PORTFOLIO THEORY AS TOOL FOR SHAREHOLDER’S HEDGE AGAINST RISK AND MAXIMIZATION OF RETURNS: A CASE STUDY

BY

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ABSTRACT

The global crisis that has ravaged economies the world over has not only dealt a big blow to many nations but also the portfolio of investors including Nigeria. A discussion on portfolio theory is appropriate at this time as investors concerned should not only be focused on returns, but also on associated risk of the investment proposed or held. The objective of this paper is to shed light on the importance of diversification of securities of investors. The data analysis used in this paper is descriptive; based on two hypothetical models A and B. The analysis of the data shows that best result is obtained when the securities are negatively correlated, as the level of risk will be at minimum.
INTRODUCTION:

The continued privatization and commercialization of government agencies by the Federal Government has generated much interest in share ownership in companies, with over-subscription for the shares so far offered in some cases. The recent N25 billion recapitalization of banks in 2005 has further heightened the interest in share ownership. Some banks have their shares oversubscribed than others. The same scenario applies to other sectors of the economy. From the point of view of an investor the likely situation is that of continuous accumulation of ordinary shares in different companies without regard to the ways in which these investments are related. The objective of this paper is to shed light on the importance of diversification of securities of investors. A discussion on portfolio theory is appropriate at this time as investors concern should not only be focused on returns, but also on associated risk of the investment proposed or held.

It is crucial at this period of global financial meltdown when prices of shares have nosedived without any guarantee that the prices of these shares will rise in the foreseeable future. However, this is the hallmark of stock trading as it is often associated with ups and down (bullish and Bearish period).

The treatment of portfolio theory in this write-up will be examined basically on the theoretical background but it is hoped that it would have stirred up much interest to encourage investors to re-examine the composition of their “basket of investments.” It will also serve as a summary of required knowledge of financial management students, lecturers, the public and references provided will serve as a good overview of the topic. Section 1 dwells on the introduction while Section 2 explains portfolio theory measurement of portfolio risk. Section 3, 4, 5 and 6 sheds light on review of literature,
methodology, data analysis and relevance of risk-free security. The paper ends with conclusion and recommendation.

PORTFOLIO THEORY MEASUREMENT OF PORTFOLIO RISK

A portfolio is merely a collection of investments which are created to diversify holdings of wealth and to achieve, simultaneously, low risk and high returns. The investors, in choosing a security, will be concerned about its expected returns and about the degree of risk associated with that return. An investor can be a risk lover, risk neutral and risk averter. The usually accepted assumption is that he will be averse to risk, but will seek to maximize his return. The risk of portfolio is a function of other securities constituting the portfolio. By selecting securities that have little relationship with each other, an investor is able to reduce relative risk. The combination of securities in such a way as to reduce risk is known as diversification. Risk can be defined as the possibility that the actual return will deviate from the expected return. It is measured by standard deviation, $\delta$ or the measure of variance $\delta^2$. This can further be validated by the coefficient of variation $(\delta/x)$.

Generally, however the effect of diversification on the risk is dependent on the correlation between the possible returns of investment comprising the portfolio. The reduction in risk achieved by diversification depends, ceteris paribus, on the coefficient of correlation. The best results are obtained when the two assets are negatively correlated with the optimum being where the assets are perfectly negatively correlated. However, an investor may concentrate his investment in a particular sector e.g bank. Most times investors do this out of emotion without any financial analysis of the share worth.
Combination of securities A and B will result in less risk than that attached to each of the securities separatively, provided that the correlation is not too high and positive. It is possible to set down some general rules, which will enable the investor to select a set of strategies, which appear better than others viz

(a) When the investor is faced with selecting two portfolios with the same return and different risk, he will choose the one with the lower risk.

(b) If two portfolios have the same risks and different returns, he will prefer the one with the higher return.

(c) If one portfolio has both a higher return and a lower risk than another, the investor will prefer the first portfolio.

Based on the foregoing assumptions and also considering the investor who is averse to risk, he will only be interested in risk and returns, and not in any further aspect of the security. In some exceptional cases, the investor will be concern about the growth of his investment rather than immediate capital (cash) that is return on his investment.

