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A modified predictive model for estimating gas flow rate in horizontal drain hole

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ABSTRACT

Accurate prediction of the flow rate of horizontal gas well is necessary for economic feasibility, planning and development of gas field. Most of the early models assumed that the production from the horizontal well is infinitely conductive except few recent models. Some recent models reported in the literature for estimating flow rate in horizontal well where the pressure losses due to friction along horizontal drain hole was considered.

An improved model that checks the impact of all possible well bore pressure losses on gas production rate of horizontal well is reported. The neglected impact of well bore pressure losses due to fluid accumulation and kinetic energy in the past models is thought to be a conceivable reason for the inconsistency between computed rates from the models and rates got from production tests. The new model was validated using the same field contextual investigation utilized by Guo et al. and outcome got from the new model yields more satisfactory results. A more realistic results that evident all flow phenomena in gas well include the initial unsteady, pseudo-steady and steady state flow condition hence flow rate at any given production time has been established for flow of gas along horizontal well. The outcomes of the study demonstrate that the percentage deviation of the new model at steady state flow condition is less than 5.0% compared with 11.05% acquired from Guo et al. model following by 259.7% from Furui's model, and 1118.2% got from Joshi's model. This work gives field operators a precise and helpful device for prediction and assessment of production in a gas horizontal well.

1. Introduction

The utilization of horizontal wells in different reservoir setting has turned into a common practice in the petroleum industry for production of all kind of reservoir setting and has been reported by several researchers. Horizontal well techniques was used to minimize gas coning in case of gas drives reservoir and water cresting in case of water drives reservoir hence, minimize water production as reported in Ref. [1]. Fleming [2] investigated the utilization of horizontal well to improve efficiency by intersecting natural fractures in naturally fractured reservoirs. The technique was also applied to thermal flooded reservoir to produce high viscous oil, reported [3,4] and to produce highly viscous bitumen, reported in Ref. [5]. Butler [6] applied the technique to tight gas reservoir (reservoir with low permeability) to improve drainage volume and hence enhancing cumulative gas production. Other applications involving miscible flooded reservoir reported in Ref. [7], water flooded reservoir investigated in Ref. [8], application in steam flooded reservoir and reported in Ref. [9] and in gas storage reservoirs documented in Ref. [10]. Fadairo et al. [11] also show the applicability of horizontal well for lessening impact of oilfield scales deposition around the well inside reservoir with incidence of scale formation.

Field histories have proven that under certain geological and reservoir characteristics, horizontal well can improve not only the production rate but also the reserves of the field. Case studies have also proven that horizontal wells can replace several conventional

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vertical wells to achieve the same production performance [12]. Therefore, the use of horizontal wells makes it possible to develop different reservoir setting which otherwise could not be economically developed under convectional well configuration technique as stated in Ref. [13].

Literature has showcased several models for predicting pressure drop inside the well bore and investigating its effect on productivity. Among others investigators Dikken [14] exhibited a basic isothermal model that shows the transition process of single phase turbulent flow to steady reservoir flow. It was recommended that the horizontal wellbore pressure loss was completely generated by wall friction. It was inferred that the stream resistance may bring about decline in drawdown at positions far from the heel-end of the well, and subsequently result in the total production levelling off as a function of well length. Landman [15] shows improvement on Dikken [14], model by adapting the model to handle specifically perforated completions and to ascertain the inflow efficiency in various sections of various perforation of the model into a reservoir simulator.

Folefac et al. [16], formulated a model for computing pressure drop considering two phase flow in the horizontal wellbore. It was deduced in their study that pressure drop in horizontal well bore segment can be essentially high when production performance from the well is high, when the flow is two phase and when there is small well radius along perforated interval.

Landman and Goldthrope [17] examined a numerical model for describing how perforation distribution influences the performance of a horizontal well delivering under consistent state single phase flow. Enhancing the perforation distribution was accomplished, efficiency was maximized, gas and water coning were retarded by assuming the uniform specific inflow along the well. A modified model for flow of fluid in horizontal perforation pipe with well mass transfer was demonstrated in Ref. [18]. Their model revealed that the higher the magnitude of turbulence the heavier the drag impact induced by perforation.

