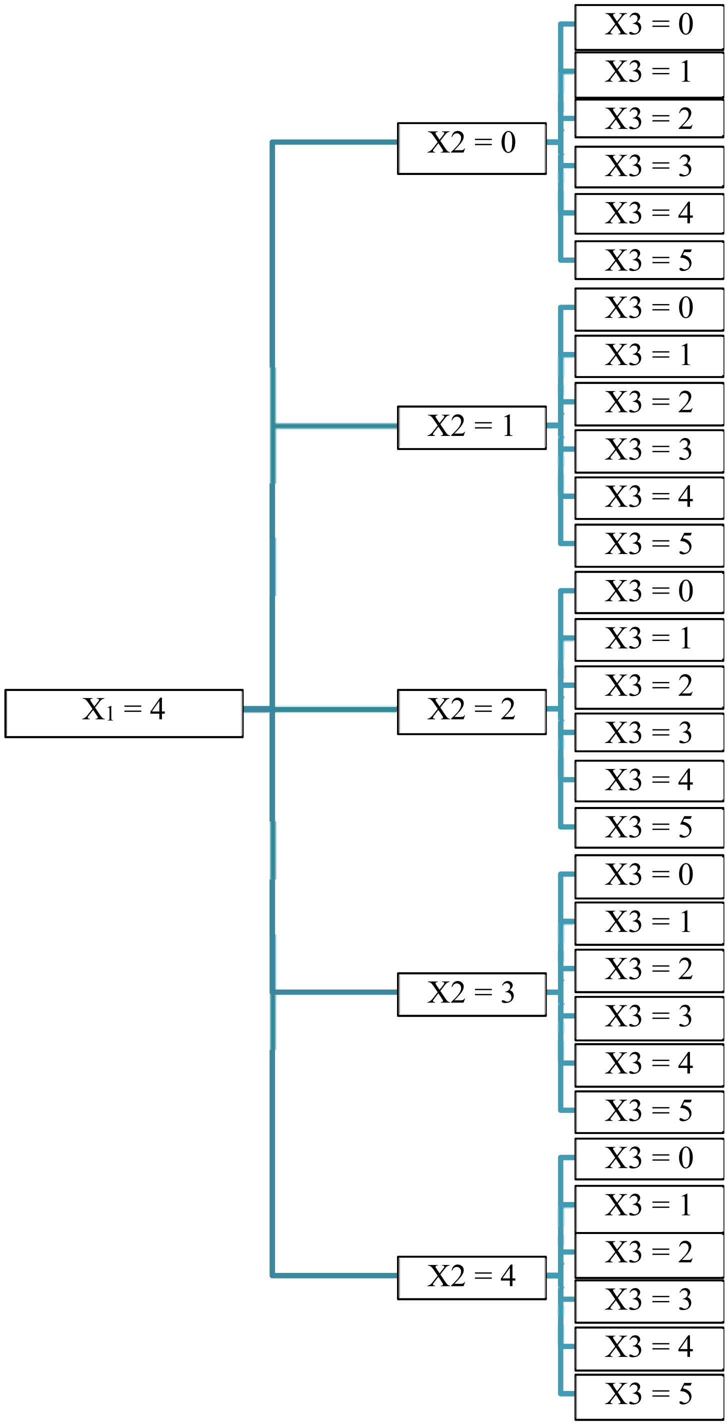

Figure 6: Component of the tree diagram for $X_1 = 4$.

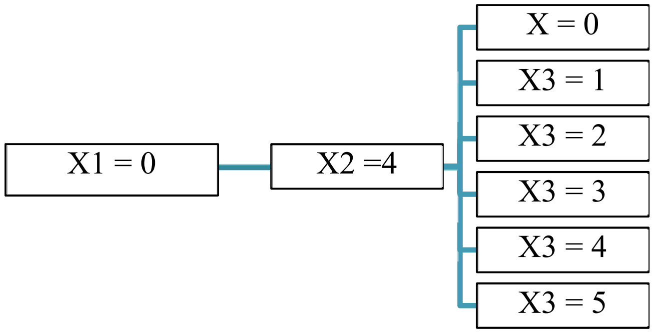

Figure 7: Reduced tree diagram.

