Asian Journal of Scientific

Research

Volume 12 (3): 298-307, 2019

Research Article

Equivalence of Picard-type Hybrid Iterative Algorithms for Contractive Mappings

Kanayo Stella Eke and Hudson Akewe

Abstract

Background and Objective: Fixed point iterative algorithms are designed to be applied in solving equations arising in physical formulation but there is no systematic study of numerical aspects of these iterative algorithms. The Picard, Mann, Ishikawa, Noor and multi step iterative algorithms are the commonly used iterative algorithms in proving fixed point convergence and stability results of different classes of mappings. The objectives of this study therefore were: (1) To develop a Picard-type hybrid iterative algorithm called Picard-Mann, Picard-Ishikawa, Picard-Noor and Picard-multistep iterative algorithms, (2) Prove equivalence of convergence theorems using these algorithms for a general class of mappings in a normed linear space and (3) Provide numerical examples to justify the applicability of the algorithms. Materials and Methods: Analytical method was used to prove the main theorem, while numerical method was to demonstrate the application of the equivalence results. Results: Strong convergence, equivalence and numerical results constitute the main results of this study. Conclusion: The results obtained from this study showed that the Picard-type hybrid iterative algorithms have good potentials for further applications, especially in terms of rate of convergence.

How to cite this article:

Kanayo Stella Eke and Hudson Akewe, 2019. Equivalence of Picard-type Hybrid Iterative Algorithms for Contractive Mappings. Asian Journal of Scientific Research, 12: 298-307.

DOI: 10.3923/ajsr.2019.298.307

URL: https://scialert.net/abstract/?doi=ajsr.2019.298.307

INTRODUCTION

The application of fixed point theory play a vital role in many areas of mathematics and some of the methods therein are used in solving problems in various branches of biology, chemistry, economics and other mathematical sciences. The existence of solution in ordinary differential equation has a close link with the fixed point of a given iterative algorithm. The convergence to the fixed point of a given iterative algorithm under some contractive conditions correspond the solution of the ordinary differential equations. The commonly used iterative algorithms introduced by notable authors in proving the convergence and stability results of different classes of mappings are: Picard1, Mann2, Ishikawa3, Noor4 and multistep5 iterative schemes.

Let (X, d) be a complete metric space and T: X→X be a selfmap of X. Assume that FT = {pεX: Tp = p} is the set of fixed points of T. For a0εX, the Picard iterative algorithm1defined by:

(1)

has been employed to approximate the fixed points of mappings satisfying the inequality relation:

(2)

for every a, b ε X.

Khan6 introduced a different perspective to fixed point iteration algorithms by presenting the Picard-Mann hybrid iterative algorithm for a single non-expansive mapping. It was shown that this type of algorithm is independent of Picard1, Mann2 and Ishikawa3 iterative algorithms since {αn} and {βn} are in (0, 1). Furthermore, he proved that the Picard-Mann hybrid algorithm6 converges faster than Picard1, Mann2 and Ishikawa3 iterative algorithms in the form of the result of Berinde7 for contractions. It also proved strong convergence and weak convergence theorems with the help of his newly introduced iterative process for the class of non-expansive mappings in a general Banach space and applied it to obtain results in a uniformly convex Banach space.

It is worthy to remark here that many researchers have proved useful results on the equivalence of the various iterations, that is, they have shown that the convergence of any of the given iterative algorithm to the unique fixed point of the contractive operator for single mapping T is equivalent to the convergence of the other iterations. Chief among these are the results of Olaleru and Akewe8, Solutuz9 and Soltuz10. However, only very few equivalence results are known of the Picard-type hybrid iterative algorithms. This study will address these areas.

The study of the Khan6 is the main motivation of this study. While the Khan6 worked on the rate of convergence of Picard-Mann iterative algorithm for non-expansive mappings, so the aim of present study was to prove the equivalence of convergence of Picard-multistep iterative algorithms for contractive mappings. Thus, this study was divided into three phases: Firstly, a Picard-multistep iterative algorithm was developed and a strong convergence result is proved for a general class of contractive mapping. Secondly, shown that the convergence of this Picard-multistep algorithm is equivalent to the convergences of Picard-Noor, Picard-Ishikawa, Picard-Mann and Picard iterative algorithms for the same class of contractive mappings. Finally, with help of numerical examples, the equivalence results were demonstrated to be applicable in the real sense.

MATERIALS AND METHODS

Relevant materials from reputable journals are used to identify open problems and possible ways of solving them6,8,10. The research methods employed in this study are both analytical and numerical. The analytical approach is used in proving the main theorem, while the numerical aspect is done in the examples. The following iterative algorithms are useful in proving the main results.

Let (E, ‖.‖) be a normed linear space and D a non-empty, convex, closed subset of E and T:D→D be a selfmap of D. Let x0ε D, then, the sequence defined by:

(3)

where,, 1< i < k-1. Equation 3 is called Picard-multistep hybrid iterative algorithm.

For an initial point c0ε D, the sequence is defined by:

(4)

where, Equation 4 is called Picard-Noor hybrid iterative algorithm.

For an initial point b0ε D, the sequence is defined by:

(5)

where, Equation 5 is called Picard-Ishikawa hybrid iterative algorithm.

For any initial point u0ε D the sequence is defined by:

(6)

where, Equation 6 is called Picard-Mann hybrid iterative algorithm6.

It shall now consider some of the contractive mappings useful in proving our main results.

Let E be a normed linear space and D a non-empty, convex, closed subset of E and T: D→D be a selfmap of. There exists a real number δ ∈ (0, 1) and all x, y ∈ D such that:

(7)

Zamfirescu11 discussed mappings T satisfying the following contractive condition:

(8)

where, δ ∈ (0,1).

Inequality Eq. 8 becomes Eq. 7 if x is a fixed point of T.