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APPROXIMATE-ANALYTICAL SOLUTIONS OF SOME CLASSICAL RICCATI DIFFERENTIAL EQUATION USING THE DAFTARDAR-GEJJI JAFARI METHOD

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Abstract: This present paper considers the approximate-analytical solution of some classical Riccati Differential Equations (RDEs). Here, an efficient numerical method referred to as Daftardar-Gejji Jafari Method (DJM) for solving the functional differential equations is applied. Three numerical examples are considered to show the accuracy of the proposed method.

Keywords: iterative methods; Riccati differential equations; DJM; nonlinear differential equations; approximate-analytical solutions.

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1. INTRODUCTION

The Riccati differential equation is one of the essential classes of differential equations, which is very useful in the area of sciences and engineering. To be considered in this work is the general Riccati differential equation (RDEs) of the form [1]:

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$$\begin{cases} Z'_y = f_1(y) z^2 + a_1 z - a_1 a_2 - a_2{}^2 f_1(y), \\ Z_0 = a_2, \end{cases} \tag{1.1}$$

such that:

$$Z = a_2 + \psi(y)\left[E - \int f_1(y)\psi(y)\,dy\right]^{-1}, \tag{1.2}$$

represents the general solution of (1.1). By definition,

$\psi(y) = \exp\left[a_1 y + 2a_2 \int f_1(y)\,dy\right]$ and E is an arbitrary constant.

This particular type of differential equation plays a very significant role in applied sciences and engineering [2]. The idea was initiated by the Italian Scholar Jacopo Francesco Riccati [3]. Riccati differential equations can be applied to different areas such as diffusion process, control theory, stochastic processes, rheology, damping laws, and so on [4-9].

Due to the nonlinear nature of the Riccati Differential equation, the general solution (1.1), may not be easily obtained. Hence, the need to apply numerical (iterative) methods for obtaining the approximate solutions [10]. This problem has drawn the attention of many researchers as widely investigated and remarked. Different numerical methods such as Adomian Decomposition, Homotopy Perturbation Variational Iteration, Differential Transform, Taylor Matrix, Chebyshev polynomials, Legendre wavelet, and He's variational methods have been applied to Riccati differential equations [11-22]. The integrability of RDEs was studied in [23]; the general solution of the RDEs was considered via the analytical method [24, 25]. Riccati differential equation was transformed from first-order into the second-order form by proposing a new and efficient transformation in [26]. Bezier Curves Method (BCM) was introduced to obtain the approximate solution of RDEs in [27], Chebyshev cardinal functions and Cubic B-spline scaling functions have been used to solve the RDE, as presented in [28].

There are so many works already in existence that discussed the application of the Daftar-Gejji Jafari method. This method was proposed in 2006 by two researchers Daftardar-Gejji and Jafari [29]. The technique is capable of handling any form of a functional differential equation (linear and nonlinear). DJM has been widely used by many researchers to solve problems relating to linear and nonlinear ODEs and PDEs, both in integer and fractional orders [30-37].

Approximate or analytical solution methods for linear and nonlinear differential models are linked to the following [38-42]. This present work considers the application of DJM for obtaining the approximate solution of some class of nonlinear Riccati differential equations.

The remaining part of this work is organized as follows: nonlinear RDE is presented in section 2. Method of the solution is discussed in section 3; numerical examples are considered in section 4, and then, the concluding remarks are made in section 5.