Stage 3: Second stage branch and bound

The tree diagram reduced to the branch of the tree shown in Fig. 1(a). Now, with X1 = 0, X2=0, gives

$$\text{Max } Z = x1 + x2 + x3 = 0 + 0 + x3$$

Subject to

$$0 + 0 \leq 4.5$$
$$0 + X3 \leq 5.5$$
$$X1, X2, X3 \geq 0$$

This implies
with X1 = 0, X2=0, gives X1=0, X2 = 0, X3 = 5.5. Which implies Z = 5.5
Similarly,
with X1 = 0, X2=1, gives X1=0, X2=1, X3=4.5. Which implies Z = 5.5
with X1 =0, X2=2, gives X1=0, X2=2, X3=5.5. Which implies Z = 7.5
with X1 =0, X2=3, gives X1=0, X2=3, X3=5.5. Which implies Z = 8.5
with X1 =0, X2=4, gives X1=0, X2=4, X3=5.5. Which implies Z = 9.5

It can be seen that the branch corresponding to X1 = 0, X2 =4, has the highest upper bound value 9.5. The branch is then scheduled for further analysis.

Stage 4: Third stage branch and bound

The reduced tree diagram of the integer program is shown in Fig. 7. Now, with X1 = 0, X2=4, X3=0 gives

$$\text{Max } Z = X_1 + X_2 + X_3 = 0 + 4 + 0 = 4$$

Subject to

$$0 + 4 \leq 4.5$$
$$0 + 0 \leq 5.5$$
$$X_1, X_2, X_3 \geq 0$$

This implies
with $X_1 = 0$, $X_2=4$, $X_3=0$ gives $X_1= 0$, $X_2 = 4$, $X_3 = 0$. Which implies Z = 4
Similarly,
with $X_1 = 0$, $X_2=4$, $X_3=1$, implies Z = 5

with X_1 =0, X_2=4, X_3=2, implies Z = 6
with X_1 =0, X_2=4, X_3=3, implies Z = 7
with X_1 =0, X_2=4, X_3=4, implies Z = 8
with X_1 =0, X_2=4, X_3=5, implies Z = 9

It can be seen that the branch corresponding to X1 = 0, X2 =4, X3=5 has the highest solution value of 9. The corresponding branch gives the optimal solution. It gives the optimal solution of the original problem.

The solution is as follows:

$$X_1 = 0, X_2=4, X_3=5, Z=9$$

5 SOLUTION DISCUSSION

From the results obtained, it indicates that the structure of the road might not be very important when compared to planned growth or land use distribution in optimizing public transport system in sub-Saharan city transportation system. For example, in Lagos Nigeria, which represents a typical sub-Saharan city, would have her traffic congestion problems solved if emphasis is placed on planned growth or land use distribution. Lagos growth rate is over five percent. Considering a city in that part of the world with available resources of ten billion naira for transport system, only nine billion might be needed to optimize the transport system if attention is paid to planned growth and land-use distribution. Specifically, if four billion naira is spent on planned growth and five billion naira is spent on land-use distribution, the optimal transport system will be achieved. X_1=0 implies that structural pattern of the roads carries no weight when considering optimizing the transport system of sub-Saharan African cities. X_2= 4 and X_3=5 means both planned growth and land-use distribution are critical when considering optimizing the public transportation system of sub-Saharan African cities.

6 CONCLUSION

This study set out to improve public transportation system in Saharan African cities by optimizing the factors involved in transport system in that part of the world. These factors, identified as structural pattern of the roads, planned growth and land-use distribution, are not of the same importance. From the result obtained, it shows that structural pattern of roads in that part of the world might not be important when considering the optimal transportation system. This is partly because the way they build does not support a particular pattern of road. For instance, most of the roads are not straight. However, planned growth and land-use distribution are very vital in optimizing the public transportation system of sub-Saharan African cities. Going by the rapid, uncontrolled population of these African cities, planned growth and land-use distribution are very important factors that must be put into consideration in order to optimize the public transportation system of these cities. In a way of comparison, land-use distribution, as a factor, is slightly more important than planned growth.

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