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# Constitutive Model to Analyse the effect of Variation in Temperature Distribution on the Rate of Heat Transfer around a Spur Gear Tooth

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## Abstract

The study employed the finite element approach for the solution of 1-dimensional steady state heat transfer to determine the temperature distributions on a spur gear tooth. The gear tooth was subjected to temperature variations both at the top and root as well as variations in the respective coordinate of the material particles. The developed model was further used to evaluate the rate of heat transfer through the gear material. The result showed that the heat flux decreases with increasing values of  $Z$  which represent the particle coordinate. However, change in  $Z$  coordinate also constitute increased random motion of the particles in the gear causing steady degradation even at decreasing heat flux. Therefore, the study has revealed that temperature variation has negative effect on the wear mode of the gear material and at higher temperature, failure of the gear tooth would set in.

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*Keywords:* Failure, Production Technology, Gears, Reliability

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## Nomenclature

- $t_1$  temperature at the top surface of the gear tooth
- $t_2$  temperature at the top surface of the gear tooth
- $Z_1$  coordinate of the particle at the top of gear tooth

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- $Z_2$  coordinate of the particle at the root of the gear tooth  
K thermal conductivity of mild steel given as 54 W/m. K  
 $Q_z$  heat transfer  
A cross sectional area

## 1. Introduction

The efficiency of spur gear tooth is a function of the load and temperature which the tooth is subjected to [1]. Variation in load and temperature resulted in thermal stress on the gear tooth [2, 3]. Assessment of temperature variation of gear in an unsteady state condition revealed flash temperature differences which is attributed to the dynamic loading [4]. Salawu *et al.* [5] reported that critical design of the gear tooth involute would give efficient gear transmission ratio and reduction in gear failure that could have resulted from increased temperature variations. Various model has been developed to address failures associated with gears. For instance, Shi *et al.* [6] developed a model to analyse the temperature variation on the gear tooth of a traction gear in the direction vertical to the gear tooth. However, there is limitation in the efficiency and applicability of the model as a result of the fact that tooth contact temperature, friction at the surface and mesh stiffness are factors which usually result to variations in temperature [7,8]. More so, Mertens and Senthilvelan [9] reported that polymer gears exhibit increased temperature variations when meshed with mild steel gears under increased load. This was attributed to area of contact and friction at the surface of the gear tooth in pair. In addition to this, it was possible to predict the temperature variation at the tooth contact in gears using finite element approach. This involved modelling of the rolling and sliding processes that gear tooth is subjected to [10]. Consequently, rise in temperature at the point of mesh can be predicted using coupled thermo-elastic finite element model. However, this also is limited by coefficient of friction which can only be determined experimentally [11]. On the same hand, Fernandez [12] suggested that finite element model could be effective in predicting the rise in temperature of gears if the load can be determined. Similarly, temperature variation in spur gear tooth is also attributed to both speed of operation and the environmental condition. In fact, vibration is a function of increased temperature [13,14]. These parameters have great effect on the tribological performance of the gear, especially the polymer gears [15]. Based on these uncertainties associated with the existing model for temperature variation on gear tooth, this study focused on developing a model to predict the temperature at the top and root of a single tooth spur gear under certain assumptions using 1-dimension approach of Fourier heat conduction. The study further evaluated the numerical values of temperature both at the top and the root of the spur gear tooth. The focus was based on the need to establish the reason for efficiency enhancement of the gear tooth during design.

## 2. Methodology

### 1.1 Problem Formulation

The study considered a spur gear in mesh as shown in Figs. 1a and b. Fig. 1a, showed the cross section of the gear in mesh during operation while Fig. 1b describes a hypothetical situation of gear tooth in service. The study assumed finite points  $Z_1$  and  $Z_2$  at the tip and root of a tooth, while  $t_1$  and  $t_2$  represent the temperature at the tip and root of the gear tooth. As the temperature increases, linear expansion occurs within the tooth signifying the material displacement of point Z from the original position to a new position and possibly induces thermal stress build up and eventual failure. Therefore, the focus of this study was to determine, theoretically by way of modelling, the effect of temperature variation on the gear tooth.