2. NONLINEAR RICCATI DIFFERENTIAL EQUATION (NRDES)

Consider the nonlinear Riccati differential equations (RDEs) of the form:

$$\begin{cases} z'(t) = Q(t)z^2 + P(t)z - h(t), \quad t_0 \le t \le t_f \\ z(t_0) = \lambda, \end{cases} \tag{2.1}$$

where $Q(t), P(t)$ and $h(t)$ are continuous, t_0, t_f and λ are arbitrary constants, and $z(t)$ is the unknown function. By comparing (2.1) and (1.1) we have:

$$\begin{cases} z'_y \Rightarrow \dfrac{dz}{dt}, \\ f_1(y)z^2 \Rightarrow Q(t)z^2, \\ a_1 z \Rightarrow P(t)z, \\ a_1 a_2 - a_2{}^2 f_1(y) \Rightarrow h(t). \end{cases} \tag{2.2}$$

As stated earlier, our approach follows the concept of using the Daftardar-Gejji Jafari method (DJM) to approximate the solution of $z(t)$, and $\bar{N}(t)$. Here, $\bar{N}(t)$ is given in detail form in section 3. Furthermore, (2.1) is widely encountered in Engineering, Physical science, and other areas.

Remark 2.1: If $t = 0$ (2.1) becomes linear. So, $t \ne 0$ for our nonlinear cases.

3. DAFTARDAR-GEJJI JAFARI METHOD (DJM)

Consider the general functional equation defined as follows

$$z = a + L(z) + N[z], \tag{3.1}$$

where a is a known function $L[\cdot]$ and $N[\cdot]$ are the linear and nonlinear operators, respectively. Suppose we define $\bar{N}[z]$ as:

$$\bar{N}[z] = L[z] + N[z], \tag{3.2}$$

then (3.1) becomes:

$$y = b + \bar{N}[z]. \tag{3.3}$$

Now considering a solution, z of (3.2) having the infinite series form:

$$\begin{cases} z = \sum_{i=0}^{\infty} z_i, \\ \bar{N}[z] = \bar{N}\left[\sum_{i=0}^{\infty} z_i\right] \end{cases}. \tag{3.4}$$

The nonlinear operator $\bar{N}$ can now be decomposed as

$$\bar{N}\left(\sum_{i=0}^{\infty} z_i\right) = \bar{N}[z_0] + \sum_{i=1}^{\infty}\left[\bar{N}\left(\sum_{i=0}^{m} z_i\right) - \bar{N}\left(\sum_{i=0}^{m-1} z_i\right)\right], m = 1,\ 2... \tag{3.5}$$

Therefore, putting (3.4) and (3.5) into (3.3), we obtain

$$\sum_{i=0}^{\infty} z_i = b + \bar{N}[z_0] + \sum_{i=1}^{\infty}\left[\bar{N}\left(\sum_{i=0}^{m} z_i\right) - \bar{N}\left(\sum_{i=0}^{m-1} z_i\right)\right] \ m = 1,\ 2,... . \tag{3.6}$$

Hence, the recurrence relation is gotten as:

$$\begin{cases} z_0 = a \\ z_1 = \bar{N}(z_0) \\ z_{m+1} = \bar{N}\left[\sum_{i=0}^{m} z_i\right] - \bar{N}\left[\sum_{i=0}^{m-1} z_i\right],\ m = 1,\ 2,... \end{cases} \tag{3.7}$$

such that:

$$z = a + \sum_{i=1}^{\infty} z_i = \sum_{i=0}^{\infty} z_i. \tag{3.8}$$

Bhaleka *et al.,* [43] discussed the convergence of this method in detail.

4. ILLUSTRATIVE EXAMPLES

This section presents some illustrative examples following the method as mentioned above. The numerical results are given in figures and tables to show the effectiveness of the proposed method.

