PAPER • **OPEN ACCESS**

SAM approximate solutions of some evolution equations

To cite this article: S. O. Edeki *et al* 2022 *J. Phys.: Conf. Ser.* **2199** 012009

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Abstract. The Successive Approximation Method (SAM) is introduced in this research to solve the evolution equations. These approximate-analytical solutions of the considered cases are computed with ease. The proposed technique is used directly, without transformation, discretization, linearization, or any restrictive assumptions.

Keywords: Non-linear evolution model**,** approximate solutions; Transform method, SAM

1. Introduction

Differential equations may be used to simulate a variety of physical processes, the majority of which are non-linear. A differential law governing a system's development (evolution) through time may be viewed as an evolution equation. The phrase (evolution equation) has no precise definition, and its meaning is determined not only by the equation but also by the description of the issue for which it is applied. The ability to construct the solution from a predefined starting condition, which may be taken as a description of the system's initial state, is typical of evolution equations [1-3].
We must rely on numerical/iterative approaches since relatively few non-linear problems have exact analytical solutions. SAM is straightforward with computer programs like Maple, SageMath, and Mathematica. It produces results that are in excellent agreement with other approaches and need few iterations in many circumstances. We employ SAM to solve some evolution equations in this article [4, 5] and compare the results to known approaches, even graphical representations.
The analytic approaches for these problems are frequently limited and difficult to evaluate. Numerical approaches have been arbitrated more efficiently and reliable in solving the dynamical models (equations) and other differential models in this regard [6-12].
For a better understanding of evolution equations, in terms of derivations and formats, readers are referred to [1, 3].
Different solution experts have recently discussed numerous methods for finding an exact or numerical solution to ordinary or partial differential models [13-30]. In this work, a novel approach termed Successive Approximation Method (SAM) is applied to some non-linear evolution models.

2. Remark on Continuity Condition and the Proposed SIM

The method of solution referred to as Successive Approximation Method (SAM) is introduced here, in line with some basic preliminaries.

2.1 Lipschitzian Continuity Condition

Let $f(t,y)$ be given function, so $f(t,y)$ satisfies a Lipchitz condition with respect to y in a certain region referred to as D in the XY-plane, if there exists a non-negative constant ζ, such that

$$|f(t,y_a)-f(t,y_b)| \le \zeta |y_a - y_b|$$

whenever (t,y_a) and (t,y_b) are in D, and ζ is called the Lipchitz constant.

2.2 Overview of the Successive Iteration Method

Suppose a first-order non-linear ordinary differential equation (ODE) is given as follows with an initial condition:

$$\begin{cases} \dfrac{dy}{dt} = g(t,y), \\ y(t_0) = y_0 \end{cases} \tag{2.1}$$

Suppose $\int_{t_0}^{t} (\cdot)\, dy = I_{t_0}^{t}(\cdot)$ denotes a one-fold integral operator w.r.t. a concerned variable; thus, by direct integration of both sides (2.1) over $(t_0,\ t)$, we have:

$$\begin{cases} I_{t_0}^{t}(dy) = I_{t_0}^{t}(g(s,y)) \\ y(t_0) = y_0 \end{cases} \tag{2.2}$$

This implies that:

$$\begin{cases} y(t) = y_0 + I_{t_0}^{t}(g(s,y)), \\ y = y(t) \end{cases} \tag{2.3}$$

By iteration, we substitute $y(t) = y_{n+1}(t) = y_{n+1}$ and $y_n(s) = y_n$

Therefore, (2.3) becomes;

$$\begin{cases} y_{n+1} = y_0 + I_{t_0}^{t}(g(s,y_n)), \\ y = y(t) \end{cases} \tag{2.4}$$

3. Applications

In this section, some case examples of the GDE are considered via the proposed method in terms of solutions. For the effectiveness and efficiency of the technique, the results are presented graphically compared to their exact solutions.

