Remarks on the solution of a one-dimensional heat model with axial symmetry using decomposition method

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Remarks on the solution of a one-dimensional heat model with axial symmetry using decomposition method

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Abstract. This paper explores the use of a new decomposition method termed-Natural Decomposition Method (NDM) on a one-dimensional heat model-equation with axial symmetry for analytical solutions. The series solutions of the sampled cases are obtained by the proposed method with ease and high precision for less computational time. Hence, this indicates the feasibility of the proposed approach.
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Keywords: Natural decomposition transform, Differential models, Approximate solution

1.0 Introduction

In science and engineering, differential equations and their implementations have been noted as building blocks. However, in most cases, it remains difficult to achieve their exact, approximate or numerical solutions [1-7]. In this study, an axial symmetry source-less heat model is considered, which describes a one-dimensional non-stable thermal process. This is generally of the form:

$$\begin{cases} \dfrac{\partial E}{\partial t} = \dfrac{a}{w}\dfrac{\partial}{\partial w}\left(w\dfrac{\partial E}{\partial w}\right) \\ E(w,0) = h(w) \end{cases} \tag{1.1}$$

where the speed and spatial order of the system is regulated by $a > 0$, body temperature at the point w, and time parameter, t is given as $E(w,t)$,

Other computational methods can be explored for approximate and exact solutions to similar models [8-15]. However, the relatively new method referred to as Natural Decomposition Method (NDM) will be adopted for the solution of the model in (1.1). Thus, the detail of the method is presented in the next section.

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2.0 Overview of the Decomposition Method

We cover the preliminaries of the Decomposition Method (NDM) as follows, and remarks are made to some references for details [14].

Suppose we define a class of functions F as:

$$F = \left\{ v(t) : \exists\, k, c_1, c_2 > 0 \ni \left| v(t) \right| < k e^{|t| c_i} \right\}, \tag{2.1}$$

then, the transform of $v(t)$ defined and denoted as:

$$N\left[v(t)\right] = M(s,\eta) = \int_0^\infty v(\eta t)\exp(-st)\,dt,\ t \in [0,\infty) \tag{2.2}$$

implies the natural transform of $v(t)$, on the ground that the existence of the integral in the ascertained. . In addition, the Inverse Natural Transform (INT) the function in (2.1-2.2) is:

$$\left.\begin{aligned} N^{-1}\left\{N\left[v(t)\right]\right\} &= N^{-1}\left\{M(s,\eta)\right\} \\ &= v(t). \end{aligned}\right\}. \tag{2.3}$$

The basic properties of NT are presented in Table 1 NT as follows.

Table 1 NT basic properties

Function $f(\bullet)$	NT of the function $N\left(f(\bullet)\right)$
t^n	$\dfrac{n!\eta^n}{s^{n+1}},\ n \geq 0$
$e^{\vartheta t}$	$\dfrac{1}{s - \vartheta\eta}$
$\cos(\vartheta t)$	$\dfrac{s}{s^2 + (\vartheta\eta)^2}$
$\sin(\vartheta t)$	$\dfrac{\vartheta\eta}{s^2 + (\vartheta\eta)^2}$
$\dfrac{\partial v}{\partial t}$	$\dfrac{s}{\eta} M(s,\eta) - \dfrac{v(x,0)}{\eta}$
$\dfrac{\partial^n v}{\partial t^n}$	$\dfrac{s^n}{\eta^n} M(s,\eta) - \sum_{j=0}^{n-1} \dfrac{s^{n-j-1}}{\eta^{n-j}} v^{(j)}(x,0)$
$\dfrac{\partial^n v}{\partial x^n}$	$\dfrac{\partial^n}{\partial x^n}\left[M(s,\eta)\right]$

2.1 The Decomposition Method

Considering the general differential equation of the form:

$$\begin{cases} Df(x,t)+Rf(x,t)+N_* f(x,t)=g(x,t) \\ f(x,0)=m(x) \end{cases} \tag{2.4}$$

where D is a differential operator in t (first order in this case), R is remaining section of the derivative or differential operator, N_* is the nonlinear operator, and $g(x,t)$ is a source term.