Example 4.1

Consider the following RDE [27, 44]

$$\begin{cases} z'(t) = z^2(t) - z(t), \\ 2z(0) = 1. \end{cases} \tag{4.1}$$

The exact solution of (4.1) was given as:

$$z^*(t) = \frac{\exp(-t)}{1+\exp(-t)}.$$

In integral form, (4.1) yields:

$$z(t) = \frac{1}{2} + \int_0^t z^2(s)\,ds - \int_0^t z(s)\,ds. \tag{4.2}$$

Now,

$$z(t) = a + \bar{N}[z].$$

This implies that:

$$\begin{cases} 2a = 1, \\ \bar{N}[z] = \int_0^t z^2(s)\,ds - \int_0^t z(s)\,ds. \end{cases} \tag{4.3}$$

For the linearity concept, the following are remarked:

$$\begin{cases} L[z] = \int_0^t z(s)\,ds, \; N[z] = \int_0^t z^2(s)\,ds. \end{cases}$$

By applying DJM to (4.2), the following is obtained:

$$2z_0 = 1$$

$$z_1 = \bar{N}[z_0] = \int_0^t z^2(s)\,ds - \int_0^t z(s)ds,$$

$$= \int_0^t \left(z_0{}^2 - z_0\right)ds.$$

$$z_3 = \bar{N}[z_0 + z_1 + z_2] - \bar{N}[z_0 + z_1],$$

$$= \bar{N}\left[\frac{1}{2} + z_1 + z_2\right] - \bar{N}\left[\frac{1}{2} + z_1\right],$$

$$= \int_0^t \left[\left(z_0 + z_1 + z_2\right)^2 - \left(z_0 + z_1 + z_2\right)\right]ds - \left[\int_0^t \left(\left(z_0 + z_1\right)^2 + \left(z_0 - z_1\right)\right)ds\right].$$

$$z(t) = \sum_{i=0}^{3} z_i.$$

Example 4.2: Consider the classical RDE [3, 27, and 45]

$$\begin{cases} z'(t) = -z^2(t) + 2z(t) + 1, \\ z(0) = 0. \end{cases} \tag{4.4}$$

The exact solution for (4.4) is given as:

$$z^*(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)^{0.5}\right).$$

Equation (4.4) yields:

$$z(t) = t - \int_0^t z^2(s)\,ds + 2\int_0^t z(s)ds, \tag{4.5}$$

in integral form.

Now,

$$z(t) = a + \bar{N}[z].$$

This implies that:

$$\begin{cases} a = t, \\ \bar{N}[z] = -\int_0^t z^2(s)\,ds + 2\int_0^t z(s)ds. \end{cases} \tag{4.3}$$

For the linearity concept, the following are remarked:

$$\left\{ L[z] = 2\int_0^t z(s)ds,\ N[z] = -\int_0^t z^2(s)\,ds \right. .$$

By applying DJM to (4.5), the following is obtained:

$$z_0 = t$$

$$z_1 = \bar{N}[z_0] = -\int_0^t z^2(s)\,ds + 2\int_0^t z(s)ds,$$

$$= -\int_0^t (z_0)^2 ds + 2\int_0^t z_0 ds.$$

$$z_2 = \bar{N}[z_0 + z_1] - \bar{N}[z_0],$$

$$= \bar{N}[t + z_1] - \bar{N}[t],$$

$$= \ -\left[\int_0^t (z_0 + z_1)^2\, ds - 2\int_0^t (z_0 + z_1)\,ds\right] - [z_1].$$

$$z_3 = \bar{N}[z_0 + z_1 + z_2] - \bar{N}[z_0 + z_1],$$

$$= \bar{N}[t + z_1 + z_2] - \bar{N}[t + z_1],$$

$$= \left[-\int_0^t (z_0 + z_1 + z_2)^2\, ds + 2\int_0^t (z_0 + z_1 + z_2)\,ds\right] - \left[-\int_0^t (z_0 + z_1)^2\, ds + 2\int_0^t (z_0 + z_1)\,ds\right].$$

$$z(t) = \sum_{i=0}^{3} z_i.$$

Example 4.3

Consider the following Riccati differential equation [3, 27, and 45].

$$\begin{cases} z'(t) = z^2(t) + 8tz(t) + 16t^2 - 5, \\ z(0) = 1. \end{cases} \tag{4.6}$$