Ozkan et al.; [19], presented a semi-analytical model that couples horizontal wellbore and reservoir hydraulics for slightly compressible fluid of constant compressibility. Pressure loss due to friction in the wellbore was considered and calculated using the frictional factor coefficient obtained by Colebrook correlation. The results emphasized the importance of the further investigation to determine the horizontal wellbore surface roughness, since increase in wellbore pressure losses induced by high wellbore surface roughness and cause decreases in horizontal well productivity.

Abdulwahid et al. [20] presented a wellbore single phase model that considered not pressure loss due to friction and acceleration but also pressure loss induced by influx. The authors concluded that fluid influx increases the apparent friction factor along the horizontal wellbore but at the same there is decline in fluid influx.

Chen et al. [21] presented a wellbore-reservoir coupled model for horizontal wells introducing gel water shutoff phenomena. A simulator is used to simulate the process of gel injection in the reservoir and then improved to calculate the performance of horizontal well after the treatment. A simulator was generated for gel degradation process by developing viscosity model and time-varying residual resistance factor model. Their method not only simulates the gel injection process but also estimate the performance of water shut off in horizontal wells. A real life scenario for horizontal water shutoff prediction demonstrates that their method can produce accurate results to handle the process of water shutoff.

Sarica et al. [22], examined the impact of wellbore hydraulic on the pressure traverse and productivity of horizontal gas wells. The gas pseudo-pressure function theory was established. The pressure drop due to friction and pressure drop due to flow acceleration were presented in their model and thought to be critical particularly in gas wells. Their results demonstrated that, at late time, the qualities of a finite conductivity horizontal well response are like those of infinite conductivity horizontal wells with a smaller dimensionless length.

Guo et al. [23] numerically communicated the impact of wellbore hydraulic on horizontal gas well productivity by discretising the horizontal wellbore into 10 portions to figure frictional pressure along the wellbore. Their work was further simplified using analytical expression for easy handling purpose and was accounted for in Ref. [24] where a basic and thorough scientific model for evaluating and predicting the productivity of horizontal well. The outcomes got from their work reported just 20% error which is closer to exact than other accessible models.

Many analytical works done in the past on horizontal productivity have reported the impact of pressure drop due to friction along the horizontal wellbore and very few reported the effect of pressure drop due to acceleration. Fadairo et al. [13] gave insight into the impact of all other pressure losses on long horizontal well performance. Their model revealed that there is an optimal effective diameter at which the productivity index response to increment in flow in a horizontal well. Without neglecting any pressure loss term in the governing flow equation in circular pipe, the first law of thermodynamics in U.S. engineering units is reported [25–27].

This new study is an improvement on Guo et al. model [24], formulating a modified model that checks the impact of all wellbore pressure losses that is pressure drop due to friction, kinetic and accumulation on horizontal well flow rate. The outcomes of this work demonstrated that the inconsistency in the results obtained by gauge and that obtained by the past developed models in the literature were not just because of the impact of pressure losses due to friction as suggested by previous authors however may likely be caused by losses due to kinetic and accumulation experienced by the flowing fluid in a channel. It was also noticed that the impact was most proclaimed at the onset of flow of fluid in conduct where initial transience at the early production time is experienced. The newly modified model was applied to the same field contextual investigation utilized in Ref. [24] and results is significantly more exact and demonstrate that the error margin of the new model is less than 5% comparing with 11.05% obtained using Guo et al. model using the gauge measurement value as the benchmark. This work gives reservoir engineer an exact and helpful device for estimating and assessment of horizontal wells gas production rate.