Applying the natural transform on (2.3) gives:

$$N\left[Df(x,t)\right]=N\left[g(x,t)\right]-N\left[\left(Rf(x,t)+N_* f(x,t)\right)\right].$$

$$\Rightarrow \frac{s}{\eta}M(s,\eta)-\frac{f(x,0)}{\eta}=N\left[h(x,t)\right]-N\left[\left(Rf(x,t)+N_* f(x,t)\right)\right] \tag{2.5}$$

Showing that:

$$M(s,\eta)=\frac{m(x)}{s}+\frac{\eta}{s}N\left[g(x,t)\right]-\frac{\eta}{s}N\left[\left(Rf(x,t)+N_* f(x,t)\right)\right]. \tag{2.6}$$

Applying $L_t^{-1}(\cdot)$ on both sides of (2.6) gives:

$$\begin{cases} p(x,t)=G(x,t)-N^{-1}\left\{\frac{\eta}{s}N\left[\left(Rp(x,t)+N_* p(x,t)\right)\right]\right\}, \\ G(x,t)=N^{-1}\left\{\frac{m(x)}{s}+\frac{\eta}{s}N\left[h(x,t)\right]\right\}. \end{cases} \tag{2.7}$$

Suppose the solution and the nonlinear term are expressed as follows according to Adomian and its polynomial:

$$\begin{cases} f(t,x)=\sum_{i=0}^{\infty} f_i(t,x), \\ Nf(t,x)=\sum_{i=0}^{\infty} A_i. \end{cases} \tag{2.8}$$

and A_i defined as:

$$A_i=\frac{1}{i!}\frac{\partial^i}{\partial\lambda^i}\left[N\left(\sum_{j=0}^{i}\lambda^j f_j\right)\right]_{\lambda=0}. \tag{2.9}$$

Thus, (2.7) becomes:

$$\sum_{i=0}^{\infty} f_i(x,t)=G(x,t)-N^{-1}\left\{\frac{\eta}{s}N\left[\left(R\left(\sum_{i=0}^{\infty} f_n(x,t)\right)+\left(\sum_{i=0}^{\infty} A_i\right)\right)\right]\right\}. \tag{2.10}$$

Therefore, the solution $f(x,t)$ is can be obtained via the recursive relation:

$$\begin{cases} f_0=N^{-1}\left\{\frac{m(x)}{s}+\frac{\eta}{s}N\left[g(x,t)\right]\right\} \\ f_{n+1}=-N^{-1}\left\{\frac{\eta}{s}N\left[\left(R(f_i)+A_i\right)\right]\right\}, i\geq 0. \end{cases} \tag{2.11}$$

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Whence, $p(x,t)$ is finalized as:

$$f(x,t) = \lim_{N \to \infty} \sum_{i=0}^{N} f_i \,. \tag{2.12}$$

3 Applications/Test Examples

Here, (1.1) is considered with some known initial data (conditions) based on the following test case-examples T1 and T2 as follows [11, 14]:

Example T1: Suppose the following 1D heat model of the form

$$\begin{cases} \dfrac{\partial E}{\partial t} = \dfrac{a}{w} \dfrac{\partial}{\partial w}\left(w \dfrac{\partial E}{\partial w} \right) \\ E(w,0) = 3\left(1 + w^2\right) \end{cases} \tag{3.1}$$

Example T2: Suppose the following 1D heat model of the form:

$$\begin{cases} \dfrac{\partial E}{\partial t} = \dfrac{a}{w} \dfrac{\partial}{\partial w}\left(w \dfrac{\partial E}{\partial w} \right) \\ E(w,0) = 2 + 3w^2. \end{cases} \tag{3.2}$$

In accordance with the proposed solution hints in section 2, the following solutions for *case T1* and *T2* are obtained and presented respectively, as follows

$$E(w,t) = 3\left\{1 + \left(w^2 + 4at\right)\right\}, \tag{3.3}$$

$$E(w,t) = 2 + 3\left(w^2 + 4at\right). \tag{3.4}$$

3.1 Numerical solutions

Here, the obtained solutions are presented graphically for different values of $a > 0$ (the regulator of speed and spatial order of the system). Figure 1a and Figure 1b are, for Example T1 for $a = 1$ and $a = 2$ respectively. Similarly, Figure 2a and Figure 2b are, for Example T2 for $a = 1$ and $a = 2$ respectively.

