Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

On the spectrum of the weighted p-Laplacian under the Ricci-harmonic flow

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Abstract

This paper studies the behaviour of the spectrum of the weighted p-Laplacian on a complete Riemannian manifold evolving by the Ricci-harmonic flow. Precisely, the first eigenvalue diverges in a finite time along this flow. It is further shown that the same divergence result holds on gradient shrinking and steady almost Ricci-harmonic solitons under the condition that the soliton function is nonnegative and superharmonic. We also continue the program in (Abolarinwa, Adebimpe and Bakare in J. Ineq. Appl. 2019:10, 2019) to the case of volume-preserving Ricci-harmonic flow.

MSC: Primary 53C21; secondary 53C44; 58C40

Keywords: Ricci harmonic flow; Laplace–Beltrami operator; Eigenvalue; Monotonicity; Ricci solitons

1 Introduction

In this paper we aim at studying the properties of the spectrum of the weighted p-Laplacian on a complete Riemannian manifold with evolving geometry. It is a well known feature that spectrum as an invariant quantity evolves as the domain does under any geometric flow. Throughout, we will consider an n-dimensional complete Riemannian manifold $(M, g, d\mu)$ equipped with weighted measure $d\mu = e^{-\phi}\, dv$ and potential function $\phi \in C^{\infty}(M, d\mu)$, whose metric $g = g(t)$ evolves along either the Ricci-harmonic flow or volume-preserving Ricci-harmonic flow. Firstly, we extend results in [8] to the case of volume-preserving Ricci-harmonic flow. We will obtain a variation formula for the first eigenvalue and show that it is monotonically increasing under this setup. Secondly, we study maximal time behaviour of the first eigenvalue. It is found that the bottom of the spectrum diverges in a finite time of the flow existence. We observe the same result for the behaviour of the evolving spectrum on a class of self-similar solutions, called gradient almost Ricci-harmonic solitons.

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