International Journal of Civil Engineering and Technology (IJCIET)
Volume 10, Issue 01, January 2019, pp. 2474–2483, Article ID: IJCIET_10_01_222
Available online at http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=10&IType=1
ISSN Print: 0976-6308 and ISSN Online: 0976-6316

ON MODELLING TAIL RISK OF ELECTRICAL ENERGY PRODUCTION LEVEL

Olumide S. Adesina
Department of Mathematical Sciences, Olabisi Onabanjo University,
Ago-Iwoye, Ogun State, Nigeria

Tolulope F. Oladeji
Department of Banking and Finance, Covenant University,
Ota, Ogun State, Nigeria

Pelumi E. Oguntunde
Department of Mathematics, Covenant University,
Ota, Ogun State, Nigeria

Remi J. Dare
Department of Mathematical Sciences, Kings University,
Ode-Omu, Osun State, Nigeria

ABSTRACT

Fitting the correct type of model to a particular set of data is key to proffering solution to bugging issues. Electricity production in Nigeria has been faced with various challenges for over ten decades since the first production and supply. Consequently, there is a need to measure electricity production risk. Extreme Value Theory (EVT) is considered sufficient in measuring such risk by modelling tails of the distribution. Adopting EVT, there is a need to measure Value-at-risk and Expected Shortfall which can be adequately done with Generalized Pareto Distribution (GPD); one of the models for extreme events. The preference for GPD is because it models the distribution of exceedances over a high threshold rather than the individual observations. In this study, diagnostics tests were carried out in order to determine the suitability of GPD for fitting the data, and GPD was found adequate modelling future risk of electricity production for the given data. The GPD was then used to fit the electricity production data in Nigeria at 1%, 0.5%, and 0.1% probability. Following the result, measures to avoid electricity production risks were recommended.

Key words: Value at Risk, Extreme Value Theory, Peak Over Threshold, Generalized Pareto distribution, Electricity.

Cite this Article: Olumide S. Adesina, Tolulope F. Oladeji, Pelumi E. Oguntunde, Remi J. Dare, On Modelling Tail Risk of Electrical Energy Production Level, *International Journal of Civil Engineering and Technology (IJCIET)* 10(1), 2019, pp. 2474–2483.
http://www.iaeme.com/IJCIET/issues.asp?JType=IJCIET&VType=10&IType=1

1. INTRODUCTION

Production and supply of electricity over the years in Nigeria has never been adequate since 1896 when electricity generation activities started. There has been high level of uncertainties as regards optimal level production of electricity despite resources and experts in the field of science and engineering. According to Apudo *et al*., (2014), Africa in general suffers uncertainty as regards electricity production; these challenges practically collapse businesses, economic and social activities attract criticism from the citizenry, and there is consistent anxiety as to when the challenges would be over. In the face of these challenges, there is a need to model the rare events which has damaging consequences; extreme value theory is found suitable to model such events. Manfred & Elvis (2006) mentioned that EVT helps to identify the extent to which a rare and damaging effect can go if it eventually happens.

Extreme value theory (EVT) explains occasion where there is likelihood of rare or damaging events occurring. Extreme value distribution arises as stochastic behaviour for maximums or minimums of some identically and independently distributed random variables. EVT is the theory of modelling and measuring events which occur with very small probability (McNeil 1997). EVT is applicable to various fields of studies; engineering (Velazquez *et al*., 2014), athletics (Einmahl & Magnus, 2008), hydrology (Smith, 2003), extreme fire (Alvarado-Celestino, 1992), and health (Zhao, 2010). In the area of risk management, finance, and insurance, we have the works of Rufino & de Guia (2011), Hu (2013), Uppal (2013), Wainnaina & Waititu, (2014), Adeleke *et al*., (2015), Adesina *et al*., (2016) and many others.

