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The gompertz inverse exponential (GoIE) distribution with applications

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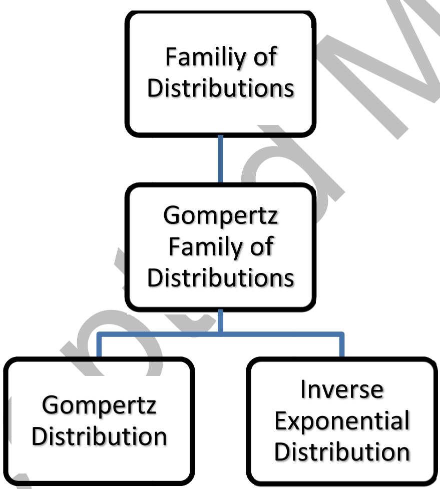
Abstract: We derived the Gompertz Inverse Exponential distribution using the Gompertz generalized family of distributions. We obtained the various basic statistical properties of the model and the model parameters were estimated using the maximum likelihood estimation method. Real life applications were provided and the Gompertz Inverse Exponential distribution provides better fits than the Gompertz Exponential, Gompertz Weibull and Gompertz Lomax distributions.



Keywords: Generalized models, Gompertz family of distributions, Inverse Exponential, Mathematical Statistics, Probability distributions

1.0 Introduction

The Inverse Exponential distribution is a special case of the Inverse Weibull distribution, it has been introduced as far back as 1982 by [1] and it is capable of modeling data sets with inverted bathtub failure rate. More details about the usefulness of the inverse exponential distribution have been discussed by [2-5] and many others.

Some attempts in the literature to increase the modeling capacity of the inverse exponential distribution include the works of [6-10]; these works used different families of distributions to extend the inverse exponential distribution, a list of these families of distributions can be found in [11-12], [13-15] and several others.

The interest of the paper is on the Gompertz family of distributions because it is relatively new and flexible. Its densities are:

$$F(x) = 1 - e^{\frac{\alpha}{\beta}\left\{1-\left[1-G(x)\right]^{-\beta}\right\}} \quad ; \quad \alpha > 0, \beta > 0 \tag{1}$$

and

$$f(x) = \alpha g(x)\left[1 - G(x)\right]^{-\beta-1} e^{\frac{\alpha}{\beta}\left\{1-\left[1-G(x)\right]^{-\beta}\right\}} \quad ; \quad \alpha > 0, \beta > 0 \tag{2}$$

Where α and β are extra shape parameters and it was developed using the following transformation:

$$F(x) = \int_{0}^{-\log\left[1-G(x)\right]} w(t) \tag{3}$$

$w(t)$ is the probability density function (pdf) of the Gompertz distribution and T is a random variable.

$G(x)$ and $g(x)$ are the cumulative distribution function (cdf) and pdf of the baseline distribution respectively. In our case, the baseline distribution is the inverse exponential distribution defined as:

$$G(x) = e^{-\frac{\theta}{x}} \tag{4}$$

and

$$g(x) = \frac{\theta}{x^2} e^{-\frac{\theta}{x}} \tag{5}$$

Where θ is regarded as the scale parameter.

We derive the cdf of the Gompertz Inverse Exponential (GoIE) distribution by substituting Equation (4) into Equation (1) and we have:

$$F(x)=1-e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}} \quad ; \quad \alpha>0, \beta>0, \theta>0 \tag{6}$$

We also derive the corresponding pdf by substituting Equations (4) and (5) into Equation (2) and we have:

$$f(x)=\alpha\frac{\theta}{x^2}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1} e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}} \quad ; \quad \alpha>0, \beta>0, \theta>0 \tag{7}$$

Where α and β are shape parameters

θ is the scale parameter

The pdf and cdf of the GoIE are represented in Figures 1 and 2 respectively using different parameter values.