**REVIEW OF EARLIER LITERATURE**

Shim and Siegel (1998) explained portfolio return as the expected return on a portfolio of assets and the weighted average return of individual assets in the portfolio is the fraction of investment in the total funds. An efficient combination of assets and its foundation lies in the work of Markowitz. According to Dobbins and Witt (1988) the assumptions underlying the model are as follows:

1. The return on an investment adequately summarizes the outcome of the investment, and investors visualize a probability distribution of rates of return
2. Investors’ risk estimates are proportional to the variance of return they perceive for a security or portfolio.

3. Investors are willing to base their decisions on just two parameters of the probability distribution function - the expected return and variance of return.

4. The investor exhibits risk aversion, so for a given expected return he prefers minimum risk. Obviously, for a given level of risk the investor prefers maximum expected return.

Correlation Coefficient is very important in the measuring of portfolio risk. Van Horne and Wachowicz (2001) posited that for standardized statistical measure of the linear relationship between two variables, it ranges from −1.0 (perfect negative correlation), through 0 (no correlation) to +1.0 (perfect positive correlation). Covariance is statistical measure of the degree to which two variables (e.g. securities’ return) move together. A positive value means that, on average, they move in the same direction. The total risk of a portfolio is measured by the standard deviation of the probability distribution of possible security returns. In their study, Bodie and Marcus (2003) observed that the low risk of portfolio is due to the inverse relationship between the performances of the two funds: Stock fund and Bond fund. In a recession, stocks fare poorly, but this is offset by the good performance of the fund. Conversely, during boom scenario, bonds fall, but stocks do well. Therefore, the portfolio of the two risky assets is less risky than either asset individually. Portfolio risk is reduced most when the returns of the two assets most reliably offset each other. In portfolio management, Strong (2000) emphasize six aspects: Learning the basic principles of finance, setting portfolio objectives,
formulate investment strategy, have a game plan for portfolio revision, performance evaluation and protect the portfolio when appropriate. Bodie and Marcus (2005) found that seemingly risky securities might be portfolio stabilizers and actually low-risk asset. Myers (2003) found that stock’s contribution to the risk of a fully diversified portfolio depends on its sensitivity to market changes. Correlation Coefficient measures the degree to which two variables move together. This sensitivity is generally known as beta $\beta$. Jordan, Westerfield and Ross (2001) in his study, observed that spreading an investment across a number of assets will eliminate some, but not all of the risk. The expected return on a risky asset depends only on that asset’s systematic risk and it is measured by Beta Coefficient- Greek Symbol $\beta$. Beta Coefficient is the amount of systematic risk present in a particular risky asset relative to that in an average risky asset. Brealey and Myers (1996) posited that lending and borrowing extend the range of investment possibilities. If you invest in portfolio $S$ and lend or borrow at the risk-free interest rate $rf$, you can achieve any point along the straight line from $rf$ through $S$. This gives you a higher expected return for any level of risk than if you just invest in common stock. In the diagram below, fig.1 at equilibrium no stock can lie below the Security Market Line (SML). For example, instead of buying stock A, investors would prefer to lend part of their money and put the balance in the market portfolio.
And instead of buying stock B, they would prefer to borrow and invest in the market portfolio. Capital Asset Pricing Model (CAPM) is a method of expressing the risk-return relationship for a security or portfolio of securities: it brings together systematic (Undiversifiable) risk and return. When CAPM equation is shown in graph form, the resultant straight line: \( y = ax + c \) is referred to as Security Market Line (SML). It is line which exhibits the positive relationship (correlation) between systematic risk of a security and its expected return. The SML represents the level of return expected in the market for each level of the share’s beta (market risk), thus explains the risk-return trade-off. Let us assume that there are three portfolios with associated risks and return characteristics:

<table>
<thead>
<tr>
<th>PORTFOLIO</th>
<th>RETURN%</th>
<th>( \delta )%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>B</td>
<td>12.0</td>
<td>6.0</td>
</tr>
<tr>
<td>C</td>
<td>12.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>
If the investor has a choice between A and B, he will choose B because it has a higher return than A even though they are at the same risk level. Choice between B and C will be in favour of C because of its lower risks. If the investor has a choice between the three portfolios he will choose C because of its higher returns and lower risk. This can be depicted graphically thus:

Fig. 2

The expected returns ($R_p$) and risk ($\delta$) of a portfolio can be calculated by the following formula, which is applicable for a simple two-asset portfolio security A, and B.

$$R_p = (R_A) + (1 - X) (R_B) \quad \text{Equation i}$$

$$P = \sqrt{X^2 \delta A^2 + (1 - X)^2 \delta B^2 + 2X (1 - X) \delta A \delta B \text{COR}_{AB}} \quad \text{Equation ii}$$

or $$\delta p = \sqrt{A^2 + B^2 + 2ABr} \quad \text{Equation iii}$$
Where:

\( \text{COR}_{AB} \) or \( r = \text{Coefficient of Correlation} \) of the possible returns from securities A and B.