A number of studies have been carried out on modelling electricity production and management. For instance, Sebetoi & Okou (2010) studied the transmission of electric energy in Sub-Saharan African and waste that follows its transmission, the authors came up with electricity tariff models on energy efficient use. Mhilu (2007) used multiple regression technique for the development of regional equations for the estimation of low flow regimes of small settlements. Mbugua *et al*., (2012) discussed how quantile estimation and extreme value theory can be used to model extreme daily rate of change in domestic consumption of electricity. Apudo *et al*., (2014) applied EVT to identify the minimum electric production level in Kenya. Chan & Gray (2005) carried out a comparative study of EVT and EGARCH to measure value-at-risk for daily electricity spot prices, the study suggests that EVT outperforms EGARCH. Rodrigo & Nicolás (2012) conducted a study on value at risk of electricity markets through modelling inter-exceedances times using the Generalized Pareto model. The authors applied the technique to four main electricity markets in Australia and results were compared with traditional models, the result was in favour of GP model.

In this present study, EVT was adopted and it requires that values be taken from the tails of its distribution in order to measure tail risk. Longin (1999) opined that over the years, extreme value theory has been a reliable tool which attempts to provide the best possible estimate of the tail area of the distribution. Following EVT technique, this study aim to adopt the value-at-Risk (VaR) and Expected Shortfall to measure risk associated with electricity production in Nigeria. A similar study by Adesina *et al*., (2016) presented the performance of EVT for VaR and ES estimation and compared with the Gaussian and Historical simulation. The result obtained by the authors showed that EVT can predict risk more accurately relative to the Gaussian and Historical simulation, particularly using the Generalized Pareto distribution. The remaining part of this paper is sectionalised as follows; in section 2, the Materials and method are presented. In section 3, application and discussion of results are made, and in section 4, the findings are summarised with recommendation given.

2. MATERIALS AND METHOD

2.1. Generalized Pareto Distribution

From the study of Pickands (1975) and Balkema & de Haan (1974), the cumulative distribution function (cdf) of a two-parameter GPD distribution; the conditional excess distribution function $F_u(y)$ for a large value of μ is given by:

$$F_\mu(y) = G_{\beta,\xi}(y) : \mu \to \infty$$

$$G_{\beta,\xi}(y) = 1 - (1 + \xi^{\frac{y}{\beta}})^{-1/\xi}; \xi \neq 0 \qquad (1)$$

$$= 1 - \xi^{-\frac{y}{\beta}}, \xi = 0$$

where $\beta > 0, y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq -\beta/\xi$ when $\xi < 0$

Maximum likelihood estimate (MLE) is approximated by:

$$L(\xi, \beta; Y_j) = -N_u In\beta + \left(\frac{1}{\xi} - 1\right)\sum_{j=1}^{N_u} \left(1 - \xi\frac{Y_j}{\beta}\right), \xi \neq 0 \qquad (2)$$

$$L(\xi, \beta; Y_j) = -N_u In\beta - \frac{1}{\beta}\sum_{j=1}^{N_u} Y_j,\ \xi = 0 \qquad (3)$$

The maximum likelihood for the parameters of GPD following computer assistance measures gives:

$$\hat{\xi}_{MLE} = -\left(\frac{1}{N_u}\right)\sum_{j=1}^{N_u} In\left(1 - \hat{\theta}_{MLE}Y_j\right)$$

and

$$\hat{\beta}_{MLE} = \frac{\hat{\xi}_{MLE}}{\hat{\theta}_{MLE}} \qquad (4)$$

The sample mean excess plot point is:

$$\left(u, \frac{1}{n_u}\sum_{i=1}^{n_u}(x_{(i)} - u)\right) \qquad (5)$$

After plotting the mean excess plot, the threshold u, should be chosen where the relationship for mean excess of all higher thresholds is linear (Hu, 2013). Cole (2001) outlined three diagnostics for the choice of threshold including the mean excess plots, parameter stability and more general model fit diagnostics plots. The mean excess plot is used to choose the threshold from either the lower tail or upper tail which will be used to fit the GP model. The shape of the “mean excess plot” can also be used to determine if GPD can be used to fit the dataset and this can be demonstrated by exhibiting a smooth curve. Choosing threshold follows that; a low threshold gives room for material in estimation of parameter, while choosing a sufficiently high threshold leads to having asymptotic exact parameters.