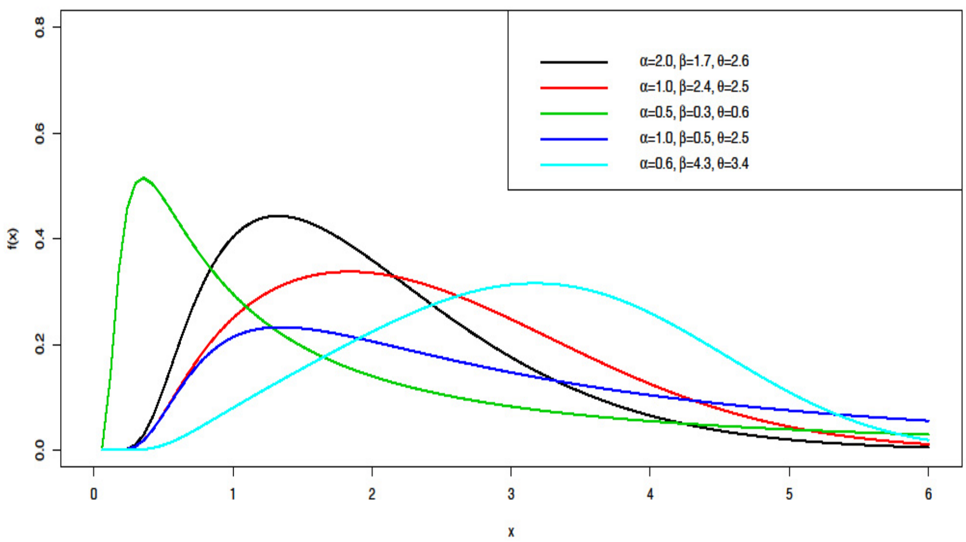


Figure 1: PDF of the GoIE distribution

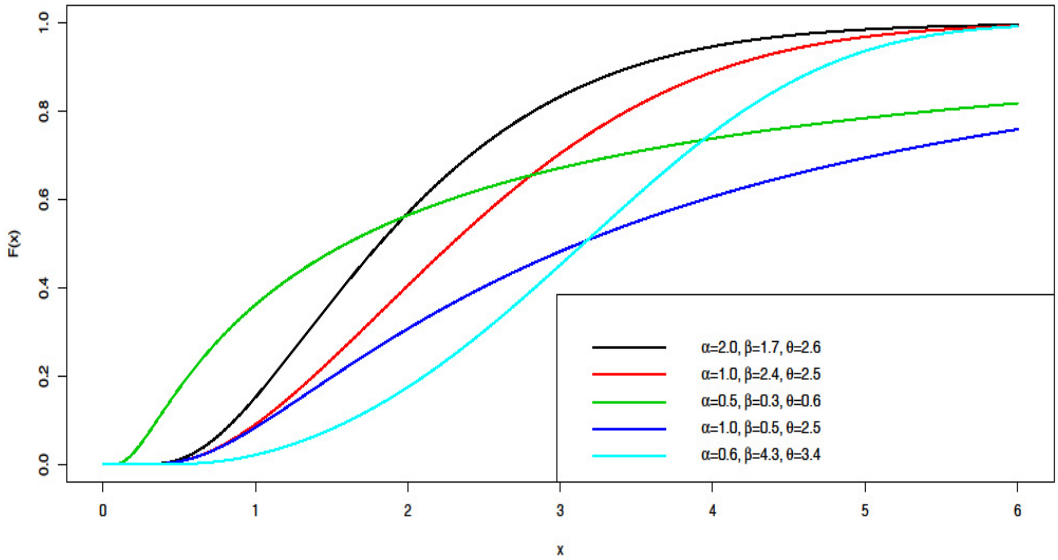


Figure 2: CDF of the GoIE distribution

Some Basic Statistical Properties of the Gompertz Inverse Exponential Distribution

The basic mathematical properties of the GoIE are obtained as follows:

First, we obtain the reliability function by using the relation:

$$S(x) = 1 - F(x)$$

So, the reliability function of the GoIE distribution is:

$$S(x) = e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}} \quad ; \quad \alpha > 0, \beta > 0, \theta > 0 \tag{8}$$

We represent this graphically in Figure 3.