\( X = \) the probability of returns A; \((1 - X) = \) the probability of return B. Equation (iii) is a simplified version of (ii) where: \( A = \delta \) of \( A \) x weight or probability of A; \( B = \delta \) of \( B \) x weight or probability of B; \( r = \) coefficient of correlation. For individual portfolio,

Expected Returns \( \bar{R} = R_1 (P) + R_2 (P) \)

Where: \( \bar{R} = \) Expected returns; \( R = \) possible returns; \( P = \) probability of possible returns

and Standard deviation \( (\bar{\delta}) = \sqrt{\sum (R-R)^2} \times P \)

**METHODOLOGY**

The method used in this paper consists of primary data generated by the author, and secondary data elicited from relevant textbooks. To summarize portfolio risk measurement, we can examine the following problem. Global Financial Meltdown Company is considering investments in one or both of two securities A and B, and we have the following information:

<table>
<thead>
<tr>
<th>Security</th>
<th>Possible %</th>
<th>Probability of Return occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40 (A₁)</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>20 (A₂)</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>60 (B₁)</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>10 (B₂)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The problem now is how we manage this investment of the investor so to maximize return and minimize risk. This can be done by:

(a) Calculating the expected return and standard deviation of each security separately, and the expected value and standard deviation of a portfolio: For example in the
investment comprising 80% of A and 20% of B, assuming no correlation between possible returns, and (b) we can calculate the expected value of portfolio return and its associated risk using an appropriate formula for cases 1-11.

i. Case I: Perfect positive correlation

ii. Case II: Perfect negative correlation

iii. Case III: Zero correlation

DATA ANALYSIS

The data analysis used in this paper is descriptive; based on two hypothetical models A and B. The expected return, expected probability of return and degree of correlation of securities A and B are shown on Tables 1, 2 and 3 below.

(a) Security A

i. \( R_A = (40\% \times .5) + (20\% \times .5) = 30\% \)

\[
\begin{array}{c|c|c|c|c}
R & R_A & (R - R_A) & (R - R_A)^2 & P \\
40 & 30 & 10 & .5 & 50 \\
20 & 30 & 10 & .5 & 50 \\
\hline
\end{array}
\]

\( \Sigma (R - R_A)^2 P = 100 \)

\( \delta A = \sqrt{100} = 10\% \)

Portfolio of 80% of A and 20% of B

ii. \( R_B = (60\% \times .5) + (10\% \times .5) = 35\% \)

\[
\begin{array}{c|c|c|c|c|c|c|c}
R & R_B & (R - R_B) & (R - R_B)^2 & P & (R - R_B) & (R - R_B)^2 & P \\
60 & 35 & 25 & 625 & .5 & 312.5 & 625 & .5 \\
10 & 35 & 25 & 625 & .5 & 312.5 & 625 & .5 \\
\hline
\end{array}
\]

\( \Sigma (R - R_B)^2 P = 625.0 \)

\( \delta B = \sqrt{625} = 25\% \)

iii. Expected return \( \bar{R}_p = x \bar{R}_A + (1 - x) \bar{R}_B \)

\( = 0.8(30\%) + 0.2(35\%) = 31\% \)

Table 1: Expected Returns of Securities

<table>
<thead>
<tr>
<th>Return</th>
<th>Return % (Rp)</th>
<th>(Rp - \bar{Rp})^2</th>
<th>Pp</th>
<th>(Rp - \bar{Rp})^2 Pp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 B1</td>
<td>50</td>
<td>361</td>
<td>0.25</td>
<td>90.25</td>
</tr>
<tr>
<td>A1 B2</td>
<td>25</td>
<td>36</td>
<td>0.25</td>
<td>9.00</td>
</tr>
<tr>
<td>A2 B1</td>
<td>40</td>
<td>81</td>
<td>0.25</td>
<td>20.25</td>
</tr>
<tr>
<td>A2 B2</td>
<td>15</td>
<td>256</td>
<td>0.25</td>
<td>64.00</td>
</tr>
</tbody>
</table>

\( \delta p = \sqrt{183.50} \)

\( = 13.55\% \)
(b) With 80% of available resources invested in A and 20% in B, four conditions are assumed to be possible, viz; for the returns of A and B: high, high (H H); low, low (L L); high, low (H L); and low, high (L H). If there is perfect positive correlation as in case I, only conditions of HH and LL can apply with 50% probability of each. In case II, only conditions of HL and LH can apply also with 50% probability of each. With case III of zero correlation, any of the four conditions may arise with an assumed equal probability of occurrence of 25%.

