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Approximate-analytical solutions of the quadratic Logistic differential model via SAM

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Abstract. This paper applies the novel Successive Approximation Method (SAM) for the solution of the quadratic Logistic Differential Model (LDM). To confirm the reliability of the method, illustrative examples are considered, and it is remarked that the approximate-analytical solutions of the considered cases are computed with ease. The proposed technique is used directly, without transformation, discretization, linearization, or any restrictive assumptions.

Keywords: Logistic differential model, non-linear model**,** approximate solutions; Transform method

1. Introduction

A logistic function is a solution to a conventionally expressed differential equation in mathematics referred to as Logistic Differential Model (LDM). In contrast to exponential functions, logistic functions take into account the restrictions that preclude unbounded resources [1-3]. This sort of model is also used in chess ratings, cancer therapy (such as the modeling of tumor progression), economics, and the study of language adoption. This concept is implausible since the environment constrains population growth. The general form of the LDM is given as:

$$\left.\begin{aligned} &\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right),\ N = N(x,t) \\ &N_0 = N(x,0) \end{aligned}\right\} \tag{1.1}$$

where $N = N(x,t)$ is the population size of the species at time t, r denotes the rate of growth in the absence of limited resources, and K denotes the carrying capacity or the maximum population that the ecosystem can support indefinitely.
This research aims to use the SAM to solve the Logistic Differential Model (LDM). Consequently, the goals are to apply SAM to the logistic differential equation and compare the results obtained using SAM to the exact solutions (if any) of the Logistic differential model. Although, in this respect, numerical techniques have been applied for solving dynamical models (equations) and other differential models [5-15].
Numerous strategies for obtaining an exact or numerical solution to ordinary or partial differential models have lately been presented by several solution specialists [16-26]. In this present work, an innovative technique known as the Successive Approximation Method (SAM) is used to solve LDM.

2. Remark on Continuity Condition and the Proposed SIM

The method of solution referred to as Successive Approximation Method (SAM) is introduced here, in line with some basic preliminaries.

2.1 Lipschitzian Continuity Condition

Let $f(t,y)$ be given function, so $f(t,y)$ satisfies a Lipchitz condition with respect to y in a certain region referred to as D in the XY-plane, if there exists a non-negative constant ζ, such that

$$|f(t,y_a) - f(t,y_b)| \le \zeta |y_a - y_b|$$

whenever (t,y_a) and (t,y_b) are in D, and ζ is called the Lipchitz constant.

2.2 Overview of the Successive Iteration Method

Suppose a first-order non-linear ordinary differential equation (ODE) is given as follows with an initial condition:

$$\begin{cases} \dfrac{dy}{dt} = g(t,y), \\ y(t_0) = y_0 \end{cases} \tag{2.1}$$

Suppose $\int_{t_0}^{t}(\cdot)\,dy = I_{t_0}^{t}(\cdot)$ denotes a one-fold integral operator w.r.t. a concerned variable; thus, by direct integration of both sides (2.1) over $(t_0,\ t)$, we have:

$$\begin{cases} I_{t_0}^{t}(dy) = I_{t_0}^{t}(g(s,y)) \\ y(t_0) = y_0 \end{cases} \tag{2.2}$$

This implies that:

$$\begin{cases} y(t) = y_0 + I_{t_0}^{t}(g(s,y)), \\ y = y(t) \end{cases} \tag{2.3}$$

By iteration, we substitute $y(t) = y_{n+1}(t) = y_{n+1}$ and $y_n(s) = y_n$

Therefore, (2.3) becomes;

$$\begin{cases} y_{n+1} = y_0 + I_{t_0}^{t}\left(g(s, y_n)\right), \\ y = y(t) \end{cases} \tag{2.4}$$

3. Method and Model Discussed

This part discusses the proposed method and the Logistic model, as formulated based on some assumptions. Case examples are also considered via the SAM.

CASE I: Consider the following version of the LDE:

$$\begin{cases} \dfrac{dP}{dt} = \dfrac{1}{4}P(1-P) \\ P(0) = \dfrac{1}{3} \end{cases} \tag{3.1}$$

whose exact solution is:

$$P(t) = \frac{e^{0.25t}}{2 + e^{0.25t}} \tag{3.2}$$

Equation (4.1) is rewritten as follows:

$$\begin{cases} dN = \dfrac{1}{4}N(1-N)dt \\ N(0) = \dfrac{1}{3} \end{cases} \tag{3.3}$$

Applying the SIM to (4.2), give the following relation:

$$\left\{ N_{j+1} = N_0 + \frac{1}{4}I_0^t\left(\left(N_j\left(1-N_j\right)\right)\right), \ \ j \geq 0. \right. \tag{3.4}$$

Thus, we obtained the following iteratively:

$$\left.\begin{aligned} N_0 &= \frac{1}{3}, \\ N_1 &= \frac{1}{3} + \frac{t}{18}, \\ N_2 &= \frac{1}{3} + \frac{1}{18}t - \frac{1}{3888}t^3 + \frac{1}{432}t^2, \\ N_3 &= \frac{1}{3} + \frac{1}{18}t - \frac{1}{423263232}t^7 + \frac{1}{20155392}t^6 + \frac{13}{11197440}t^5 - \frac{1}{46656}t^4 - \frac{1}{5184}t^3 + \frac{1}{432}t^2 \end{aligned}\right\}$$