Case I: Here, we consider a evolution equation [2]

$$\begin{cases} w_t + e^t w w_x + w w_{xx} = 0, \\ w(x,0) = x,\ w = w(x,t) \end{cases} \tag{3.1}$$

Equation (3.1) is rewritten as follows:

$$\begin{cases} w_t = -\left(e^t w w_x + w w_{xx}\right), \\ w(x,0) = x,\ w = w(x,t) \end{cases} \tag{3.2}$$

Applying the SIM to (3.2), give the following relation:

$$\left\{ w_{j+1} = w_0 - I_0^t\left(\left(e^t w_j \left(w_j\right)_x + \left(w_j\right)\left(w_j\right)_{xx}\right)\right),\ j \geq 0. \right. \tag{3.3}$$

Thus, we obtained the following iteratively:

$$w_1 = 2x - \mathrm{e}^t x$$

$$w_2 = \frac{10x}{3} - \frac{x\mathrm{e}^{3t}}{3} + 2x\mathrm{e}^{2t} - 4x\mathrm{e}^t$$

$$w_3 = \frac{323x}{63} - \frac{100\mathrm{e}^t x}{9} + \frac{41x\mathrm{e}^{4t}}{9} - \frac{88x\mathrm{e}^{3t}}{9} + \frac{40x\mathrm{e}^{2t}}{3} - \frac{x\mathrm{e}^{7t}}{63} + \frac{2x\mathrm{e}^{6t}}{9} - \frac{4x\mathrm{e}^{5t}}{3}$$

$$\left.\begin{aligned} w_4 = {} & \frac{1793989x}{238140} - \frac{104329\mathrm{e}^t x}{3969} - \frac{10037x\mathrm{e}^{9t}}{1701} + \frac{209x\mathrm{e}^{12t}}{3402} - \frac{212x\mathrm{e}^{11t}}{567} \\ & + \frac{4796x\mathrm{e}^{10t}}{2835} - \frac{x\mathrm{e}^{15t}}{59535} + \frac{2x\mathrm{e}^{14t}}{3969} - \frac{4x\mathrm{e}^{13t}}{567} - \frac{47060x\mathrm{e}^{7t}}{1323} + \frac{106496x\mathrm{e}^{6t}}{1701} \\ & - \frac{250486x\mathrm{e}^{5t}}{2835} + \frac{85825\mathrm{e}^{8t} x}{5292} + \frac{56212x\mathrm{e}^{4t}}{567} - \frac{147520x\mathrm{e}^{3t}}{1701} + \frac{32300x\mathrm{e}^{2t}}{567} \end{aligned}\right\}$$

$$\left.\begin{aligned} w_5 = {} & \frac{158667664590038921x}{14763939698944800} + \frac{81625772191\mathrm{e}^{8t} x}{21003948} + \frac{7885724887429x\mathrm{e}^{10t}}{2362944150} \\ & - \frac{373846811591x\mathrm{e}^{15t}}{1181472075} + \frac{49673592803x\mathrm{e}^{14t}}{78764805} - \frac{659728799909x\mathrm{e}^{13t}}{585109980} - \frac{1445126508395x\mathrm{e}^{9t}}{378071064} \\ & + \frac{645840983x\mathrm{e}^{12t}}{357210} - \frac{48714154862x\mathrm{e}^{11t}}{18753525} - \frac{823817582788x\mathrm{e}^{7t}}{236294415} + \frac{924026475509x\mathrm{e}^{6t}}{337563450} \\ & - \frac{14946354554x\mathrm{e}^{5t}}{8037225} + \frac{21780451582x\mathrm{e}^{4t}}{20253807} + \frac{187165078381x\mathrm{e}^{2t}}{945177660} - \frac{73215712013x\mathrm{e}^{3t}}{141776649} \\ & - \frac{295781x\mathrm{e}^{25t}}{675126900} + \frac{134x\mathrm{e}^{26t}}{2679075} - \frac{466x\mathrm{e}^{27t}}{101269035} + \frac{48479771x\mathrm{e}^{22t}}{472588830} - \frac{3218396532121\mathrm{e}^t x}{56710659600} \\ & - \frac{136408419877x\mathrm{e}^{17t}}{2380447440} - \frac{201462923x\mathrm{e}^{19t}}{30541455} + \frac{12019649x\mathrm{e}^{24t}}{3780710640} - \frac{1535749x\mathrm{e}^{23t}}{78764805} \\ & + \frac{5380847873291\mathrm{e}^{16t} x}{37807106400} + \frac{309145231x\mathrm{e}^{18t}}{15002820} - \frac{2213286169x\mathrm{e}^{21t}}{4725888300} + \frac{210629969x\mathrm{e}^{20t}}{112521150} \\ & - \frac{x\mathrm{e}^{31t}}{109876902975} + \frac{2x\mathrm{e}^{30t}}{3544416225} - \frac{4x\mathrm{e}^{29t}}{236294415} + \frac{929x\mathrm{e}^{28t}}{2835532980} \end{aligned}\right\}$$

