Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Iterative Method for Approximate-Analytical Solutions of Linear Schrödinger Equation

Iterative Method for Approximate-Analytical Solutions of Linear Schrödinger Equation

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Abstract: In this study, the modified Picard Iterative Method (MPIM) is used to provide analytic and numerical solutions to linear Schrödinger Equations. These approximate-analytical solutions for the examples under consideration are easily computed. The suggested method is employed without any transformation, discretization, linearization, or limiting assumptions. The obtained results are similar to their exact forms. As a result, the approach is highly suggested for both linear and non-linear time-space fractional partial differential models with applications in various applied disciplines.

Keywords: Picard Iteration, Schrödinger Equations; Analytical solutions.

1.0 Introduction

The Schrödinger equation is a special partial differential equation (PDE) that appears in non-linear optics, superconductivity, plasma physics, and quantum mechanics, among other fields. The time-independent Schrödinger model or equation (TISE) and the time-dependent Schrödinger wave equation (TDSWS) are two versions of this equation [1, 2]. The typical form of a Schrödinger model is as follows (1.1).

$$\left.\begin{array}{l} i\dfrac{\partial w}{\partial t}+\lambda\dfrac{\partial^2 w}{\partial x^2}+p(x)w+\xi|w|^2 w=0 \\ g(x)=w(x,0),\ \ i^2=-1 \end{array}\right\} \tag{1.1}$$

where $w=w(x,t)$ is a complex, $p(x)$ is a function of x and λ, ξ are constants.

For the solution of such a model, many methods have been used [3-7]. Other approximate methods can be used in the same way [8-11]. In this research, we solve the linear form of Schrödinger equations using an iterative approach termed modified Picard Iterative Method (MPIM).

Journal of Physics: Conference Series **2199** (2022) 012005 doi:10.1088/1742-6596/2199/1/012005

2.0 Picard Iterative Method and the Model

The method of solution referred to as Successive Approximation Method (SAM) is introduced here, in line with some basic preliminaries.

2.1 Lipschitzian Continuity Condition

Let $f(t,y)$ be given function, so $f(t,y)$ satisfies a Lipchitz condition with respect to y in a certain region referred to as D in the XY-plane, if there exists a non-negative constant ζ , such that

$$|f(t,y_a)-f(t,y_b)|\le\zeta|y_a-y_b|$$

whenever (t,y_a) and (t,y_b) are in D, and ζ is called the Lipchitz constant.

2.2 Overview of the Successive Iteration Method

Suppose a first-order non-linear ordinary differential equation (ODE) is given as follows with an initial condition:

$$\begin{cases}\dfrac{dy}{dt}=g(t,y),\\ y(t_0)=y_0\end{cases} \tag{2.1}$$

Suppose $\int_{t_0}^{t}(\cdot)\,dy=I_{t_0}^{t}(\cdot)$ denotes a one-fold integral operator w.r.t. a concerned variable; thus, by direct integration of both sides (2.1) over $(t_0,\ t)$, we have:

$$\begin{cases}I_{t_0}^{t}(dy)=I_{t_0}^{t}(g(s,y))\\ y(t_0)=y_0\end{cases} \tag{2.2}$$

This implies that:

$$\begin{cases}y(t)=y_0+I_{t_0}^{t}(g(s,y)),\\ y=y(t)\end{cases} \tag{2.3}$$

By iteration, we substitute $y(t)=y_{n+1}(t)=y_{n+1}$ and $y_n(s)=y_n$

Therefore, (2.3) becomes:

$$\begin{cases}y_{n+1}=y_0+I_{t_0}^{t}(g(s,y_n)),\\ y=y(t)\end{cases} \tag{2.4}$$

3.0 Applications

Here, the proposed method (MPIM) to a case study of a linear Schrödinger equation.

Case 3.1: In (1.1), we take $\lambda=-1,\ p(x)=0,$ and $\xi=0$, then the following is considered:

$$\begin{cases} h_t + ih_{xx} = 0 \\ h(x,0) = 1+\sinh 2x \end{cases}, \tag{3.1}$$

Equation (3.1) is rewritten as follows:

$$\begin{cases} h_t = -ih_{xx} \ , h = h(x,t) \\ h(x,0) = 1+\sinh(2x) \\ h(x,t) = 1+e^{-4it}\sinh(2x) \end{cases} \tag{3.2}$$

Applying the SIM to (3.2), give the following relation:

$$\left\{ h_{j+1} = h_0 - I_0^t\left(\left(i\left(h_j\right)_{xx}\right)\right), \ j \geq 0. \right. \tag{3.3}$$