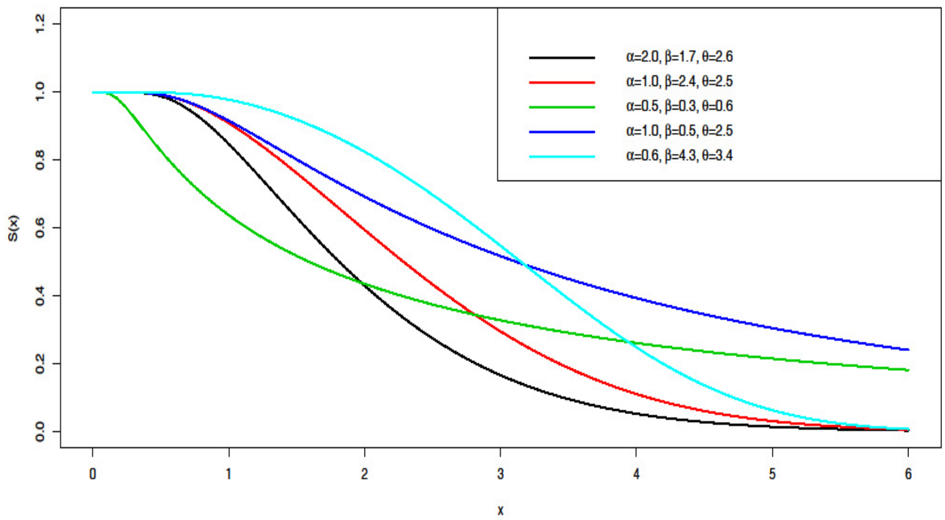


Figure 3: Reliability function of GoIE distribution

We obtain the failure rate of the GoIE distribution by dividing the pdf in Equation (7) by the reliability function in Equation (8) and we have:

$$h(x)=\frac{\alpha\frac{\theta}{x^2}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1}e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}{e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}$$

$$h(x)=\alpha\frac{\theta}{x^2}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1} \quad ; \quad \alpha>0,\beta>0,\theta>0 \qquad (9)$$

We represent this graphically in Figure 4.

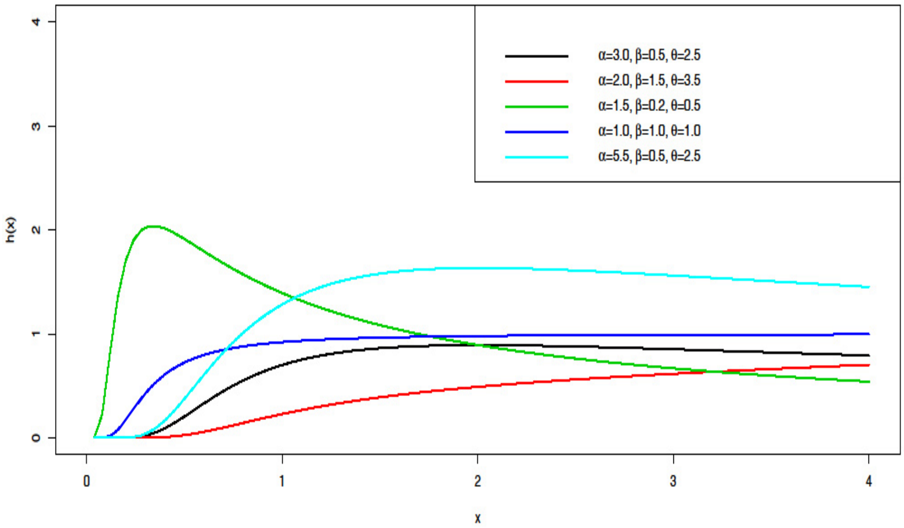


Figure 4: Failure rate of the GoIE distribution

We obtain the reversed hazard function of the GoIE distribution by dividing the pdf in Equation (7) by the cdf in Equation (6) and we have:

$$r(x)=\frac{\alpha\frac{\theta}{x^{2}}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1}e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}{1-e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}; \qquad \alpha>0,\beta>0,\theta>0 \qquad (10)$$

Also, we obtain the odds function by dividing the cdf in Equation (6) by the reliability function in Equation (8) and we have:

$$O(x)=\frac{1-e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}{e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}}; \qquad \alpha>0,\beta>0,\theta>0 \qquad (11)$$

Quantile Function and Median

We derive the quantile function $Q(u)$ from the relation:

$$Q(u)=F^{-1}(u)$$

So, we derive the quantile function of the GoIE distribution as:

$$Q(u) = -\theta \ln\left\{1-\left[1-\frac{\beta\ln(1-u)}{\alpha}\right]^{-1/\beta}\right\}^{-1} \tag{12}$$

where $u \sim Uniform(0,1)$

In other words, random samples from the GoIE distribution can be generated using:

$$x = -\theta \ln\left\{1-\left[1-\frac{\beta\ln(1-u)}{\alpha}\right]^{-1/\beta}\right\}^{-1}$$

We can conveniently derive the median of the GoIE distribution by making the substitution of $u = 0.5$ in Equation (12) to have:

$$Median = -\theta \ln\left\{1-\left[1-\frac{\beta\ln(0.5)}{\alpha}\right]^{-1/\beta}\right\}^{-1} \tag{13}$$

We can also obtain the first quartile and third quartile by making the substitution of $u = 0.25$ and $u = 0.75$ respectively into Equation (12).