$$\vdots$$

The result obtained in (4.6) is consistent with the analytical solution in [2]. However, the technique shown here seems to be simpler and easy. Figures 1-2 illustrate the approximate and exact solutions

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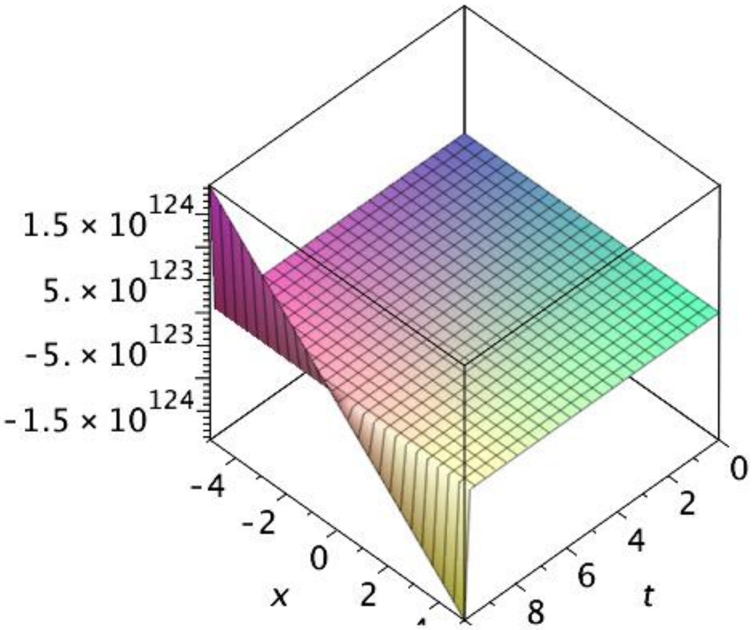

Figure 1: SIM 5-term Approximate solution (Case I)

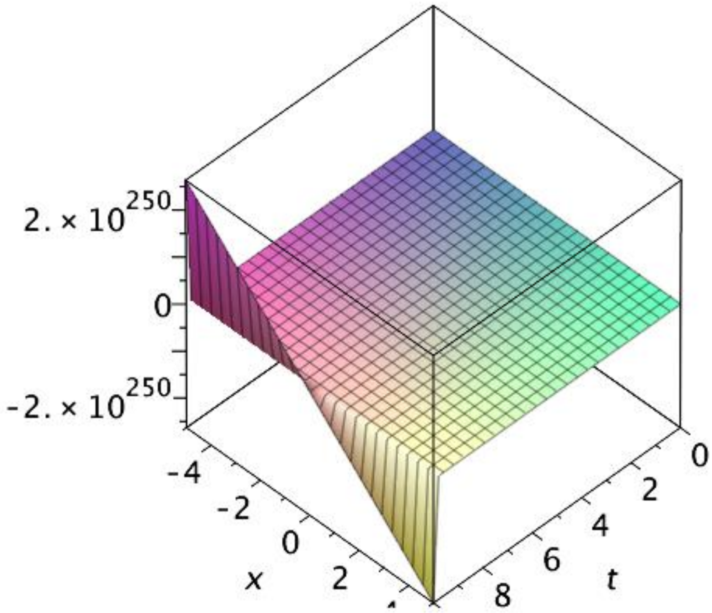

Figure 2: SIM 6-term Approximate solution (Case I)