Using the GPD and peak-over threshold approach, the Value-at-Risk can be given as;

$$V\hat{a}R_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{np}{N_u}\right)^{\hat{\xi}} - 1\right) \qquad (6)$$

n is number of observations, N_u is number of tail observations with parameters of GPD. The expected shortfall follows that:

$$ES_p = V\hat{a}R_p + E(X - V\hat{a}R_p \mid X > V\hat{a}R_p) \quad (7)$$

3. EMPIRICAL APPLICATION AND RESULTS

Dataset of daily energy production in Nigeria from January 1st, 2015 to June 30th 2016 containing 547 observations was used. Also, dataset for first quarter of 2017 was collected containing 90 observations and modelled. The software by R core team (2018) was used to implement the analysis and “fExtremes” package in R by Wuertz *et al*., (2013) was used for Extreme VaR. Table 1 shows the descriptive statistics of the total daily energy generated in Megawatt Hour (MWH) and its returns for January 1st, 2015 to June 30th, 2016. Raw datasets were obtained from National Bureau of Statistics (NBS): https://www.proshareng.com/admin/upload/reports/DailyEnergGenerationQ12017.pdf and www.nigerianstat.gov.ng respectively.

The maximum and minimum electricity productions are 109400 and 1180 MW respectively. 1st quarter (Q1) of 2017 has maximum and minimum productions of 140,300 and 39,840 respectively. However, the descriptive statistics shows that there is slight improvement in electric power generation in first Quarter of 2017 relative to year 2015 and 2016. Table 1 represents information for the period of 2015 and part of 2016, there is a negative asymmetry and the kurtosis value obtained is platykurtic (since the kurtosis value was less than 3). The returns show a positive asymmetry and kurtosis value obtained is Leptokurtic (greater than 3). Table 2 is explained later as it relates to Figure 2. For the raw data, Table 3 shows that there is a negative asymmetry and the kurtosis value obtained is platykurtic. But for the returns, there is a negative asymmetry and the kurtosis value obtained is Leptokurtic.

Table 1 Descriptive Statistics I

Total daily Energy Generated MWH						
1st Qrt.	Median	Mean	3rd Qrt	STD DEV	Skewness	Kurtosis
75200	87770	83720	95020	15613.65	-1.271696	2.075322
Returns of Total daily Energy Generated MWH						
-0.0403800	-0.0000197	-0.0005786	0.0374600	0.2011833	0.1597479	148.779

Table 2 Descriptive Statistics II

Total daily Energy Sent out MWH (547 valid observations)						
Minimum	Median	Mean	Maximum	STD DEV	Skewness	Kurtosis
1108.74	85794.6	81967.5	107106.	15303.4	-1.26343	2.06282
Total daily Energy Generated MW per Hour						
49.1863	3657.02	3488.19	4557.17	650.569	-1.27519	2.09393
Total Energy Sent out MW per Hour						
46.1975	3574.78	3415.31	4462.74	637.642	-1.26343	2.06282

Table 3 Descriptive Statistics III

Total daily Energy Generated MWH (Q1 2017)						
1st Qrt.	Median	Mean	3rd Qrt	STD DEV	Skewness	Kurtosis
80550	88520.0	85240.0	93490.0	14690.90	-0.55847	2.58565
Returns of Total daily Energy Generated MWH						
-0.0403800	0.00000	0.00023	0.03327	0.16548	-0.386903	9.6686

The time series plot for the daily energy production for the first quarter of 2017 is displayed in Figure 1