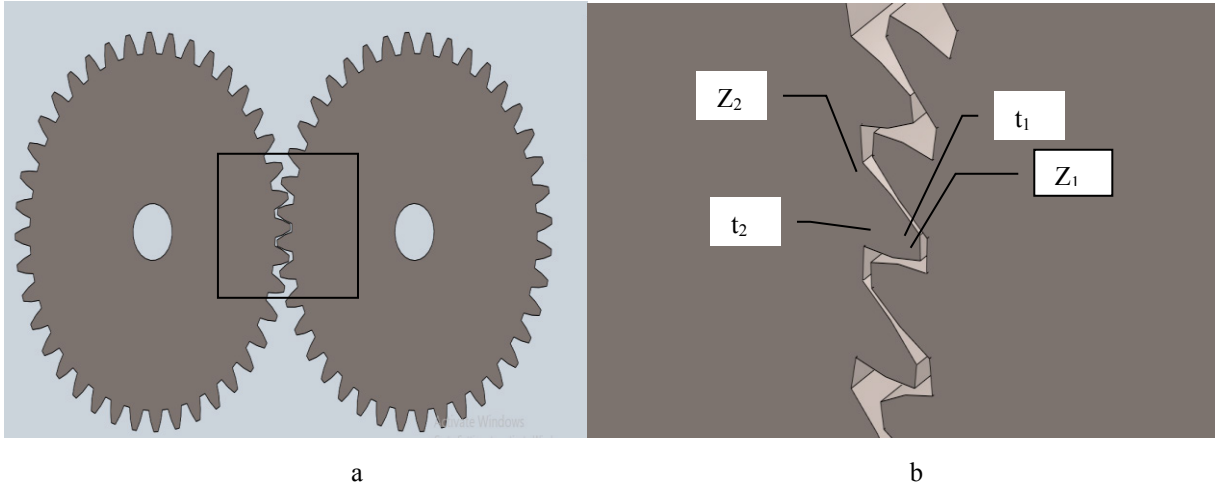


Fig.1: Diagrammatic representation of a Spur gear tooth

## 2.2 Model Development

Applying the Fourier law of heat conduction in 1-dimension to model the heat distribution on the gear tooth gives:

$$\frac{dt}{dz} = \frac{Q_{zg}}{K_g A_g} \quad [16]: \quad (1)$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (bz)^2}{4} = \frac{\pi b^2 z^2}{4} \quad (2)$$

Putting equation (2) into equation (1) gives:

$$4Q_{zg} dz = -K_g \pi b^2 z^2 dt \quad (3)$$

From which,

$$\frac{4Q_{zg} dz}{\pi b^2 z^2} = -K_g dt \quad (4)$$

Integrating both sides of equation (4) at the interval of  $z_1$  to  $z$  and  $t_1$  to  $t$  along the wall of the gear tooth gives:

$$\int_{z_1}^z \frac{4Q_{zg} dz}{\pi b^2 z^2} = \int_{t_1}^t -K_g dt \quad (5)$$

$$\frac{4Q_{zg}}{\pi b^2} \int_{z_1}^z \frac{dz}{z^2} = -K_g \int_{t_1}^t dt \quad (6)$$

$$\frac{4Q_{zg}}{\pi b^2} \left[ -\frac{1}{z} \right]_{z_1}^z = -K_g [t]_{t_1}^t \quad (7)$$

$$\frac{4Q_{zg}}{\pi b^2} \left[ \frac{1}{z} + \frac{1}{z_1} \right] = -K_g [t - t_1] \quad (8)$$

Equ (8) gives;

$$-\frac{4Q_{zg}}{\pi b^2 K_g} \left[ \frac{1}{z_1} - \frac{1}{z} \right] = t - t_1 \quad (9)$$

$$t = -\frac{4Q_{zg}}{\pi b^2 K_g} \left[ -\frac{1}{z} + \frac{1}{z_1} \right] + t_1 \quad (10)$$

$$t(z) = t_1 - \frac{4Q_{zg}}{\pi b^2 K_g} \left[ \frac{1}{z_1} - \frac{1}{z} \right] \quad (11)$$