The exact solution of (4.6) is given as:

$$z^*(t) = 1 - 4t.$$

In integral form, (4.6) yields:

$$z(t) = 1 + \frac{16}{3}t^3 + 5t + \int_0^t z^2(s)\,ds + 8\int_0^t tz(s)ds \tag{4.7}$$

Now,

$$z(t) = a + \bar{N}[z].$$

This implies that:

$$\begin{cases} a = 1 + \frac{16}{3}t^3 + 5t, \\ \bar{N}[z] = \int_0^t z^2(s)\,ds + 8\int_0^t tz(s)ds. \end{cases} \tag{4.8}$$

For the linearity concept, the following are remarked:

$$\left\{ L[z] = 8\int_0^t tz(s)ds,\ N[z] = \int_0^t z^2(s)\,ds \right. .$$

By applying DJM to (4.7), the following is obtained:

$$z_0 = 1 + \frac{16}{3}t^3 + 5t,$$

$$z_1 = \bar{N}[z_0] = \int_0^t z^2(s)\,ds + 8\int_0^t tz(s)ds,$$

$$= \int_0^t (z_0)^2 ds + 8\int_0^t tz_0 ds.$$

$$z_2 = \bar{N}[z_0 + z_1] - \bar{N}[z_0],$$

$$= \int_0^t (z_0 + z_1)^2 ds + 8\int_0^t t(z_0 + z_1)\,ds - [z_1],$$

$$z_3 = \bar{N}[z_0 + z_1 + z_2] - \bar{N}[z_0 + z_1],$$

$$= \int_0^t (z_0 + z_1 + z_2)^2 ds + 8\int_0^t t(z_0 + z_1 + z_2)\,ds - \left[\int_0^t (z_0 + z_1)^2 ds + 8\int_0^t t(z_0 + z_1)\,ds \right],$$

$$\vdots$$

$$z_5 = \bar{N}\left[z_0 + z_1 + z_2 + z_3 + z_4\right] - \bar{N}\left[z_0 + z_1 + z_2 + z_3\right],$$
$$= \int_0^t \left(z_0 + z_1 + z_2 + z3 + z_4\right)^2 ds + 8\int_0^t t\left(z_0 + z_1 + z_2 + z_3 + z_4\right) ds$$
$$- \left[\int_0^t \left(z_0 + z_1 + z_2 + z_3\right)^2 ds + 8\int_0^t t\left(z_0 + z_1 + z_3\right) ds\right].$$

$$z(t) = \sum_{i=0}^{5} z_i.$$

4.1 Numerical Results

Here, the results are presented in tabular and graphical forms, as shown in Tables 4.1-4.3 and Figure 4.1-4.3.

Table 4.1: Error Analysis of $z(t)$ and $z^*(t)$ for example 4.1

t	Approximate Solution $z(t)$	Exact Solution $z^*(t)$	$\lvert z(t) - z^*(t)\rvert$
0.0	5.000000000000000E-01	5.000000000000000E-01	0.000E+00
0.1	4.750208125062004E-01	4.750208125210600E-01	1.490E-11
0.2	4.501660007936508E-01	4.501660026875221E-01	1.890E-09
0.3	4.255574510602679E-01	4.255574831883410E-01	3.213E-08
0.4	4.013121015873016E-01	4.013123398875480E-01	2.383E-07
0.5	3.775395469060020E-01	3.775406687981455E-01	1.122E-06
0.6	3.543397357142857E-01	3.543436937742045E-01	3.958E-06
0.7	3.318007937934028E-01	3.318122278318340E-01	1.143E-05
0.8	3.099970031746032E-01	3.100255188723876E-01	2.852E-05
0.9	2.889869688058035E-01	2.890504973749960E-01	6.353E-05
1.0	2.688120039682540E-01	2.689414213699951E-01	1.294E-04