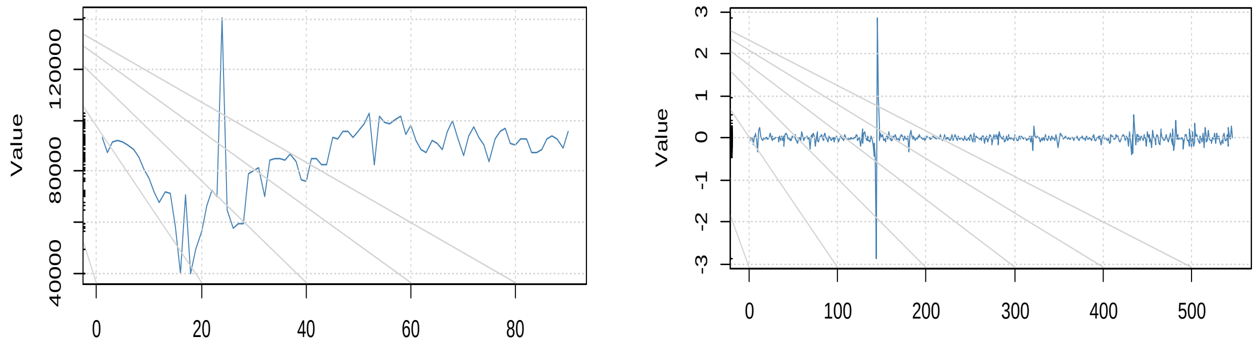
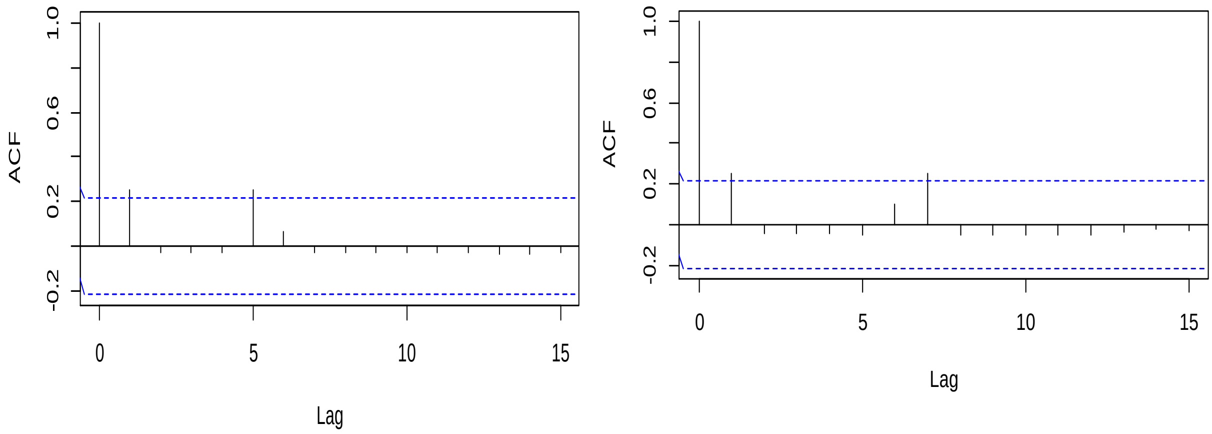


Figure 1 Time series plot of daily Energy Production in Nigeria between January 1st, 2017 and March 31st 2017

From the plot on the left hand side of Figure 1, it can be observed that the electricity production over the 90 days has not increased steadily. Figure 1 also shows that the data is non-stationary over the period under observation. This is an indication that there is risk of having the power drop below expectation in the near future, as demand for electricity is on the increase both for domestic and industrial consumption. Table 2 shows the descriptive statistics of the total daily energy sent out in Megawatt Hour, total daily energy generated in Megawatt per hour and total energy sent out in Megawatt per hour.

The serial autocorrelation function of the transformed data shows a trend that after every lag1, there is dependences and so on as illustrated in Figure 2.



ACF before transformation **ACF of transformed data**

Figure 2 Autocorrelation plots showing series Distances

The correlation of energy production is obviously high; that is why volatility clustering is observed, where there is a tendency of large values being followed by other large values and small being followed by the smaller ones. Exceedances of high or low threshold appear in clusters, indicating that there is dependence in the tails. The plots for the blocks and scatter residuals using the Generalised Extreme Value (GEV) distribution are presented in Figure 3.