Order Statistics

The pdf of the jth order statistic for a random sample of size n from a distribution function $F(x)$ and an associated pdf $f(x)$ is given by:

$$f_{j:n}(x) = \frac{n!}{(j-1)!(n-j)!} f(x)\left[F(x)\right]^{j-1}\left[1-F(x)\right]^{n-j} \tag{14}$$

Where $f(x)$ and $F(x)$ are the pdf and cdf of the GoIE respectively. The pdf of the jth order statistics for a random sample of size n from the GoIE distribution is however given as:

parameter values.

$$f_{j:n}(x) = \frac{n!}{(j-1)!(n-j)!}\left\{\alpha\frac{\theta}{x^2}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1} e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right\}\left[1-e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right]^{j-1}\left[e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right]^{n-j} \tag{15}$$

So, the pdf of minimum order statistics is obtained by substituting $j=1$ in Equation (15) and we have:

$$f_{1:n}(x)=n\left\{\alpha\frac{\theta}{x^{2}}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1}e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right\}\left[e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right]^{n-1} \quad (16)$$

While the corresponding pdf of maximum order statistics is obtained by making the substitution of $j=n$ in Equation (15) as:

$$f_{n:n}(x)=n\left\{\alpha\frac{\theta}{x^{2}}e^{-\frac{\theta}{x}}\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta-1}e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right\}\left[1-e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x}}\right]^{-\beta}\right\}}\right]^{n-1} \quad (17)$$

Estimation

We use the method of maximum likelihood estimation to estimate the parameters of the GoIE distribution. For a random sample $x_1, x_2, ..., x_n$ distributed according to the cdf of the GoIE distribution, we obtain the log-likelihood function as:

$$f(x_1,x_2,...,x_n;\alpha,\beta,\theta)=\prod_{i=1}^{n}\left[\alpha\frac{\theta}{x_i^{2}}e^{-\frac{\theta}{x_i}}\left[1-e^{-\frac{\theta}{x_i}}\right]^{-\beta-1}e^{\frac{\alpha}{\beta}\left\{1-\left[1-e^{-\frac{\theta}{x_i}}\right]^{-\beta}\right\}}\right] \quad (18)$$

We obtain the log-likelihood function L as;

$$L=n\ln(\alpha)+n\ln(\theta)-2\sum_{i=1}^{n}\ln(x_i)-\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)-(\beta+1)\sum_{i=1}^{n}\ln\left[1-e^{-\frac{\theta}{x_i}}\right]+\frac{\alpha}{\beta}\sum_{i=1}^{n}\left\{1-\left[1-e^{-\frac{\theta}{x_i}}\right]^{-\beta}\right\} \quad (19)$$

Differentiate L with respect to parameters α, β and θ and equate the results to zero, solve the resulting equations simultaneous to obtain the parameter estimates. The solution may not be obtained in closed form, so, software can be used to obtain the estimates numerically.

2.0 Model Validation and Application

We applied the GoIE distribution to two data sets and we make comparisons with the Gompertz Exponential, Gompertz Weibull and Gompertz Lomax distributions. We used R software for the analysis and the criteria used are Akaike Information Criteria (AIC), Consistent Akaike Information Criteria CAIC), Bayesian Information Criteria (BIC), Negative Log-likelihood (NLL) and Hannan and Quinn Information Criteria (HQIC).

First Illustration: We use a data that relates to the strengths of 1.5cm glass fibres. The data set has been analyzed previously by [12] and [16-18]. The observations are:

0.55, 0.74, 0.77, 0.81, 0.84, 1.24, 0.93, 1.04, 1.11, 1.13, 1.30, 1.25, 1.27, 1.28, 1.29, 1.48, 1.36, 1.39, 1.42, 1.48, 1.51, 1.49, 1.49, 1.50, 1.50, 1.55, 1.52, 1.53, 1.54, 1.55, 1.61, 1.58, 1.59, 1.60, 1.61, 1.63, 1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68, 1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82, 1.84, 1.84, 2.00, 2.01, 2.24

The result of the analysis is displayed in Table 1.