2. Derivation of mathematical model

Considering Furui et al.'s oil well model [28] that was modified in Ref. [29] to handle the gas stream in horizontal well and reported in U.S. field units as:

$$Q_g(x) = \int_0^x J_{sp}\left[p_e^2 - p_w^2(x)\right]dx \tag{1}$$

The specific productivity index is given as [28].

$$J_{sp} = \frac{k_H}{142.4\mu_g zT\left\{I_{ani}\ln\left[\frac{h}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{h} - I_{ani}(1.224 - s)\right\}} \tag{2}$$

Guo et al. [24] reported that the average velocity of gas (ft/s) at point x in the horizontal

$$v_g(x) = \frac{4.17*10^{-4} Q_g(x) T}{p_{whH} D_h^2} \tag{3}$$

Taking assumption that the gas compressibility is constant and substituting equation (1) into (3) the equation yields [24]:

$$v_g(x) = \frac{4.17*10^{-4} T}{p_{wH} D_h^2} \int_0^x J_{sp}\left[p_e^2 - p_w^2(x)\right] dx \tag{4}$$

Without neglecting any pressure loss term in the governing flow equation in circular pipe, the first law of thermodynamics gives the following relation in U.S. engineering units is reported in Refs. [25–27]:

$$-dp_w(x) = \frac{2 f_f \rho v_g^2(x)}{g_c D_h} dx + \frac{2 \rho v_o(x)}{g_c \Delta t} dx + \frac{2 \rho v_o^2(x)}{g_c} \tag{5}$$

Incorporating the gas density into equation (5), we have [24]:

$$\rho_g = \frac{2.7 \gamma_g p_{wH}}{T} \tag{6}$$

Substituting equation 6 into 5, yields

$$\mathrm{dp_w}(x) = -\frac{0.014 f_f \gamma_g p_{wH} v_g^2(x)}{d_h T} dx + \frac{1.165x10^{-3} \gamma_g p_{wH} v_g(x)}{T \Delta t} dx + \frac{1.165x10^{-3} \gamma_g p_{wH} v_g^2(x)}{T} \tag{7}$$

For simplicity sake, equation (7) can be simply re-arrange as

$$-\frac{\mathrm{dp_w}(x)}{dx} = v_g^2(x) \left(\frac{0.014 f_f \gamma_g p_{wH}}{d_h T} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{v_g^* T \Delta t} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{TL} \right) \tag{8}$$

$$\text{Let } v_g^* = \frac{0.06 T J_{sp}}{p_{wH} d_h^2} \Delta p^{*2} \mathrm{L} \text{ and } \Delta p^{*2} = p_e^2 - p_{wH}^2 \tag{9}$$

Substituting equation (4) into (8) and re-arranging, we have

$$\int_0^x \left[p_e^2 - p_w^2(x)\right] dx = \frac{16.65 p_{wH} d_h^2}{T J_{sp}} \sqrt{\left(\frac{0.014 f_f \gamma_g p_{wH}}{d_h T} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{v_g^* T \Delta t} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{TL} \right)^{-1} \left(-\frac{dp_w(x)}{dx} \right)} \tag{10}$$

Let define

$$C = \frac{16.65 p_{wH} d_h^2}{T J_{sp} \sqrt{\left(\frac{0.014 f_f \gamma_g p_{wH}}{d_h T} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{v_g^* T \Delta t} + \frac{1.165x10^{-3} \gamma_g p_{wH}}{TL} \right)}} \tag{11}$$

Equation (11) can be written as

$$\int_0^x \left[p_e^2 - p_w^2(x)\right] dx = C \sqrt{-\frac{dp_w(x)}{dx}} \tag{12}$$

Equation (12) can be solved, following the procedure reported in Ref. [24], we have

$$p_w(x) = p_e \left[1 - \frac{1}{\left\{ \frac{p_e}{3} \left[C_2 - \left(\frac{3}{C} \right)^{2/3} x \right] \right\}^3 + \frac{1}{3}} \right] \tag{13}$$

$$\text{Where}: C_2 = \left(\frac{3}{C} \right)^{2/3} L + \frac{3}{p_e} \left(\frac{p_e - \frac{1}{3} p_d^*}{p_d^*} \right)^{1/3} \tag{14}$$

Substituting equation (13) into equation (1), gives

$$Q_g(x) = J_{sp} p_e^2 \int_0^x \left[1 - \left[1 - \frac{1}{\left\{ \frac{p_e}{3} \left[C_2 - \left(\frac{3}{C} \right)^{2/3} x \right] \right\}^3 + \frac{1}{3}} \right]^2 \right] dx \tag{15}$$

Equation (15) can be solved using numerical method or following the analytical procedures presented in Ref. [24] as:

$$Q_g(z) = \frac{3 J_{sp} p_e}{\left(\frac{3}{C} \right)^{2/3}} \{ 2[F_1(z_0) - F_1(z)] - [F_2(z_0) - F_2(z)] \} \tag{16}$$

The worked calculation sample using Guo et al. model [24] and newly improved model at steady state condition after 780 days of production are presented respectively in the appendix A of this paper.