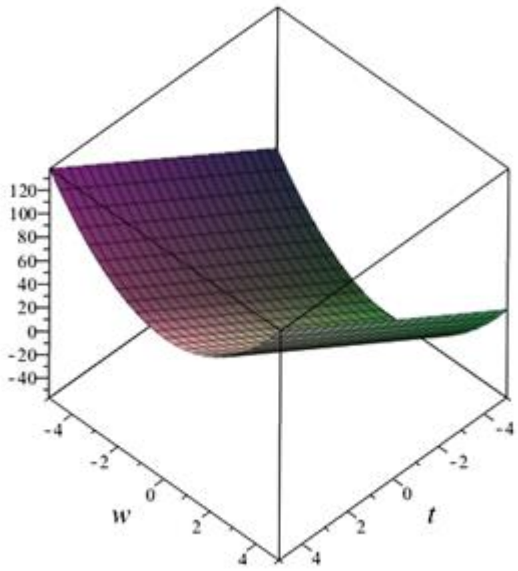

Figure 1a for Example T1 with $a = 1$

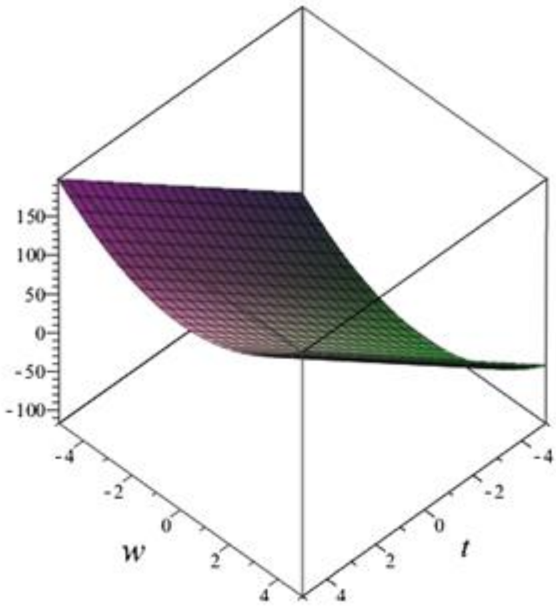

Figure 1b for Example T1b with $a = 2$

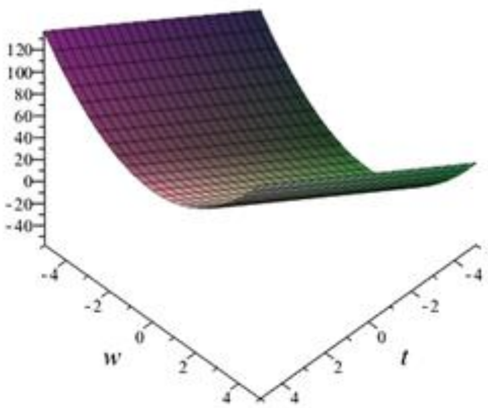

Figure 2a for Example T2 with $a = 1$

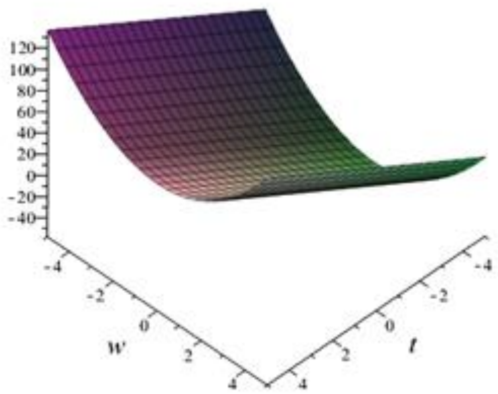

Figure 2a for Example T2 with $a = 1$

4.0 Conclusion

The implementation of the Natural Decomposition Method (NDM) has been successfully considered in the search for an analytical solution of the one-dimensional heat equation with axial symmetry. Based on this relatively new proposed approach, the solutions were easily achieved with less computing time. Therefore, it is noted for reliability and effectiveness; hence, the NDM can be adopted for higher-order versions of differential models.

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