Table 1: Table of result

Models	Estimates	NLL	AIC	CAIC	BIC	HQIC
GoIE	$\hat{\alpha} = 0.2030548$ $\hat{\beta} = 11.5541435$ $\hat{\theta} = 2.0003185$	14.04093	34.08187	34.48865	40.51127	36.61059
GoExp	$\hat{\alpha} = -0.004768848$ $\hat{\beta} = -1.810999364$ $\hat{\theta} = -1.987714978$	14.81765	35.6353	36.04208	42.06471	38.16402
GoWei	$\hat{\alpha} = 0.228488761$ $\hat{\beta} = 0.009628097$ $\hat{\theta} = 0.794918813$ $\hat{\lambda} = 5.612111282$	15.18847	38.37694	39.06659	46.94948	41.74856
GoLom	$\hat{\alpha} = 0.004592168$ $\hat{\beta} = 8.179090955$ $\hat{\theta} = 0.506999370$ $\hat{\lambda} = 1.515829085$	14.50274	37.00548	37.69513	45.57802	40.3771

The histogram of the data with the competing distributions is displayed in Figure 5.

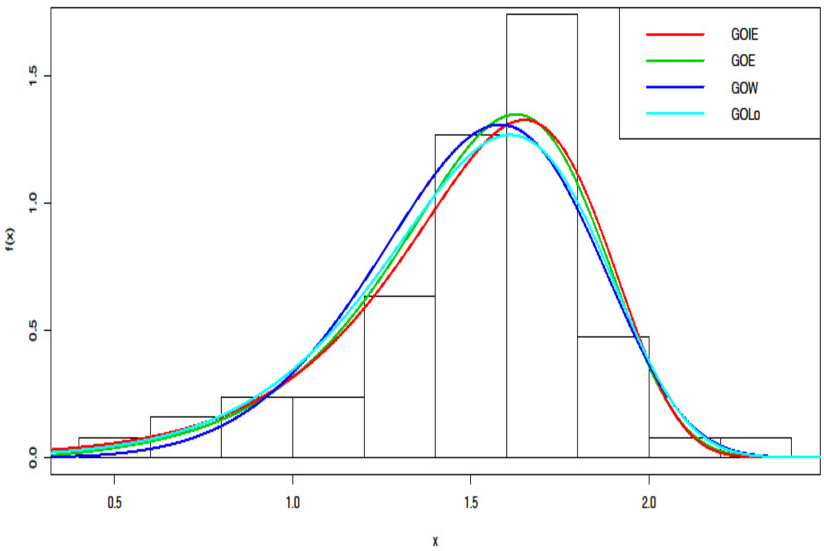


Figure 5: Histogram of the data with the competing distributions

The empirical cdf of the competing distributions with respect to the data set used is displayed in Figure 6.

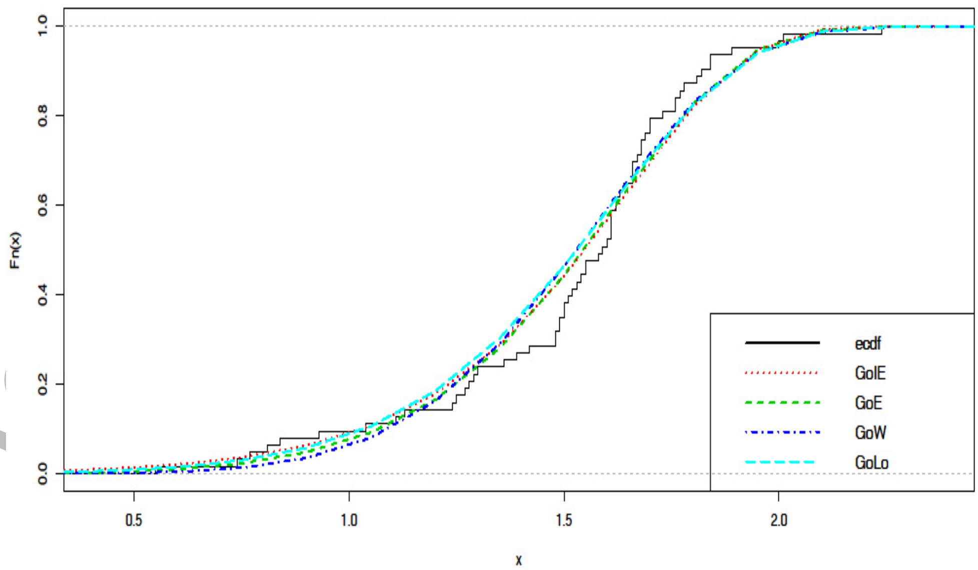


Figure 6: The empirical cdf of the data together with the competing distributions