Case II: Here, we consider a evolution (regularized long-wave) equation [2, 4]

$$\begin{cases} w_t - w_{xxt} + \left(\dfrac{w^2}{2}\right)_x = 0, \ x \in (-\infty, \infty), \ t > 0, \\ w(x,0) = x, \ w = w(x,t). \end{cases} \tag{3.4}$$

Equation (3.4) is rewritten as follows:

$$\begin{cases} w_t = w_{xxt} - \left(\dfrac{w^2}{2}\right)_x \\ w(x,0) = x, \ w = w(x,t). \end{cases} \tag{3.5}$$

Applying the SIM to (3.5), give the following relation:

$$\left\{ w_{j+1} = w_0 - I_0^t\left(\left((w_j)_{xxt} - \left(\frac{(w_j)^2}{2}\right)_x\right)\right), \ j \geq 0. \right. \tag{3.6}$$

Thus, we obtained the following iteratively:

$$\left.\begin{aligned} &w_0 = x, \\ &w_1 = x(-t+1), \\ &w_2 = x\left(1 - \frac{1}{3}t^3 + t^2 - t\right), \\ &w_3 = x\left(1 - \frac{1}{63}t^7 + \frac{1}{9}t^6 - \frac{1}{3}t^5 + \frac{2}{3}t^4 - t^3 + t^2 - t\right), \end{aligned}\right\}$$

$$w_4 = x\left(\begin{aligned} &-\frac{1}{59535}t^{15} + \frac{1}{3969}t^{14} - \frac{1}{567}t^{13} + \frac{1}{126}t^{12} - \frac{5}{189}t^{11} + \frac{22}{315}t^{10} \\ &-\frac{86}{567}t^9 + \frac{71}{252}t^8 - \frac{29}{63}t^7 + \frac{2}{3}t^6 - \frac{13}{15}t^5 + t^4 - t^3 + t^2 - t + 1 \end{aligned}\right)$$

Thus, the five-term solution of case II is given as:

$$w(x,t) = x\left(\begin{aligned} &\frac{13}{315059220}t^{28} - \frac{2}{6751269}t^{27} + \frac{1}{595350}t^{26} - \frac{5309}{675126900}t^{25} + \frac{16927}{540101520}t^{24} \\ &-\frac{2447}{22504230}t^{23} + \frac{557}{1666980}t^{22} - \frac{207509}{225042300}t^{21} + \frac{16511}{7144200}t^{20} - \frac{162179}{30541455}t^{19} \\ &+\frac{2588}{229635}t^{18} + \frac{43363}{1058400}t^{16} - \frac{1080013}{48580560}t^{17} - \frac{1}{109876902975}t^{31} + \frac{1}{3544416225}t^{30} \\ &-\frac{1}{236294415}t^{29} - \frac{63283}{893025}t^{15} + \frac{1019}{8820}t^{14} - \frac{13141}{73710}t^{13} + \frac{17779}{68040}t^{12} - \frac{1477}{4050}t^{11} + \frac{27523}{56700}t^{10} \\ &-\frac{3497}{5670}t^9 + \frac{943}{1260}t^8 - \frac{13}{15}t^7 + \frac{43}{45}t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 \end{aligned}\right)$$

which agrees with the solution obtained using the new iteration method-NIM [2], and the variational iteration method-VIM [4], even in a faster convergence manner.

4. Conclusions

In this article, the Successive Approximation Method (SAM) was used successfully to solve a number of non-linear (evolution) equations. The presented SAM is computer-friendly and has a simple concept. The findings of SAM are quite similar to those of other iterative approaches in the literature. In addition, for non-linear problems, SAM provides more accurate numerical solutions. It also does not need a lot of computer memory, stringent or restricting assumptions, or discretization techniques.

Acknowledgment

CUCRID unit of Covenant University is highly cherished for all forms of support.

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