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Coupled method for the solution of a one-dimensional heat equation with axial symmetry

S. O. Edeki ^{1*}, P. O. Ogunniyi¹, O. F. Imaga¹

¹Department of Mathematics, Covenant University, Ota, Nigeria

Contact e-mails: *soedeki@yahoo.com

Abstract. In this article, we implement the Projected Differential Transform Method (PDTM) coupled with Laplace Transform Method (LTM), hereby referred to as LPDTM, to solve a one-dimensional heat model (equation) with axial symmetry. Using the proposed framework (LPDTM), the exact solution (results) are obtained with convenience. It is noted that the suggested methodology performs very well and gives rapidly converging series solutions with less computational activities.

Keywords: Laplace transform, PDTM, Differential models, approximate solution

1.0 Introduction

Differential equations are keys tools both in pure and applied research. Their simplicity makes them relevant to various fields ranging from physics to engineering and other applied sciences.. The concepts of differential models are very helpful, even in simulation and forecasting of activities around us, such as predicting likely results and analyzing the spread of disease or improvements in species population over time. If an unknown incidence varies regarding time or space, it replicates a differential equation built on phenomena, which entails the processing speed and spatial order [1-4]. In most science fields, the heat equation is extremely significant. The Heat Equation is used as the Fokker Planck equation in finance and statistics. The diffusion equation occurring in the sample for chemical diffusion and other processes is a more general heat equation variant. A one-dimensional hyperbolic heat conductor may influence the thermal equilibrium of the superconductor when used. Efforts were made to consider the physical and chemical impact of soil temperature. Modeling the water and heat transport in soils and researching temperature impact on soil physical and chemical characteristics were the subject of recent efforts [5-8]. The temperature depends partially on the soil hydraulic properties due to the temperature influence on water viscosity. Soils may be modeled numerically or analytically for water and heat conversion. Numerical techniques have become the main study on water modelling and heat transport in soils [3, 4].

The Axial Symmetries (ASs) are referred to as isometric transformations, but they are in the opposite direction since they preserve distances from the corresponding pictures. There is axial symmetry if the points of origin match the points of the object. The symmetries are connected to most differential



models (equations) [6,7]. Here, a source-less heat equation/model will be considered as follows. Thus, a one-dimensional unsteady thermal system (process) with axial symmetry takes the form:

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\delta}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \\ u(\xi, 0) = h(\xi). \end{cases} \quad (1.1)$$

In (1.1), body temperature at the point ξ , and time parameter, t is given as $u(t, \xi)$, where $\delta > 0$, regulates the speed and spatial order of the system. For approximate solutions to related models, computational methods are being sought [9-22]. In this case, a one-dimensional version of the heat equation/model with axial symmetry is used for the approximate LPDTM process.

2.0 Laplace transform and the Projected DTM [18, 21]

Definition 1: Let $I = [a, b]$ be a time interval such that $t \in I = [a, b]$ and $h(t)$ a continuous function in $I = [a, b]$, the integral transform of $h(t)$ is denoted as:

$$I[h(t)] = \int_a^b h(t) \kappa(s, t) dt \quad (2.1)$$

where $\kappa(s, t)$ denotes the kernel of the transformation. Though, it depends on the differential types and their properties. Suppose the kernel for Laplace transform:

$$\kappa(s, t) = \begin{cases} \exp(-st), & 0 < t < \infty \\ 0, & -\infty < t < 0, \end{cases}$$

then,

$$L[\eta(t)] = \int_0^{\infty} \exp(-st) \eta(t) dt \quad (2.2)$$

is called the Laplace transform (LT) of $\eta(t) = h(t)$.

The key components of LT and their proof are can be seen in standard text. However, the following are noted [21].

$$\begin{cases} L\{c\} = \frac{c}{s}, & s \neq 0, k, \\ L\{\exp(at)\} = \frac{1}{s-a}, \\ \frac{1}{m!} L\{t^m\} = s^{-(m+1)}, & s > 0 \end{cases} \quad (2.3)$$

2.1 Remarks on the PDTM

The underlying principles and strategies of the proposed approach (PDTM) are illustrated in this section [10, 12, 18]. Let $h(x, t)$ defined on a given domain, G , be an analytic function, at a specified point (x_0, t_0) , such that the Taylor series expansion of $h(x, t)$, is ascertained. Then, the projected

differential transform of $h(x, t)$ and its inverse projected differential transform are defined and represented respectively as:

$$\left. \begin{aligned} H(x, m) &= \frac{1}{m!} \left[\frac{\partial^m h(x, t)}{\partial t^m} \right]_{t=t_0} \\ h(x, t) &= \sum_{m=0}^{\infty} H(x, m) (t - t_0)^m \end{aligned} \right\} \quad (2.7)$$