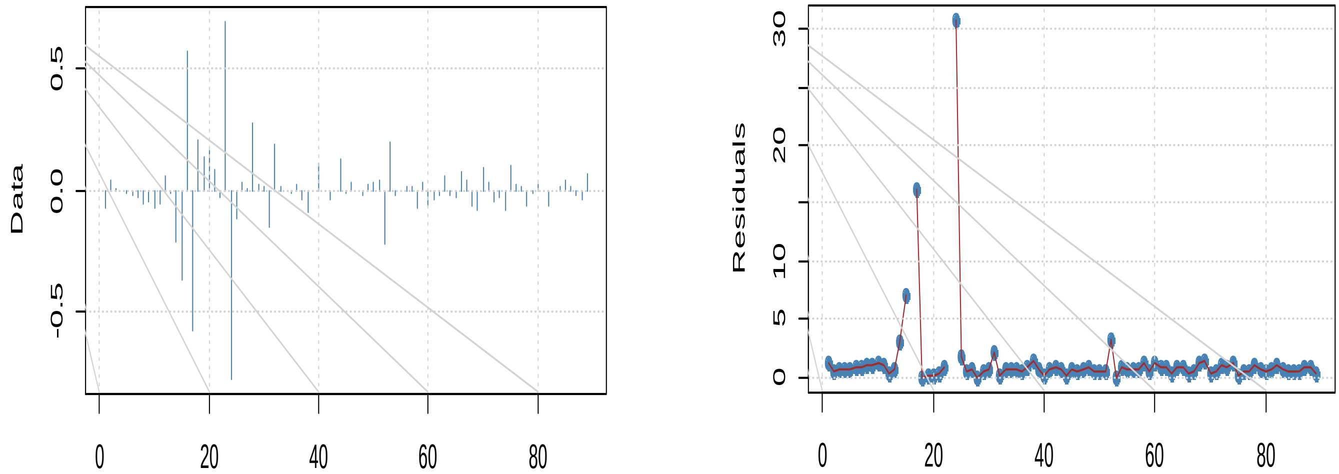


Figure 3 Blocks and scatter residuals following the GEV distribution

The distribution of the block maxima was first considered which allows the determination of the production level. The minimal productions in each of the block constitute the data points of the sample of the minimum which was used to estimate the generalised extreme value distribution. Probability weighted moment method was used to obtain the parameter estimates for the shape, location and scale parameters. The shape parameter in Table 4 corresponds to that of negative Weibull distribution. Weibull distribution and other related distributions are well explained in Oguntunde *et al*., (2014; 2017).

Table 4 GEV Estimated Parameters

Shape	Location	Scale
-0.28958055	-0.04322777	0.12472061

Figures 4 to 7 represent the diagnostic plots for accessing the suitability of the GPD model in fitting the data.