3. Model analysis

Using the same data (Table 1) provided in the literature [24], MS EXCEL software was used to calculate production rate of a horizontal well considering all possible forms of losses such as kinetic energy change and fluid accumulation The results obtained were compared with the results obtained from other existing models in the literature, as reported in Table 2. Also, the period of transition from the non-stabilised state flow to stabilised state condition in horizontal drain hole from the reservoir was established.

4. Discussion of results

Fig. 1 shows the variation of flow rate with production time for

Table 1
Fluid and reservoir parameters.

Pay zone thickness, h	60	ft
Boundary distance, yb	1472	ft
Horizontal Permeability, kH	6192	md
Vertical Permeability, kV	619	md
Skin Factor, s	0	
Gas compressibility factor, z	0.91	
Gas gravity, yg	0.7	
Reservoir Temperature, T	580	oR
Reservoir Pressure, Pe	740.67	psi
Drain hole radius, rw	0.354	ft
Effective drain hole Diameter, dh	2.85	in
Drain hole Pressure at heel, pwH	729.5	psi
Drain hole Length, L	311	ft
Drain Hole wall roughness, e	0.01	in
Gas viscosity, µg	0.012	cp
Porosity, ϕ	0.4	
Production time, t	780	Days
Gas compressibility, Cg	0.00135	psi-1

Source: Guo et al. [24].

Table 2
Final model comparison.

Data Source	Measurement (MMscf/D)	% deviation
Real time record [24]	9.95	0
Furui's equation [28]	35.79	259.7
Joshi's equation [1]	121.21	1118.2
Guo's equation [24]	11.05	11.04
This Study	10.39	4.42

horizontal wells. The figure depicts that the flow rate obtained from modified model increases from 0 to approximately 780 days and then stabilizes above 780 days of production time. The newly modified model shows the evidence that there exist an initial transience at the onset of flow which later constant with time, matching up with gauge measurement results obtained at steady state condition. When a gas well is first produced after being shut in for a period of time, a typical flow behaviour of gas passes through a short transition period, after which it attains a steady-state or semisteady (pseudosteady)-state condition. Though Guo et al. model [24] gave better results among other existing models (that is Dikken model [14], Furui et al model [28] and Penmatcha et al model [30]) reported in the literature at steady flow condition while the newly developed model gave more realistic results that evident both the unsteady and steady flow phenomena typical experiences in reality. Guo et al.'s model [24] among the most recent and accurate models stabilize from the beginning of production till the end which is resulted in higher percentage of error when compared with newly developed model using field gauge measurement result as benchmark at the steady flow condition, reported in Table 2. It is generally phenomenon that gas flow rate requires sufficient time to stabilise, that is to reach the pseudo steady or steady state condition which is evident in the new study as shown in Fig. 1.

It is reported in Fig. 2 that as the well bore length increases; an increasing deviation in production rate results obtained by modified model from the Guo et al. model [24] results was obtained with a larger deviation at the longer well bore length. The deviation from

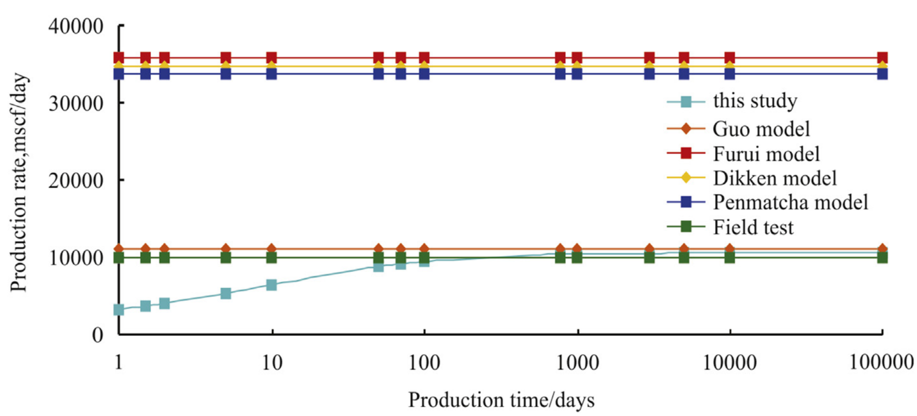

Fig. 1. Plot of flow rate against production time for the existing model [24] and the new model.