To find the heat distribution on the gear tooth, it is worthwhile to note that  $Q_z$  is constant and independent of  $z$

Thus, assume that  $z = z_2$  and  $t(z_2) = t_2$

$$t_2 = t_1 - \frac{4Q_{zg}}{\pi b^2 K_g} \left[ \frac{1}{z_1} - \frac{1}{z_2} \right] \quad (12)$$

Solving gives:

$$t_2 \pi b^2 K_g = \pi b^2 K_g t_1 - 4Q_{zg} \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \quad (13)$$

$$\frac{\pi b^2 K_g (t_2 - t_1)}{4 \left( \frac{1}{z_1} - \frac{1}{z_2} \right)} = -Q_{zg} \quad (14)$$

$$Q_{zg} = - \frac{\pi b^2 K_g (t_2 - t_1)}{4 \left( \frac{1}{z_1} - \frac{1}{z_2} \right)} \quad (15)$$

*Determination of fractional change in length*

This is given by:

$$\frac{\Delta L}{L_0} = \alpha \Delta T \quad (16)$$

Knowing that the coefficient of linear temperature expansion for all steels is between  $(11 \text{ and } 12.5) \times 10^{-6} \text{m}/(\text{m k})$ . The fractional change in length falls between  $(1.1 \text{ and } 1.25) \times 10^{-3}$

Equation (15) therefore gives the variation in the heat distribution along the path of travel of the finite points  $z_1$  and  $z_2$ . The relationship between the heat transfer rate and temperature variation as well as the temperature along the path of travel are displayed in Figs. 2 to 6.

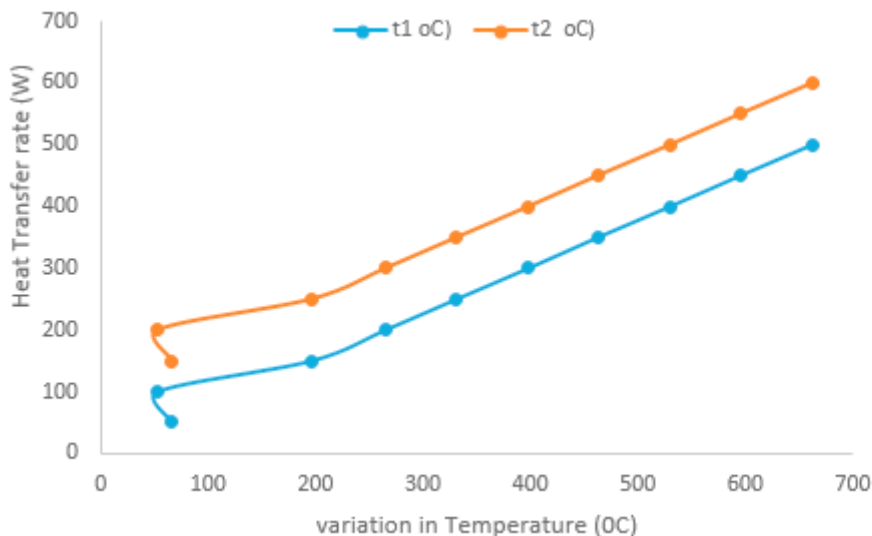
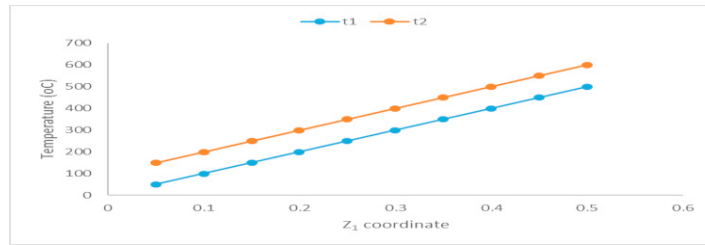
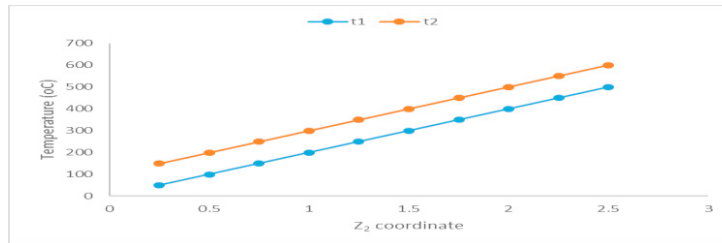
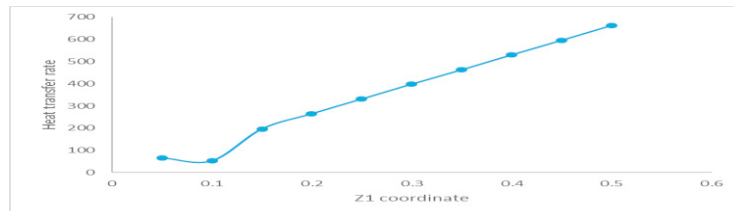
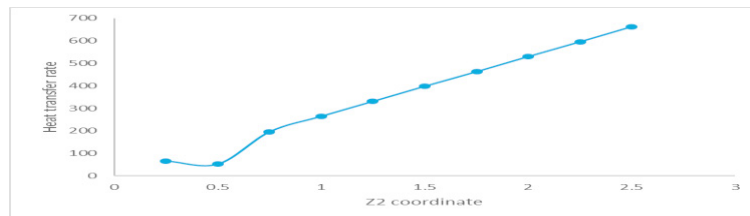