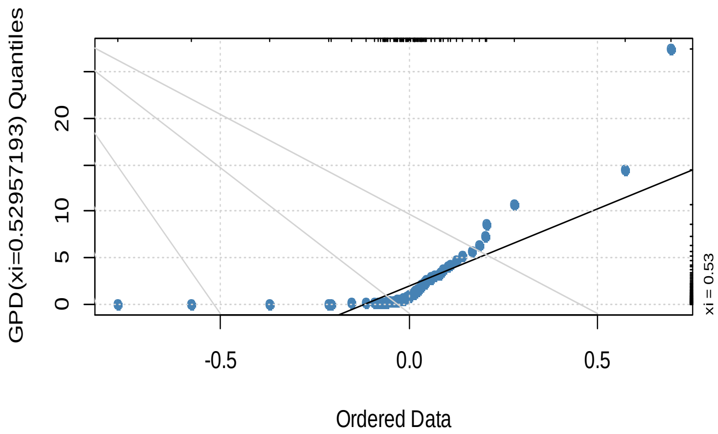


Figure 4 Quantile-Quantile plot for the data set

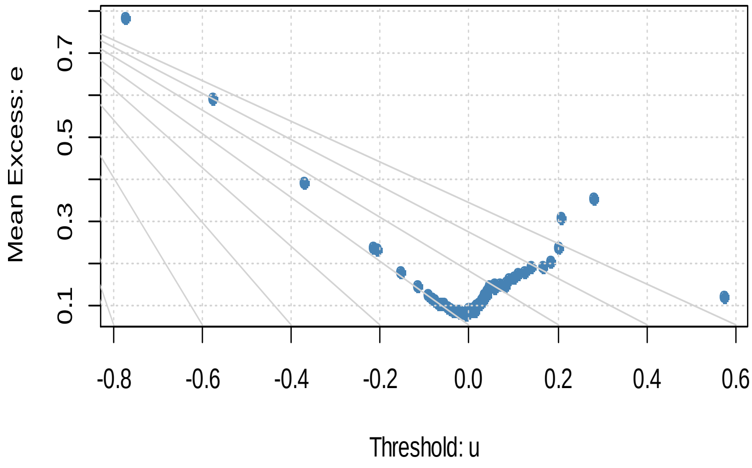


Figure 5 Quantile-Quantile plot of ordered data

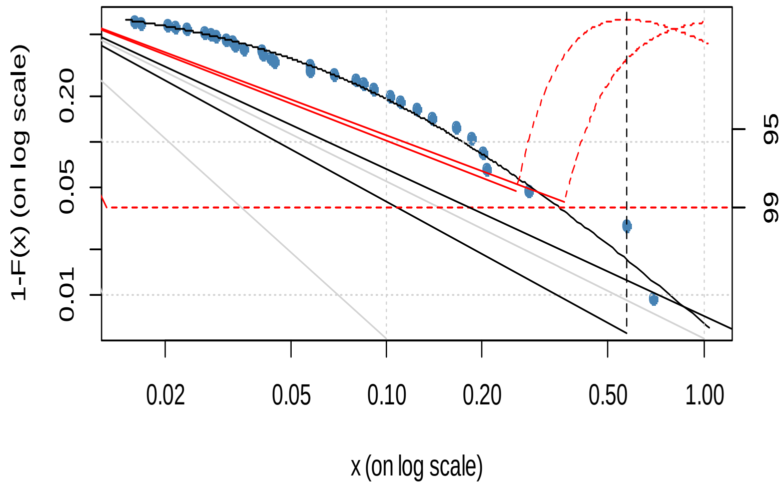


Figure 6 Tail Estimate plot for the data set

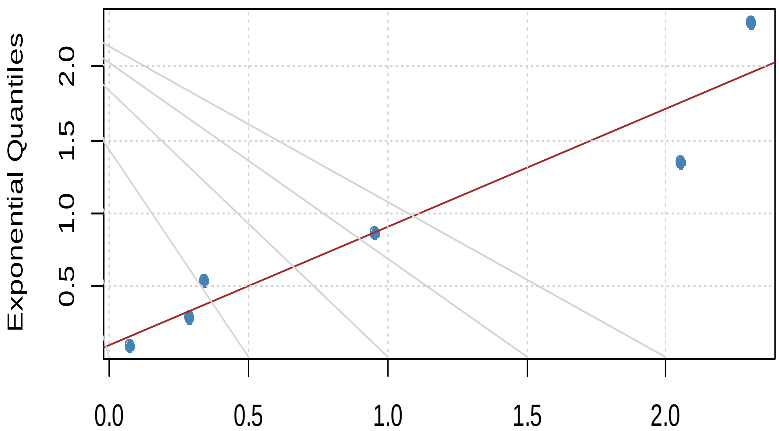


Figure 7 Q-Q residual plot of ordered data

For the Peak-Over Threshold (POT) model, the maximum likelihood approach was adopted to estimate the shape and scale parameters.

The mean excess plot is used to determine the threshold used to fit the model, the distribution for the excesses shows a smooth curve meaning GDP is a good fit for the data. GPD's Q-Q plot and QQ-plot of Residual is linear, showing that GPD is suitable; the tail plot equally exhibit the same. From Figure 5, threshold of 0.0150 was chosen, parameter estimates $\xi = 0.529571$ and $\beta = 0.05104$. Since ξ is greater than 0, the data has heavy tailed distribution, indicating that the higher the value of the shape parameter, the higher the accompanying returns.

