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Research Article

Successive Approximation of Implicit Multistep Type Iterative Algorithms in Locally Convex Spaces

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Abstract

Background and Objectives: The application of implicit fixed point iterative algorithms have been greatly employed in many physical systems as the implicit algorithms provide better approximation than their corresponding explicit algorithms and are very efficient in reducing the computational cost of the fixed point problems. The objectives of this study, therefore; were in three

fold: (1) To develop implicit hybrid Jungck-Kirk multistep iterative algorithms in a metrizable locally convex space, (2) Prove its convergence to the unique common fixed point of a pair of weakly compatible generalized contractive-type operators (S, T) and (3) Demonstrate the application of the convergence results with some examples. **Materials and Methods:** Analytical method was used to prove the main theorem, while numerical method was to demonstrate the application of the convergence result. **Results:** Strong convergence analytical and numerical results constitute the main results of this work. **Conclusion:** The results obtained from this study showed that the implicit hybrid Jungck-Kirk multistep iterative algorithms have good potentials for further applications, especially in relation to rate of convergence.

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INTRODUCTION

A locally convex space (X, u) with topology u is a topological vector space which has a local base of convex neighborhood of zero. It is metrizable if it is Hausdorff and has countable zero basis.

Consequently, it is metrizable if u can be described by a countable family of continuous seminorms¹. The X is Hausdorff if and only if for each non-zero $x \in X$, there is some² $p \in Q$ with $p(x) > 0$. To each absolutely convex absorbent subset U of X corresponds a seminorm p , called the gauge of U defined by $p(x) = \inf \{ \lambda : \lambda > 0, x \in \lambda U \}$ and with the property that $\{x : p(x) < 1\} \subseteq U \subseteq \{x : p(x) \leq 1\}$, U is a neighbourhood of zero if and only if p is continuous.

Many researchers have worked on the approximation of fixed point of different classes of operators in literature. For instance³⁻⁵. The Kirk-Mann, Kirk-Ishikawa and Kirk-Noor iterative algorithms are the commonly used schemes for approximating the fixed point of a given operator. Various authors have written very inspiring papers on Kirk-type iterative algorithms, worthy to mention are the following: the explicit Kirk-Mann, explicit Kirk-Ishikawa⁵, Kirk-Noor and Kirk-multistep⁶ iterative schemes. The rate of convergence of Kirk-type schemes for single mappings was proved in Hussain *et al.*⁷.

Akewe *et al.*⁶ proved strong convergence and stability results for explicit Kirk-multistep iterative schemes by employing a contractive-like operator in a normed linear space through useful theorems and numerical examples. For explicit iterative scheme