Second Illustration: We use a sport data which is downloadable at [19]. The observations are:

19.75, 21.30, 19.88, 23.66, 17.64, 15.58, 19.99, 22.43, 17.95, 15.07, 28.83, 18.08, 23.30, 17.71, 18.77, 19.83, 25.16, 18.04, 21.79, 22.25, 16.25, 16.38, 19.35, 19.20, 17.89, 12.20, 23.70, 24.69, 16.58, 21.47, 20.12, 17.51, 23.70, 22.39, 20.43, 11.29, 25.26, 19.39, 19.63, 23.11, 16.86, 21.32, 26.57, 17.93, 24.97, 22.62, 15.01, 18.14, 26.78, 17.22, 26.50, 23.01, 30.10, 13.93, 26.65, 35.52, 15.59, 19.61, 14.52, 11.47, 17.71, 18.48, 11.22, 13.61, 12.78, 11.85, 13.35, 11.77, 11.07, 21.30, 20.10, 24.88, 19.26, 19.51, 23.01, 8.07, 11.05, 12.39, 15.95, 9.91, 16.20, 9.02, 14.26, 10.48, 11.64, 12.16, 10.53, 10.15, 10.74, 20.86, 19.64, 17.07, 15.31, 11.07, 12.92, 8.45, 10.16, 12.55, 9.10, 13.46, 8.47, 7.68, 6.16, 8.56, 6.86, 9.40, 9.17, 8.54, 9.20, 11.72, 8.44, 7.19, 6.46, 9.00, 12.61, 9.03, 6.96, 10.05, 9.56, 9.36, 10.81, 8.61, 9.53, 7.42, 9.79, 8.97, 7.49, 11.95, 7.35, 7.16, 8.77, 9.56, 14.53, 8.51, 10.64, 7.06, 8.87, 7.88, 9.20, 7.19, 6.06, 5.63, 6.59, 9.50, 13.97, 11.66, 6.43, 6.99, 6.00, 6.56, 6.03, 6.33, 6.82, 6.20, 5.93, 5.80, 6.56, 6.76, 7.22, 8.51, 7.72, 19.94, 13.91, 6.10, 7.52, 9.56, 6.06, 7.35, 6.00, 6.92, 6.33, 5.90, 8.84, 8.94, 6.53, 9.40, 8.18, 17.41, 18.08, 9.86, 7.29, 18.72, 10.12, 19.17, 17.24, 9.89, 13.06, 8.84, 8.87, 14.69, 8.64, 14.98, 7.82, 8.97, 11.63, 13.49, 10.25, 11.79, 10.05, 8.51, 11.50, 6.26

The result of the analysis is displayed in Table 2.

Table 2: Table of result

Models	Estimates	NLL	AIC	CAIC	BIC	HQIC
GoIE	$\hat{\alpha}=4.636970$ $\hat{\beta}=1.583631$ $\hat{\theta}=25.794220$	629.335	1264.67	1234.791	1274.595	1268.686
GoExp	$\hat{\alpha}=5.330925317$ $\hat{\beta}=20.978768343$ $\hat{\theta}=0.004836174$	663.8036	1333.607	1333.728	1343.532	1337.623
GoWei	$\hat{\alpha}=11.22264866$ $\hat{\beta}=-1.29977338$ $\hat{\theta}=0.02630932$ $\hat{\lambda}=2.54531004$	641.4217	1290.843	1291.046	1304.076	1296.197
GoLom	$\hat{\alpha}=0.006519876$ $\hat{\beta}=3.370057658$ $\hat{\theta}=0.714843759$ $\hat{\lambda}=0.747124582$	645.6623	1299.325	1299.528	1312.558	1304.679

The histogram of the second data with the competing distributions is displayed in Figure 7.