Thus, we obtained the following iteratively:

$$\left.\begin{array}{l} h_1 = 1+\sinh(2x) - 4i\sinh(2x)t \\ h_2 = 1+\sinh(2x) - 8\sinh(2x)t^2 - 4i\sinh(2x)t \\ h_3 = 1+\sinh(2x) + \dfrac{32i\sinh(2x)t^3}{3} - 8\sinh(2x)t^2 - 4i\sinh(2x)t \\ h_4 = 1+\sinh(2x) + \dfrac{32\sinh(2x)t^4}{3} + \dfrac{32i\sinh(2x)t^3}{3} - 8\sinh(2x)t^2 - 4i\sinh(2x)t \\ h_5 = 1+\sinh(2x)\left(1 - \dfrac{128it^5}{15} + \dfrac{32\sinh(2x)t^4}{3} + \dfrac{32it^3}{3} - 8t^2 - 4it\right) \\ \quad \vdots \end{array}\right\}$$

$$\left.\begin{array}{l} h_9 = 1+\sinh(2x)\left(-\dfrac{2048it^9}{2835} + \dfrac{512t^8}{315} + \dfrac{1024it^7}{315} - \dfrac{256t^6}{45} - \dfrac{128it^5}{15} + \dfrac{32t^4}{3} + \dfrac{32it^3}{3} - 8t^2 - 4it + 1\right) \\ \quad = 1+e^{(-4it)}\sinh(2x). \end{array}\right\} \tag{3.4}$$

The exact and 9-term approximate results are presented in Figure 1 and Figure 2, respectively.

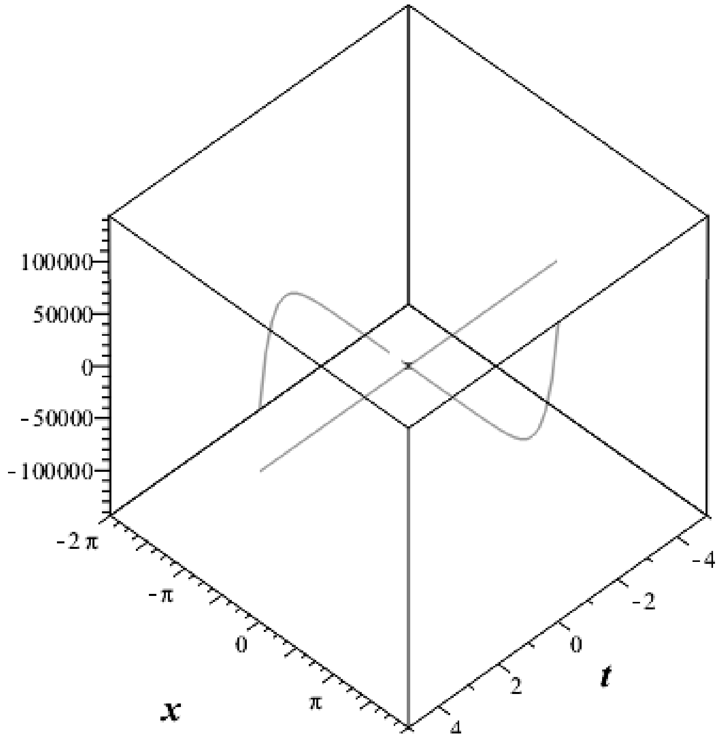

Figure 1: Exact solution of (4.1)

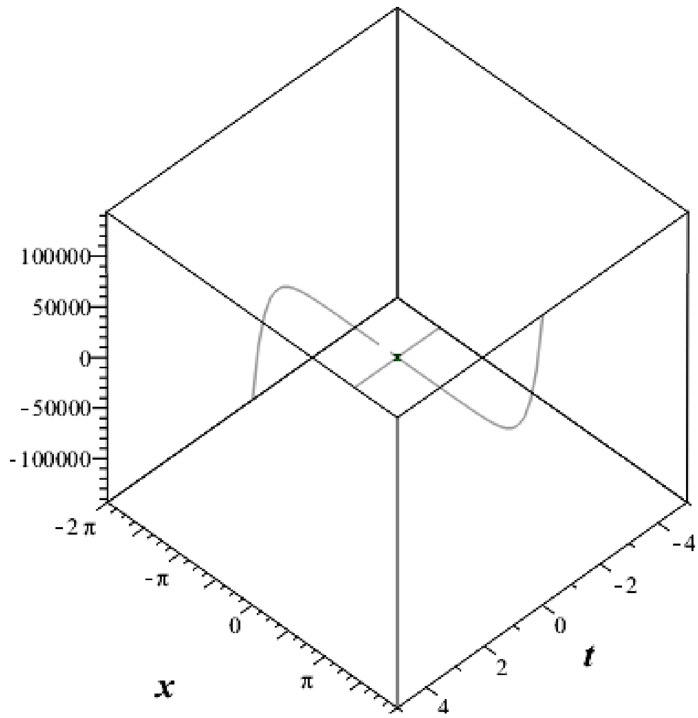

Figure 2: 9-term approximate solution of (4.1)

4.0 Concluding Remarks

This research successfully employed the modified PIM (MPIM) to solve a linear Schrödinger. The iterative method suggested is computer-friendly and has a straightforward foundation. The MPIM's results are very comparable to those of other iterative methods available in the literature. MPIM also allows more accurate numerical solutions for non-linear problems. It also does not need much computer memory, stringent or constraining assumptions, or discretization procedures.

Acknowledgment

Covenant University- CUCRID unit is well recognized for all sorts of assistance. We would also like to express our gratitude to the anonymous reviewer(s) for their insightful and helpful feedback.

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