$$\left.\begin{aligned} N_4 = \frac{1}{3} + \frac{1}{18}t &- \frac{1}{107491058137851494400}t^{15} + \frac{1}{238869018084114432}t^{14} + \frac{43}{739356484546068480}t^{13} \\ &- \frac{19}{4212857461800960}t^{12} - \frac{349}{1158535801995264000}t^{11} + \frac{73}{365699432448000}t^{10} + \frac{13}{21941965946880}t^9 \\ &- \frac{1013}{1625330810880}t^8 - \frac{61}{33861058560}t^7 + \frac{41}{268738560}t^6 + \frac{1}{2239488}t^5 - \frac{5}{248832}t^4 - \frac{1}{5184}t^3 + \frac{1}{432}t^2 \end{aligned}\right\}$$

$$\vdots \tag{3.5}$$

The result obtained in (3.5) is a five-term approximate solution of the considered case. This is consistent with the analytical solution in [2]. However, the technique shown here seems to be simpler and easy. Figures 1-2 illustrate the approximate and exact solutions.

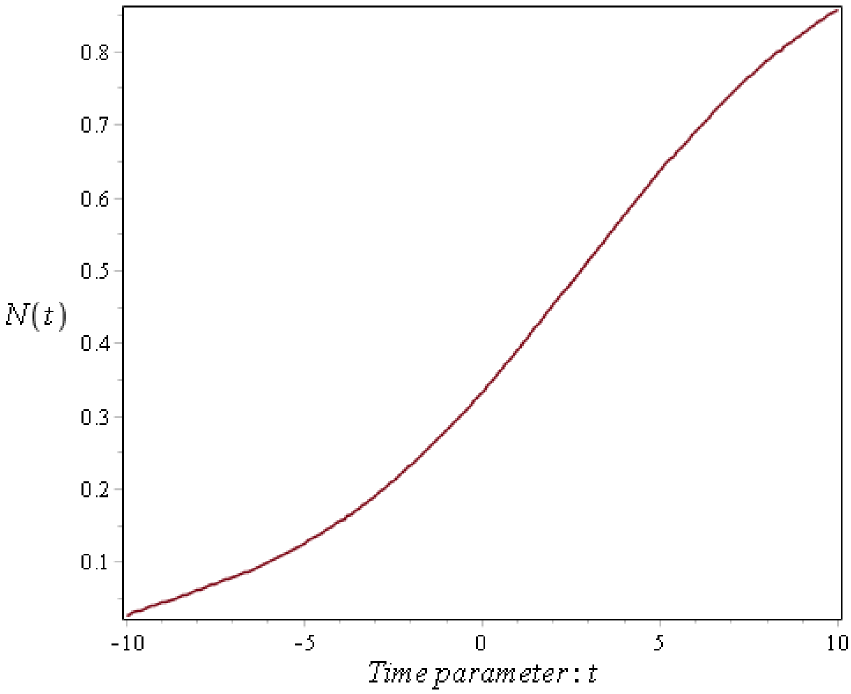

Figure 1: SIM 8-term Approximate solution (Case I)

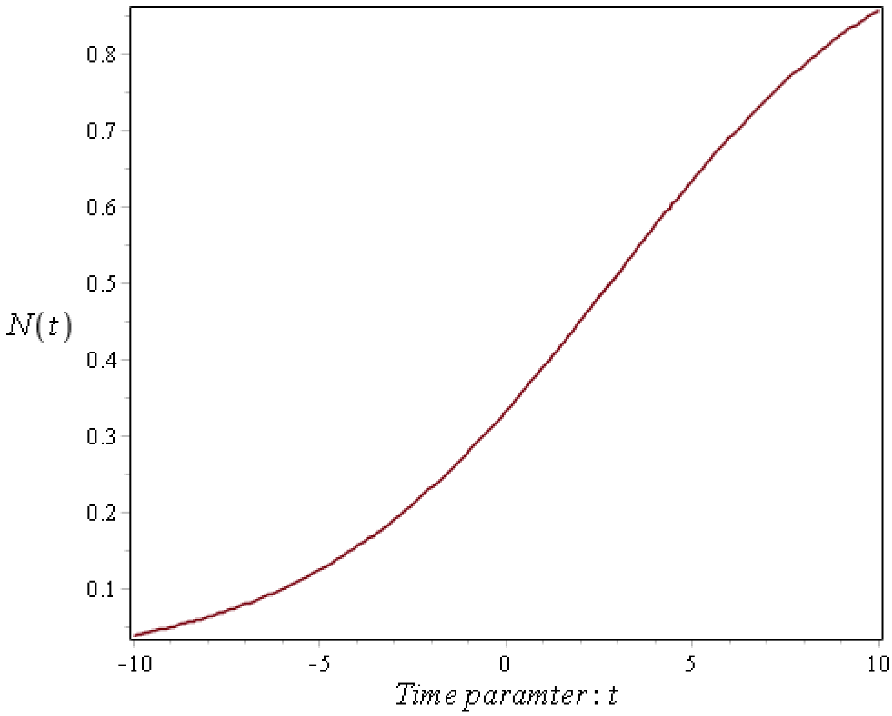

Figure 2: Exact solution (Case I)

4. Conclusions

The Successive Approximation Method (SAM) was successfully employed to solve a variety of Logistic Differential Models in this article. The SAM that is being provided is computer-friendly and has a straightforward premise. SAM's results are quite comparable to those of other iterative techniques that have been studied in the literature. SAM also delivers more accurate numerical solutions for non-linear situations. The SAM is a handy tool in finding the solutions of the Logistic Differential Model since it does not need a lot of computer memory, rigorous, restrictive assumptions, or discretization processes.

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