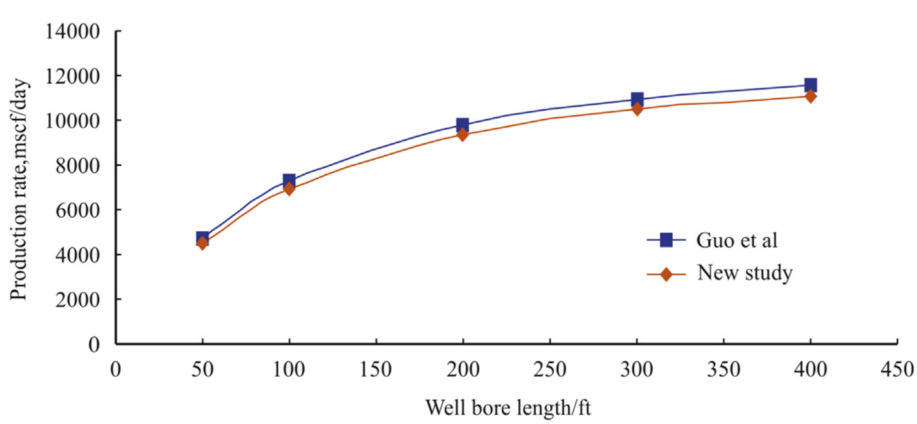

Fig. 2. Graph of flow rate against horizontal well length for the existing model [24] and the new model.

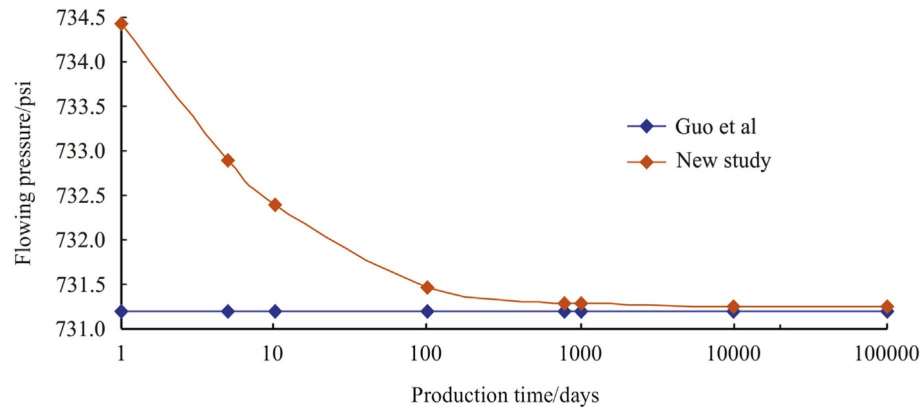

Fig. 3. Plot of flowing pressure against production time for the existing model [23] and the new model.

Guo et al. model [24] implies that their model over-predicts the production rate in gas horizontal drain-hole more due to its failure in considering all possible pressure losses in the governing fundamental thermodynamic equation.

Fig. 3 reports the variation of pressure transverse in a flowing horizontal gas well draining from gas reservoir with production time. The figure depicts that the pressure drop decreases from 0 to approximately 780days and then stabilizes above 780days of production time. The difference in result obtained in Guo model et al. [24] and the new model is the amount of flow restricted by both kinetic energy change and fluid accumulation. This difference is less significant at the later time of production. Thus, it is apparent that there exists an initial transience at the onset of flow which later constant all through with time.