Table 2: Expected Value of Portfolio Return

<table>
<thead>
<tr>
<th>Assumed condition</th>
<th>Expected value of portfolio return 80% invested in A, 20% invested in B</th>
<th>Probability of return according to correlation assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perfectly positive I</td>
<td>Perfectly negative II</td>
</tr>
<tr>
<td>HH High for A and B</td>
<td>A 0.8x40% = 32%</td>
<td>B 0.2x60% = 12%</td>
</tr>
<tr>
<td>LL Low for both A and B</td>
<td>A 0.8x20% = 16%</td>
<td>B 0.2x10% = 2%</td>
</tr>
<tr>
<td>HL High for A Low for B</td>
<td>A 0.8x40% = 32%</td>
<td>B 0.2x10% = 2%</td>
</tr>
<tr>
<td>LH Low for A high for B</td>
<td>A 0.8x20% = 16%</td>
<td>B 0.2x60% = 12%</td>
</tr>
</tbody>
</table>

The expected value of portfolio returns will now be used to compute the expected returns and standard deviation thus:

\[ R_P, R_P - (R - \bar{R}), (R - \bar{R})^2 P, \]
Case I  Perfect positive correlation

<table>
<thead>
<tr>
<th></th>
<th>HH</th>
<th>LL</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>44%</td>
<td>18%</td>
<td>31%</td>
</tr>
<tr>
<td>LL</td>
<td>18%</td>
<td>34%</td>
<td>31%</td>
</tr>
</tbody>
</table>

\[ \delta = \sqrt{169} = 13\% \]

Case II  Perfect negative correlation

<table>
<thead>
<tr>
<th></th>
<th>HL</th>
<th>LH</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL</td>
<td>34%</td>
<td>28%</td>
<td>31%</td>
</tr>
<tr>
<td>LH</td>
<td>28%</td>
<td>34%</td>
<td>31%</td>
</tr>
</tbody>
</table>

\[ \delta = \sqrt{9} = 3\% \]

Case III  Zero

<table>
<thead>
<tr>
<th></th>
<th>HH</th>
<th>LL</th>
<th>HL</th>
<th>LH</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>44%</td>
<td>18%</td>
<td>34%</td>
<td>28%</td>
</tr>
<tr>
<td>LL</td>
<td>18%</td>
<td>34%</td>
<td>28%</td>
<td>44%</td>
</tr>
</tbody>
</table>

\[ \delta = \sqrt{89} = 9.4\% \]

(C) The formula earlier given can now be used i.e.

\[ \delta_p = \sqrt{x^2\sigma^2_A + (1-X)^2\sigma^2_B + 2x(1-x)\delta A \delta B \ CORAB} \]

i. For perfect positive correlation the corr coefficient = +1

\[ \delta_p = \sqrt{(0.8)^2 (0.1)^2 + (0.2)^2 (0.25)^2 +2 (0.8) (0.2) (0.1) (0.25) (+1)} \]
\[ = 0.0064 +0.0025 +0.008 \]
\[ = \sqrt{0.0169} = 0.13 \text{ or } 13\% \]

ii. For perfect negative correlation the corr. coefficient = -1

\[ \delta_p = \sqrt{(0.8)^2 (0.1)^2 + (0.2)^2 (0.25)^2 +2 (0.8) (0.2) (0.1) (0.25) (-1)} \]
\[ = 0.0064 + 0.0025 - 0.008 \]
\[ \sqrt{0.0009} = 0.03 \text{ or } 3\% \]

iii. For Zero correlation, the correlation = 0

\[ \delta_p = \sqrt{(0.8)^2 (0.1)^2 + (0.2)^2 (0.25)^2 +2 (0.8) (0.2) (0.1) (0.25) (0)} \]
\[ = 0.0064 + 0.0025 - 0 \]
\[ \sqrt{0.0089} = 0.094 \text{ or } 9.4\% \]
Hence, answers here agreed in all respect with those obtained in (b). Whenever possible always use the formula method as it is faster once the correlation coefficient are given or assumed. In summary, the individual and portfolio results are:

**Table 3: The Degree of Correlation**

<table>
<thead>
<tr>
<th>Expected Returns ($R_p$)</th>
<th>Security A</th>
<th>Security B</th>
<th>80% A+</th>
<th>Degree of correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk ($\sigma_p$)</td>
<td>30%</td>
<td>35%</td>
<td>31%</td>
<td>-Ve</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>25%</td>
<td>13.55%</td>
<td>O</td>
</tr>
</tbody>
</table>

The best result is obtained when the securities are negatively correlated, as the level of risk will be at minimum. Portfolio theory makes us aware that an investor should not be concerned with the risk attached to an individual but rather with the overall risk of the portfolio. It is possible by diversification to reduce the overall risk of an investment portfolio.

**PORTFOLIO ANALYSIS AND SELECTION**

**Fig 3**

![Graph](image-url)
To obtain the best combination of securities will depend on the investor calculation of expected value of return and standard deviation and this is a function of his utility as shown in Fig 3. The curves are known as indifference curves, the investor is different between any combination of expected value of return and standard deviation on a particular curve. The combination of expected return and standard deviation result in a fixed level of expected utility. The greater the slope of the indifference curve the more averse the investor to risk. As we now move to the left in Fig 3, each successive curve represents a higher level of expected utility.

The individual investor will want to hold that portfolio of securities that places him on the highest indifference curve, choosing it from the opportunity set of available portfolios Fig 4. This opportunity set reflects all possible portfolios of securities as envisioned by the investor. Every point in the shaded area represents a portfolio that is attainable by the
investor. The double line, FF, at the top of set is the line of efficient combinations or the efficient frontier. It depicts the trade off between risk and expected value of return.

The objective of the investor is to choose the best portfolio from those that lie on efficient frontier. The portfolio with the maximum utility is the one at the point of tangency of the efficient frontier with the highest indifference curve, Fig 5, point M.

**PRESENCE AND RELEVANCE OF RISK-FREE SECURITY**

If a risk- free security exists, and the investor is able not to lend but to borrow it as well, then to determine the optimal portfolio under these conditions, we first draw a line from the risk-free rate RF in Fig 6, through its point of tangency with the efficient frontier FF. This line then becomes the new efficient frontier. Any point on the straight line tells us the proportion of the risky portfolio, M, and the proportion of loans or borrowings at the risk free rate. To the left of point M, the investor will hold both risk-free security and portfolio M. To the right he would hold only portfolio M and would borrow funds in order to invest further in it. The capital market line is an expression of the optimal relationship between risk and the expected rate of return on what Markowitz referred to as efficient portfolios. An efficient portfolio is one which maximizes return for a given level of risk. The optimal investment policy of tangency is between the straight line RF-M-CML and the highest indifference curve. If borrowing were prohibited, the efficient frontier will no longer be a straight line through out but would consist of line RF- M-F1. The straight line is called Capital market line (CML) and it describes the tradeoff between expected return and risk for various holdings of the risk-free security and the market portfolio. The slope of the CML represents the market price of risk.
CONCLUSION AND RECOMMENDATION

In the construction of the CML, the utility preferences of the individual affect only the amount that is borrowed or loaned; they do not affect the optimal portfolio of risky assets. Thus, the individual’s utility preferences are independent or separate from the optimal portfolio of risky assets. This condition is known as the separation theorem; it states that the determination of an optimal portfolio of risky assets is independent of the individual’s risk preferences. Looked at from another angle, the introduction of risk-free security in Fig.6 implies that only M amongst all the previously efficient set of risky portfolio has survived the introduction in analysis of the risk-free security. The selection of a risky portfolio is, thus, separated from the problems of selecting a portfolio of risk-free and risky securities because there is only one optimal portfolio, which is the market portfolio at M.

In order for the investor to hedge against risk and also maximize return of his investments, the key fundamental remains the price of purchase of the securities which also should determine time and price of sale of such investment. This is very pertinent because investors purchase securities at different times (e.g. year). In the risk-return trade off, emotional buying and selling of securities should as much as possible be avoided in the process of diversifying investment. The rationale investor should spread his portfolios amongst all sectors of the economy to minimize risk. In addition, investors are advised to consult a financial expert to assist in analyzing the financial statements of the organization before putting their money in securities.
REFERENCES