The following properties (P1-P5) and theorems associated with the method of solution are noted as follows in Table 1:

Table 1: Some Basic Properties of the PDTM

Property	Original function form	Projected Transform form
P1	$h(x, t) = \alpha h_a(x, t) + \beta h_b(x, t)$	$Q(x, m) = \alpha Q_a(x, m) + \beta Q_b(x, m)$
P2	$h(x, t) = \alpha \frac{\partial^n h_*(x, t)}{\partial t^n}$	$m! z(x, m) = \alpha (m+n)! H_*(x, m+n)$
P3	$h(x, t) = \alpha \frac{\partial h_*(x, t)}{\partial t}$	$m! Q(x, m) = \alpha (m+1)! H_*(x, m+1)$
P4	$h(x, t) = f(x) \frac{\partial^n h_*(x, t)}{\partial x^n}$	$H(x, m) = f(x) \frac{\partial^n H_*(x, m)}{\partial x^n}$
P5	$h(x, t) = f(x) h_*^2(x, t)$	$H(x, m) = f(x) \sum_{i=0}^m H_*(x, i) H_*(x, m-i)$

2.2 Laplace transform and the projected transformation method

The LPTDM is both the mechanism of transformation of Laplace and the project. With a few iterations, the process presents the solution clearly.

Considering the general partial differential equation (PDE):

$$\begin{cases} Dg(x, t) + Rg(x, t) + Ng(x, t) = h(x, t) \\ g(x, 0) = g_1^*, g = g(x, t) \end{cases} \quad (2.3)$$

such that D , R , N , and $g = g(x, t)$ n -th order derivative operator, remaining section of the derivative or differential operator, non-linear differential operator, and source term, respectively. Equation (2.3) is re-expressed as:

$$Dg(x, t) = h(x, t) - (Rg(x, t) + Ng(x, t)). \quad (2.4)$$

Then, by the Laplace and inverse Laplace transforms of (2.4), we have:

$$\begin{cases} g = g(x, 0) + L^{-1} \left\{ s^{-1} L \{ g \} \right\} - L^{-1} \left\{ s^{-1} L \{ Ng + Rg \} \right\}, \\ g = g(x, t). \end{cases} \quad (2.5)$$

Therefore, applying PDTM to (2.5) gives the following recurrence relation:

$$\begin{cases} G(x, k+1) = -L^{-1} \left\{ s^{-1} L \{ NG(x, k) + RG(x, k) \} \right\}, \\ g(x, 0) = \phi(x, t), g = g(x, t). \end{cases} \quad (2.6)$$

Equivalently, we have:

$$\begin{cases} G_0 = L^{-1} \{s^{-1} L \{G(x, t)\}\} + G(x, 0) \\ G_{k+1} = -L^{-1} \left\{ \frac{1}{s} L \{ (R(G_k) + NA_k) \} \right\}, k \geq 0 \\ g(x, t) = \phi(x, t) + \sum_{i=0}^{\infty} G_{i+1} \end{cases} \quad (2.7)$$

where A_k implies Adomian polynomials.

3 Test Examples/Applications

Considering (1.1) with some known initial data (conditions) for case-example IA and IB as follows [22]:

Case IA: Suppose the following 1D heat model of the form:

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\delta}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \\ u(\xi, 0) = 1 + 7\xi^2. \end{cases} \quad (3.1)$$

Case IB: Consider the 1D heat model of the form:

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\delta}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \\ w(\xi, 0) = \lambda \xi^2. \end{cases} \quad (3.2)$$

According to the proposed method in section 2, the solutions for *case IA* and *IB* are obtained and presented respectively, as follows:

$$u(\xi, t) = 1 + 7(4\delta t + \xi^2), \quad (3.3)$$

$$w(\xi, t) = \lambda(4\delta t + \xi^2). \quad (3.4)$$

4.0 Conclusion

The LPDTM, as a combination of Laplace Transformation (LT) and the Projected Differential Transform Method (PDTM), has been successfully used to study a one-dimensional heat equation (model) with axial symmetry. With less time and few iterations in terms of computation, the solutions were obtained with ease. Therefore, the LPDTM is recommended for the solutions of other linear and non-linear models of higher-orders; since the proposed algorithm has been ascertained for suitability with rapidly convergent series solutions.

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