5. Conclusions

This new study is an improvement on Guo model et al. [24], formulating a modified model that checks the impact of all wellbore pressure losses that is pressure drop due to friction, kinetic and accumulation on horizontal well gas flow rate. The accompanying conclusions can be drawn from this undertaking:

(1) The consequences of the new model yields a more exact result (less than 5.0%) of the expected gas production flow capacity from the well than the past models. This fact show that neglected pressure drop due to fluid accumulation and kinetic terms are major factors which cause the discrepancy in the actual gas production rate obtained by measurement and that obtained by existing models in the literature.
(2) Application of the model to a field contextual investigation demonstrates that the results of the new model is significantly more precise than past developed models which did not consider pressure losses due to fluid accumulation and kinetic terms in the governing equation.
(3) A more realistic result that include the initial unsteadiness phenomenon hence predict production rate at any given production time has been established for flow of gas along horizontal well. It was also noticed that the impact was most proclaimed at the onset of flow of fluid in conduct where initial transience at the early production time is experienced.
(4) The newly modified model was applied to the same field contextual investigation utilized [24] and results is significantly more exact and demonstrate that the error margin of the new model is less than 5% comparing with 11.05% obtained in Ref. [24] using the gauge measurement value as the benchmark.
(5) This work gives reservoir engineer an exact and helpful device for estimating and assessment of horizontal wells gas production rate.

6. Recommendation

Comparing the results obtained from the new model with the gauge results reported in Ref. [24] were not perfectly agree at the transient flow period because formulation of Guo et al. model [24] was based on the assumption of steady state flow. They might have reported steady state flow period for their validation. It is therefore highly recommends that the newly developed model should be further validated with field gauge results that will include transient flow period.

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Appendix B. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.petlm.2018.05.001.

Nomenclature

A_h	cross-sectional area, in^2
C_2	constant defined by Eq. (14)
C	constant defined by Eq. (11)
d_h	diameter of drain hole, in
D_h	diameter of drain hole, ft
f_f	Fanning friction factor, dimensionless
g	gravitation due to acceleration, 32.17 ft/s^2
g_c	gravitation due to acceleration, 32.17 ft/s^2
h	net pay zone thickness, ft
I_{ani}	anisotropy factor, dimensionless
J_{sp}	specific productivity index, Mscf/d-psi-ft
k_V	vertical permeability, md
k_H	horizontal permeability, md
L	length of horizontal section, ft
p_{wH}	pressure at the heel of horizontal well, psi
P_e	pressure at reservoir boundary, psi
$p_{w(x)}$	pressure in wellbore at any point x, psi

Q_{gH}	total gas production rate from the well, Mscf/d
$Q_{g(x)}$	flow rate in wellbore at location x, Mscf/d
r_w	wellbore radius, ft
s	skin factor, dimensionless
T	temperature, °R
t	production time, days
$v_{g(x)}$	gas velocity at point x, ft/s
x	distance from the toe of drain hole, ft
y_b	distance of boundary from drain hole, ft
z	the average z-factor, dimensionless

Greeks

ρ_g	gas density, lbm/ft^3
μ_g	the average gas viscosity, cp
ε	wall roughness, in
γ_g	gas specific gravity, air = 1.0

Appendix A. Sample Calculation

Aim:To predict the horizontal well gas flow rate at steady state condition considering all accessible losses in horizontal wellbore using equations derived with the given reservoir and fluid data as shown in Table 1.

Solution procedure for the Guo et al. [24] and newly improved models are presented below:

Guo et al. Model [24].

The specific productivity index is given [28].

$$J_{sp} = \frac{k_H}{142.4\mu_g zT\left\{I_{ani}\ln\left[\frac{h}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{h} - I_{ani}(1.224 - s)\right\}}$$

$$= 0.00808314\text{Mscf/d} - \text{psi}$$

C = constant defined [24]

$$\mathrm{C} = \frac{140.86}{Jsp}\sqrt{\frac{P_{wh}dh^5}{f_f y_g T}} = 3874224.14$$