Log returns indicate the change in production level of electricity over a period of time, loss occur when the returns are negative. Electricity production can drop at any given time by VaR= production capacity times VaR of log return series. That is,

$$VaR_p = value \times \ VaR_q(of \log returns)$$

From Table 3, the average energy production for the first Quarter of 2017 is 84,240 MWH; with 1% alpha level. This result shows that energy production capacity will not exceed 34, 225 MWH. If that happens, average production will be 26,431.00, calculated as follows:

VaR= 0.598484 $\times$ 85,240.0 = 51,014.7847

85,240 - 51014.78 = 34.225.22

Shortfall = 0.689923 $\times$ 85,240 = 58809.01

85,240.00−58809.01 = 26,431.00

The risk measures are obtained and presented in Table 4.

Table 4 Risk measures computed via peak over threshold measures at 1%, 0.5% , and 0.1% probability

Probability	Quantile	Shortfall
.9900	.6543	.6802
.9950	.6834	.6899
.9990	.6927	.6930

4. SUMMARY AND CONCLUSION

In this study, electricity production risk in Nigeria has been estimated using the Generalized Pareto distribution. Diagnostics plots in Figures 3 and 4 show that the model is suitable in fitting the data and the risk was measured accordingly. This study has shown that Extreme Value Theory is useful for accessing the size of extreme events and choice can be made using the Peak over-threshold, Frechet, Weibull, Gumbel, Block Maxima (BMM) approach following Generalized Extreme Value model (GEVD). However, this study has proven that the Peak Over-Threshold method is the best method as it pools information in the sample data and the computed risk measure provides better understanding on the restructuring of electricity production. The risk of having a low electricity production level may be as a result of unstable economy. Electricity production can be at optimum level if the factors resulting in low production are checked. For example, curbing vandalising of gas pipeline, also, the need to drive alternative electrical energy generation means such as wind, and solar should be driven at all levels of Government.

ACKNOWLEDGEMENTS

The authors thank Covenant University for supports.

REFERENCES

[1] Adesina O. S., Adeleke I., Oladeji T. F. (2016). Using Extreme Value Theory to Model Insurance Risk of Nigeria Motor Industrial Class of Business, *The Journal of Risk Management and Insurance, Assumption University*, 20(1), 40-51

[2] Adeleke I., Adesina O. S., Oladeji, T. F. (2015). Estimating Nigeria (NGN) Foreign Exchange Risk with Extreme Value Theory, Botswana Journal of Business, 8(1), 61-80

[3] Alvarado-Celestino E. (1992). Largest forest fires: An analysis using extreme value theory and robust statistics, University of Washington, USA.

[4] Apudo B. O., Mwita P. N., Mbugua L. N., Machuke G.W., Kiche J. (2014). An Application of Extreme Value Theory in Modelling Electricity Production In Kenya, *Mathematical Theory and Modeling*, 4(4), 151-162

[5] Balkema A. A., de Haan L. (1974). Residual Life time at Great Age, *Annals of Probability*, 2(5), 792-804.

[6] Cole S. G. (2001). An Introduction to Statistical Modelling of Extreme Values, Springer Series in Statistics, Springer-Verlag London, eBook ISBN: 978-1-4471-3675-0, DOI: 10.1007/978-1-4471-3675-0

[7] Chan K. F., Gray P. (2006). Using extreme value theory to measure value-at-risk for daily electricity spot prices, *International Journal of Forecasting*, 22, 283-300

[8] Einmahl, J. H. J., Magnus J. R. (2008). Records in Atlethics through Extreme- Value Theory, *Journal of American Statistical Association*, 103(484), 1382-1391

[9] Longin F. M. (1999). Optimal marginal level in future markets, Extreme price movements, *Journal of Future Markets*, 19(2)

[10] Gilli M., Kellezi E. (2006). An application of extreme value theory for measuring financial risk, *Computational Economics*, 27(2-3), 207-228