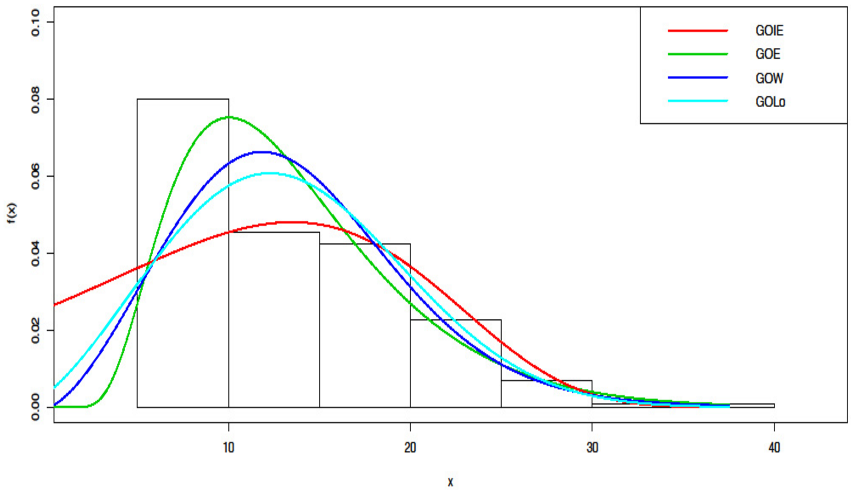


Figure 7: Histogram of the data with the competing distributions

The empirical cdf of the competing distributions with respect to the second data set is displayed in Figure 8.

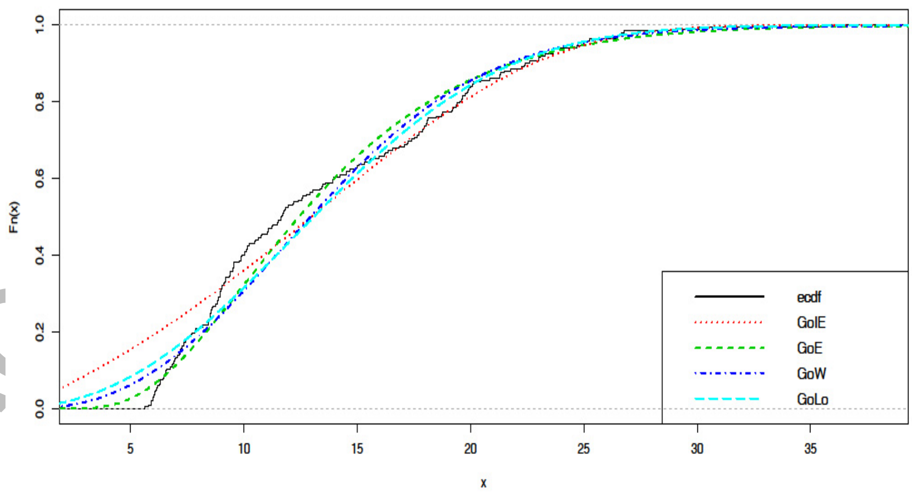


Figure 8: The empirical cdf of the data together with the competing distributions

The results from Tables 1 and 2 show that the GoIE has the lowest values for all the criteria used. So, we consider the GoIE the best model to fit the data set among the other distributions used. This is also evident in the plots provided in Figures 5 to 8.

Conclusion

The Gompertz Inverse Exponential distribution has been successfully introduced in this paper and we have provided its structural properties. The pdf of the model and its failure rate have unimodal shapes; so we conclude that the model would be useful to fit real life events with unimodal failure rates. The model is flexible and shows high modeling capability as it performs better than the Gompertz Exponential, Gompertz Weibull and Gompertz Lomax distributions.

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Public interest statement

Extending existing probability distributions became neccessary in recent years because of the need for models that can adequately fit or describe real life events. The three-parameter probability model that was introduced in this article is however an extension of the one-parameter Inverse Exponenial distribution. The extra two parameters are shape parameters due to the Gompertz family of distribution that was used. The role of these two extra parameters is to induce skewness into the Inverse Exponential distribution. As a consequence, the Gompertz Inverse Exponential distribution appears better than the Gompertz Exponential, Gompertz Weibull and Gompertz Lomax distributins when applied to real life datasets.

Dr. Pelumi E. Oguntunde is a Statistician and a faculty in the Department of Mathematics, Covenant University, Nigeria. He has Bachelor of Science (B.Sc) and Master of Science (M.Sc) degrees in Statistics from University of Ilorin and University of Ibadan respectively. He earned his Ph.D degree in Industrial Mathematics (Statistics Option) from Covenant University, Nigeria. His area of specialization is Mathematical Statistics. He teaches Statistics courses and he has made noticable contributions to several scholarly journals especially in the area of probability distributions.