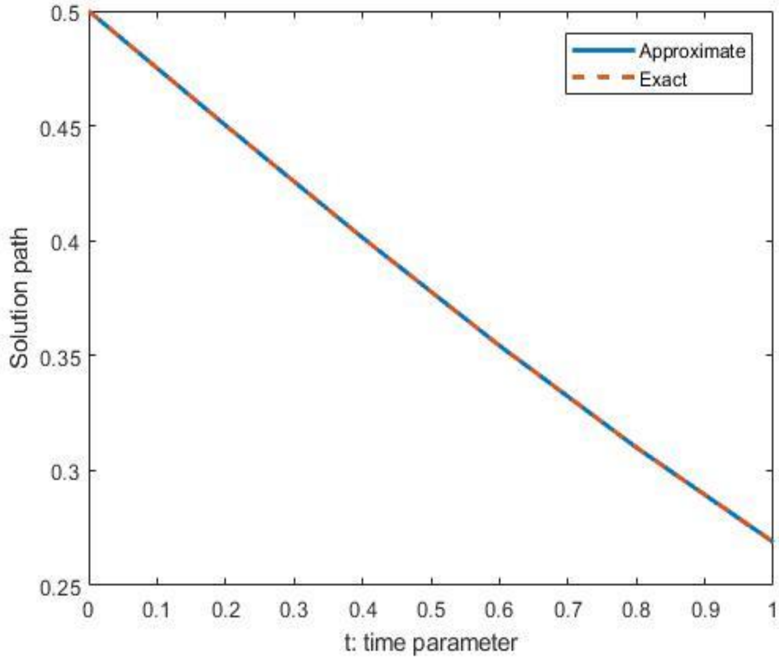

Figure 4.1: Graphs of the approximate and exact solution for Example 4.1

Table 4.2: Error Analysis of $z(t)$ and $z^*(t)$ for example 4.2

t	Approximate Solution $z(t)$	Exact Solution $z^*(t)$	$\lvert z(t)-z^*(t)\rvert$
0.0	0.000000000000000000	0.000000000000000000	0.000000
0.1	1.102951630311075E-01	1.102951969169624E-01	3.389E-08
0.2	2.419752508705418E-01	2.419767996211093E-01	1.549E-06
0.3	3.950932307796952E-01	3.951048486603785E-01	1.162E-05
0.4	5.677733163369734E-01	5.678121662929388E-01	3.885E-05
0.5	7.559368137511863E-01	7.560143934313760E-01	7.758E-05
0.6	9.534634383426247E-01	9.535662164719230E-01	1.0278E-04
0.7	1.152856119841550E+00	1.152948966979624E+00	9.285E-05
0.8	1.346306868262017E+00	1.346363655368376E+00	5.679E-05
0.9	1.526893826443628eE+00	1.526911313280625E+00	1.749E-05
1.0	1.689551055683199E+00	1.689498391594383E+00	5.266E-05