[11] Mbugua L., Mwita P., Mwalili S., Mhilu C. F. (2012). Statistical models for estimation of Extreme daily rate of change in energy use, 2nd International conference on Mechanical and Industrial Engineering; Dar es Salaam University (174-185)

[12] McNeil A. J. (1997). Estimating the tail loss severity distributions using extreme value theory, *Astin Bulletin*, 27(1), 117-137

[13] Mhilu C. F. (2007). Development of regional equations for estimation of low flow regimes of small catchments on Pemba Island, Zanzibar, *Journal of Environmental Hydrology*, 15, Paper 7, 1-7, March 2007.

[14] Nigm A., El-Habashi M., Hamdy H. (1987). The effect of risk parameters on decision making, *Insurance, Mathematics and economics*, 6, 237-244

[15] Pickands, J (1975), Statistical Inference using Extreme Value Order Statistics, *Annals of Statistics*, 3, 119-131

[16] R Core Team (2018). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL: https://www.R-project.org/

[17] Rodrigo H. L., Nicolas G. O. (2012). Computing Value At Risk On Electricity Markets through Modeling Inter-Exceedances Times, *Congreso Latino-Iberoamericano de investiacion operative, Rio de Janeiro, Brazil,* September 24-28, 2012, pp. 1150- 1160

[18] Rufino C. C., de Guia E. (2011). Empirical Comparison of Extreme Value Theory Vis-à-vis Other Methods of VaR Estimation Using ASEAN+3 Exchange Rate, *DLSU Business & Economics Review*, 20.2, 9-22

[19] Sebetosi, A. B., Okuo R. (2010). Rethinking the power transmission model for sub Saharan Africa, *Energy policy*, 38, 1448-1454

[20] Smith, R.L. (2003). Statistics of extremes, with applications in environment, insurance and finance. Chapter 1 of Extreme Values in Finance, Telecommunications and the Environment, edited by B. Finkenstadt and H. Rootzen, Chapman and Hall/CRC Press, London, pp. 1-78. (http://www.stat.unc.edu/postscript/rs/semstatrls.pdf)

[21] Uppal Y. J (2013). Measures of Extreme Loss Risk- An Assessment of Performance during the Global Financial Crisis, *Journal of Accounting and Finance*, 13(3), 105-127

[22] Velazquez J. C., Van Der Weide J .A. M., Hernandez E., Hernandez H. H. (2014). Statistical Modelling of Pitting Corrosion: Extrapolation of the Maximum Pit Depth-Growth, *International Journal of Electrochemical Science*, 9, 4129-4143

[23] Wainnaina H. W., Waititu A. G. (2014). Modelling Insurance Returns with Extreme Value Theory (A Case study for Kenya's Fire Industrial Class of Business), *Mathematical Thoery and Modelling*, 4(2), 48-64

[24] Wuertz D., Setz T., Chalabi Y. (2017). fExtremes:Rmetrics – Modelling Extreme Events in Finance. R package version3042.82. https://CRAN.R-project.org/package=fExtremes

[25] Hu Y. (2013). Extreme Value Modelling with Simulation Study and Application in Finance and Insurance. A thesis submitted for the award of MSc degree at University of Canterbury, New Zealand.

[26] Zhao X. (2010). Extreme value modelling with application in finance and neonatal research. PhD Thesis, The University of Canterbury. Available at: http://ir.canterbury.ac.nz/bitstream/10092/4024/1/thesis_fulltext.pdf

[27] Oguntunde P. E., Odetunmibi O. A, Adejumo A. O. (2014). A Study of Probability Models in Monitoring Environmental Pollution in Nigeria, *Journal of Probability and Statistics*, Article ID 864965, 6 pages

[28] Oguntunde P. E., Adejumo A. O., Owoloko E. A. (2017). The Weibull-Inverted Exponential Distribution: A generalization of the Inverse Exponential Distribution. *Lecture Notes in Engineering and Computer Science: Proceedings of the World Congress on Engineering*, 2017, 5-7 July, London, U.K., pp16-19.