C_2 = constant defined by Eq. (14)

$$C_2 = \left(\frac{3}{C}\right)^{2/3} L + \frac{3}{p_e}\left(\frac{p_e - \frac{1}{3}p_d^*}{p_d^*}\right)^{1/3} = 0.0426$$

$$z = \frac{p_e}{3}\left[C_2 - \left(\frac{3}{C}\right)^{2/3} x\right] = 4.041$$

$$z_0 = \frac{p_e C_2}{3} = 10.52$$

$$F_1(z) = 3^{-1/3}\left\{\log\left(z + 3^{-1/3}\right) - \frac{1}{2}\log\left(z^2 - 3^{-1/3}z + 3^{-2/3}\right.\right.$$
$$\left.\left. + 3^{-1/3}\arctan\left[\frac{3^{-1/3}}{3}\left(2{*}3^{-1/3}z - 1\right)\right]\right\}$$
$$= 97.08$$

$$F_1(z_0) = 3^{-1/3}\left\{\log\left(z_0 + 3^{-1/3}\right) - \frac{1}{2}\log\left(z_0^2 - 3^{-1/3}z_0 + 3^{-2/3}\right.\right.$$
$$\left.\left. + 3^{-1/3}\arctan\left[\frac{3^{-1/3}}{3}\left(2{*}3^{-1/3}z_0 - 1\right)\right]\right\}$$
$$= 104.05$$

$$F_2(z) = 2F_1(z) + \frac{3z}{3z^3 + 1} = 194.21$$

$$F_2(z_0) = 2F_1(z_0) + \frac{3z_0}{3z_0^3 + 1} = 208.12$$

$$Q_g(z) = \frac{3J_{sp}p_e}{\left(\frac{3}{C}\right)^{2/3}}\{2[F_1(z_0) - F_1(z)] - [F_2(z_0) - F_2(z)]\}$$
$$= 11054\text{Mscf/D}$$

Newly Improved Model

The specific productivity index is given [28].

$$J_{sp} = \frac{k_H}{142.4\mu_g zT\left\{I_{ani}\ln\left[\frac{h}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{h} - I_{ani}(1.224 - s)\right\}}$$

$$= 0.00808314\text{Mscf/d} - \text{psi}$$

C = constant defined in equation (11)

$$C = \frac{16.65 p_{wH} d_h^2}{TJ_{sp}\sqrt{\left(\frac{0.014 f_f \gamma_g p_{wH}}{d_h T} + \frac{1.165x10^{-3}\gamma_g p_{wH}}{v_g^* T \Delta t} + \frac{1.165x10^{-3}\gamma_g p_{wH}}{TL}\right)}}$$

$$= 3565855.503$$

C_2 = constant defined by Eq. (14)

$$C_2 = \left(\frac{3}{C}\right)^{2/3} L + \frac{3}{p_e}\left(\frac{p_e - \frac{1}{3}p_d^*}{p_d^*}\right)^{1/3} = 0.0462$$

$$z = \frac{p_e}{3}\left[C_2 - \left(\frac{3}{C}\right)^{2/3} x\right] = 4.041$$

$$z_0 = \frac{p_e C_2}{3} = 10.88$$

$$F_1(z) = 3^{-1/3}\left\{\log\left(z + 3^{-1/3}\right) - \frac{1}{2}\log\left(z^2 - 3^{-1/3}z + 3^{-2/3}\right.\right.$$
$$\left.\left. + 3^{-1/3}\arctan\left[\frac{3^{-1/3}}{3}\left(2{*}3^{-1/3}z - 1\right)\right]\right\}$$
$$= 97.16$$

$$F_1(z_0) = 3^{-1/3}\left\{\log\left(z_0 + 3^{-1/3}\right) - \frac{1}{2}\log\left(z_0^2 - 3^{-1/3}z_0 + 3^{-2/3}\right.\right.$$
$$\left.\left. + 3^{-1/3}\arctan\left[\frac{3^{-1/3}}{3}\left(2*3^{-1/3}z_0 - 1\right)\right]\right\}$$
$$= 104.2$$

$$F_2(z) = 2F_1(z) + \frac{3z}{3z^3 + 1} = 194.38$$

$$F_2(z_0) = 2F_1(z_0) + \frac{3z_0}{3{z_0}^3 + 1} = 208.4$$

$$Q_g(z) = \frac{3J_{sp}p_e}{\left(\frac{3}{C}\right)^{2/3}}\{2[F_1(z_0) - F_1(z)] - [F_2(z_0) - F_2(z)]\}$$
$$= 10390\mathrm{Mscf/D}$$

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