Fig.2: Variation of Temperature with Heat transfer rate

Fig.3: Variation in Temperature with Z<sub>1</sub> co-ordinateFig.4: Variation in Temperature with Z<sub>2</sub> co-ordinateFig.5: Variation of Heat Transfer rate with Z<sub>1</sub> Co-ordinateFig.6: Variation of Heat Transfer rate with Z<sub>2</sub> Co-ordinate

### 3. Results and Discussion

Fig. 2 presents the result of temperature variation with rate of heat transfer. From the result, at temperatures of 50°C and 150°C, the magnitude of heat transfer rate at both the tip and the root of gear tooth was 66.3 W. Further increase in temperature resulted in sharp increase in heat transfer rate. This represent the phenomenon of the initial state of the gear tooth in operation. Also, at constant temperature increase, the rate of heat transfer increased. Going by the value of the fractional change in length (equation (16)) and the result thereby, the deformation of the gear tooth would not be obvious at this stage due to the steady state conduction through the material. Although, fatigue would be inevitable when the material is subjected to operation under such condition at longer hours.

Similarly, Figs. 3 - 4 are the variation of temperature with the Z co-ordinates.  $t_1$  displayed linearity in temperature variation while there was non-linearity in the variation in  $t_2$  at 150°C and 200°C respectively. This was due to slight variation in temperature. However, linearity in temperature variation was later maintained by  $t_2$  just as it was in  $t_1$ . More so, the z coordinate increases with corresponding temperature variations. Thus, it is worthy of note that increase in temperature will cause increase in displacement of the particles from their initial positions. In the same vein, Figs. 5 to 6 depict the variation in z coordinate with the rate of heat transfer. The figures show a linear relationship between heat transfer rate and the Z co-ordinates. However, there was a noticeable dip at onset of the heat

transfer after which the heat transfer rate increased with corresponding increase along the path of travel of the finite points (i.e. Z co-ordinates).

## Conclusion

The study employed the finite element approach for the solution of 1-dimensional steady state heat transfer to determine the temperature distributions on a spur gear tooth. The gear tooth was subjected to temperature variations both at the top and root as well as variations in the respective coordinate of the material particles. The developed model was further used to evaluate the rate of heat transfer through the gear material. The result showed that the heat flux decreases with increasing values of Z which represent the material position. However, change in position Z also constitute increased random motion of the particles in the gear, causing steady degradation even at decreasing heat flux. Therefore, the study has revealed that temperature variation has adverse effect on the wear mode of the gear material.

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