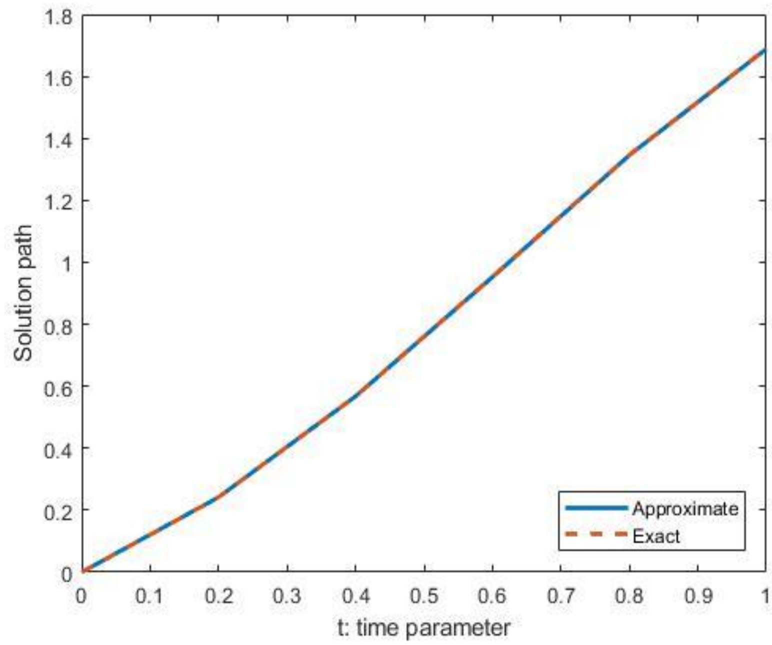

Figure 4.2: Graphs of the approximate and exact solution for Example 4.2

Table 4. 3: Error Analysis of $z(t)$ and $z^*(t)$ for example 4.3

t	Approximate Solution $z(t)$	Exact Solution $z^*(t)$	$\lvert z(t)-z^*(t)\rvert$
0.0	1.0000000000000000	1.00000000000000000	0.000000
0.1	5.999999564819216E-01	6.000000000000000E-01	4.352E-08
0.2	1.999973086173199E-01	2.000000000000000E-01	2.691E-06
0.3	-2.000292456117776E-01	-2.000000000000000E-01	2.925E-05
0.4	-6.001547258640714E-01	-6.000000000000001E-01	1.547E-04
0.5	-1.000547997417370E+00	-1.00000000000000000	5.480-04
0.6	-.401494265494364E+00	-1.400000000000000E+00	1.494E-03
0.7	-1.803366280340690E+00	-1.800000000000000E+00	3.3662E-03
0.8	-2.206474911496222E+00	-2.200000000000000E+00	6.475E-03
0.9	2.610561005274426E+00	-2.600000000000000E+00	1.056E-02
1.0	3.012510754974513E+00	-3.00000000000000000	1.251E-02

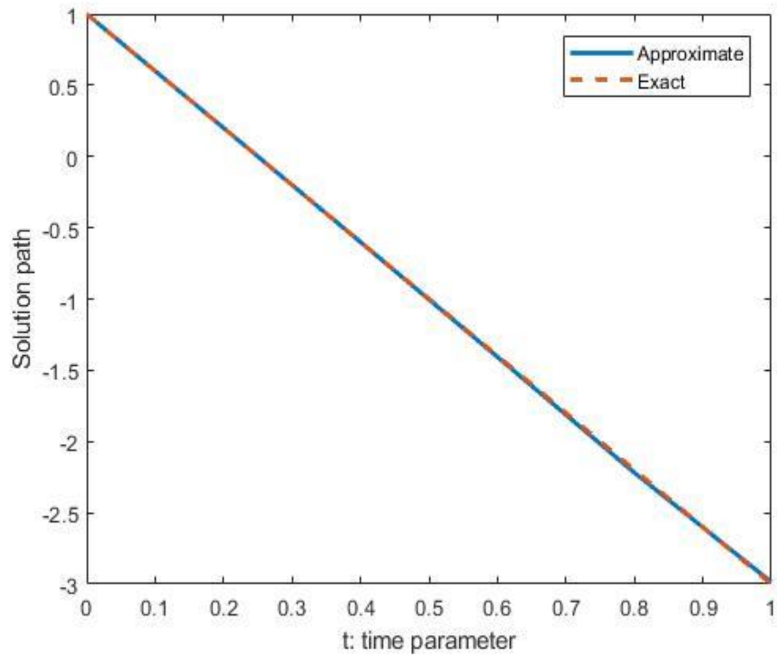

Figure 4.3: Graphs of the approximate and exact solution for Example 4.3

5. CONCLUSION

This work considered the application of the Daftardar-Gejji Jafari method for the approximate solution of some classical Riccati differential equations (RDES). This method is direct in terms of application, easy to use, and reduces computational stress. Three numerical examples were investigated to test the accuracy and efficiency of the proposed method. The results converged faster to the exact solutions when compared with some already existing methods.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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