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APPROXIMATE ANALYTICAL SOLUTIONS OF LINEAR STOCHASTIC DIFFERENTIAL MODELS BASED ON KARHUNEN-LOÉVE EXPANSION WITH FINITE SERIES TERMS

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Abstract: Stochastic Differential Equations (SDEs) as particular forms of Differential Equations (DEs) play immense roles in modeling of various phenomena with applications in physical sciences, and finance- such as stock option practices due to thermal and random fluctuations. The solutions of these SDEs, if they exist, are difficult to obtain, unlike those of the Differential Equations. In this paper, the white noise terms of the linear SDEs in Stratonovich forms are considered on the basis of Karhunen-Loéve Expansion finite series while Daftardar-Jafari Integral Method is proposed for approximate analytical solution of the linear Stratonovich Stochastic Differential Equations. Three numerical examples are considered to test the accuracy and effectiveness of this proposed method. The results obtained show clearly that the approximate solutions converge faster to the exact solutions even with fewer terms; though, higher terms increase the accuracy. The method is direct in terms of application. Thus, it is recommended for nonlinear financial models such as Ito Stochastic Differential Equations.

Keywords: Stratonovich SDEs; population dynamics; Karhunen-Loéve expansion; approximate solution; differential models; option pricing; Daftardar-Jafari method.

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1. LINEAR INTRODUCTION

Stochastic Differential Equations (SDEs) play a critical role in the field of Mathematical Finance, Biological Sciences (Population Dynamics Modelling), Physics, Engineering, and other areas. They are used to model physical phenomena, biological systems, and real-life situations [1]. Probability theory stands as a link connecting SDEs to Ordinary Differential Equations (ODES) and Partial Differential Equations (PDEs). Several authors have established that most well-known SDEs are not solvable using the analytical approaches, hence the development of numerical schemes to solve such problems [2, 3, 14].

Considering the Stratonovich SDE of this form:

$$\begin{cases} dY = a_0(Y,t)dt + \sum_{i=1}^{d} b_i(Y,t) \circ dB^i(t), \\ Y(0) = Y_0, \end{cases} \tag{1.1}$$

with, $Y \in \mathbb{R}^n$, $a_0 = a_0\left(Y,t\right)$, $b_0 = b_0\left(Y,t\right)$ are the drift and diffusion coefficients respectively, and $B\left(t\right)$ is the standard Brownian motion (Wiener process), and d represents the dimension of the Brownian Motion,

$B = \left\{B\left(t\right), 0 \le t \le T\right\}$ is a 1-dimensional Brownian motion [4].

The Itô form of the SDE in (1.1) is:

$$\begin{cases} dY = a(t,Y_i)dt + \sum_{i=1}^{d} b(t, y = Y_i) dB^i(t) \\ Y(0) = Y_0. \end{cases} \tag{1.2}$$

In integral form, (1.1) is written as:

$$Y(t) = Y_0 + \int_0^t a(s,Y(s))ds + \int_0^t b(s,Y(s)) \circ dB(s). \tag{1.3}$$

The first integral in (1.3) is known as the Riemann-Stiltjes and the second is a stochastic integral attached to the Wiener process, $B\left(t\right)$. Many researchers have used different numerical methods to solve SDEs [5-14]. In the process, many of these methods generate complex situations that cannot be accommodated effectively, most notably when dealing with the nonlinear SDEs, which makes most researchers to streamline their researches to linear SDE, ODE and PDE type. But recently, precisely in 2006, two researchers Daftardar-Gejji and Jafari [15] proposed a new iterative scheme that is capable of solving linear and nonlinear ODEs, PDEs. The method was named after them as Daftardar-Jafari Method (DJM). This is also referred to as New Iterative Method (NIM). This method has been widely accepted and used by many scholars for the solution of linear and nonlinear ODEs and PDEs in integer and fractional orders [16-22]. Azodi [23] used the method to obtain the solution of Itô-SDEs, and it was seen to produce better results than those obtained by ADM, HPM, and VIM [23].

The aim of this research is to apply the DJM to solve Stochastic Differential Equations (SDEs) of the Sratonovich form. This form of SDE plays a dominant role in mathematical finance, physics, engineering, biology, and other related fields via the transformation of the “white noise” into an ODE form using the Karhunen-Loéve expansion.

The remaining part of the work is divided into five sections. Section 2 is devoted to a brief discussion of the linear SDEs. The proposed method is discussed in section 3; numerical examples are considered in section 4, and lastly, conclusion is made in section 5.

2. LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS (LSDES)

Consider a linear Stochastic Differential Equation (LSDE) that follows a Geometric Brownian Motion (such as stock price) given price:

$$dY\left(t\right)=eY\left(t\right)dt+\sigma Y(t)dB(t), \tag{2.1}$$

where e is the stock drift, $\sigma>0$ is the stock volatility or diffusion, $dB(t)$ is the “white noise” and $Y\left(t\right)$ is a Stochastic Process. The process $(B(t))_{t\geq 0}$ is the Brownian Motion defined in $\mathbb{R}$ on some probability space (Ω, f, P). We refer the readers to [6] for the existence and uniqueness of the solution.

3. THE METHOD OF SOLUTION

This section presents the concepts of New Iterative Method (NIM) and Karhunen-Loeve Expansion (K-L Expansion).

3.1 DAFTAR-GEJII-JAFARI METHOD (DJM)

Consider the general functional equation defined as follows

$$y = b + L(y) + N[y], \tag{3.1}$$

where b is a known function, $L[\cdot]$ and $N[\cdot]$ are the linear and nonlinear operators respectively. Suppose we define $\bar{N}[y]$ as:

$$\bar{N}[y] = L[y] + N[y], \tag{3.2}$$

then (3.1) becomes:

$$y = b + \bar{N}[y]. \tag{3.3}$$

Now considering a solution, y of (3.2) having the infinite series form:

$$\begin{cases} y = \sum_{i=0}^{\infty} y_i, \\ \bar{N}[y] = \bar{N}\left[\sum_{i=0}^{\infty} y_i\right]. \end{cases} \tag{3.4}$$

The nonlinear operator $\bar{N}$ can now be decomposed as

$$\bar{N}\left(\sum_{i=0}^{\infty} y_i\right) = \bar{N}[y_0] + \sum_{i=1}^{\infty}\left[\bar{N}\left(\sum_{i=0}^{m} y_i\right) - \bar{N}\left(\sum_{i=0}^{m-1} y_i\right)\right], m = 1,\ 2... \tag{3.5}$$

Therefore, putting (3.4) and (3.5) into (3.3), we obtain

$$\sum_{i=0}^{\infty} y_i = b + \bar{N}[y_0] + \sum_{i=1}^{\infty}\left[\bar{N}\left(\sum_{i=0}^{m} y_i\right) - \bar{N}\left(\sum_{i=0}^{m-1} y_i\right)\right] \ m = 1,\ 2, \dots , \tag{3.6}$$

Hence, the recurrence relation is:

$$\begin{cases} y_0 = b \\ y_1 = \bar{N}(y_0) \\ y_{m+1} = \bar{N}\left[\sum_{i=0}^{m} y_i\right] - \bar{N}\left[\sum_{i=0}^{m-1} y_i\right], \ m = 1, \ 2, \ldots \end{cases} \tag{3.7}$$

such that:

$$y = b + \sum_{i=1}^{\infty} y_i = \sum_{i=0}^{\infty} y_i. \tag{3.8}$$

The convergence of this method has been discussed by many researchers. We refer the readers to [17, 24-26].

In another dimension of applications, numerical or exact solution methods for linear and or nonlinear differential models including SDEs are linked to the following references and the link therein [27-40].

3.2 Karhunen-Loéve Expansion (K-L E) of Brownian Motion

A Brownian Motion (also called Wiener Process) is a Stochastic Process $\{B(t)\}_{t\geq 0}$ indexed by nonnegative real numbers, t with the following properties listed below:

i. $B(0) = 0,$

ii. the probability that a randomly generated Brownian path be continuous is one,

iii. the process $\{B(t)\}_{t\geq 0}$ has stationary, independent increment,

iv. the increments: $B(t+v) - B(t)$ are normally distributed (Gaussian Process) with zero (0) and v as the expected value and variance respectively.

The Karhunen-Loéve Expansion (K-LE) of Brownian motion was used by [41] to characterize the Wiener Process. Let $Y(t)$ denote the trajectory of a random process, $Y(t,\omega)$ for a given ω. The Wiener Process $B(t)$ has trajectories belonging to $L^2([0,T])$ for almost all $\omega's$, and this space K-L expansion takes the form:

$$\begin{cases} B(t) = B(t,\omega) = \sum_{i=0}^{\infty} z_j(w)\Phi_j\left(t\right),\ 0 \le t \le T, \\ \\ \Phi_j\left(t\right) = \dfrac{2\sqrt{2T}}{(2j+1)}\sin\left(\dfrac{(2j+1)\pi t}{2T}\right), \end{cases} \tag{3.9}$$

where z_j is a sequence of identically, independent Gaussian random variables, and $\Phi_j(t)$ form a basis of orthogonal function [41, 42].

Therefore, simplifying (3.9) with $\boldsymbol{T} = 1$ gives;

$$B(t) = \sqrt{2}\sum_{j=0}^{\infty}\left[\frac{\sin\left[\left(j+0.5\right)\pi t\right]}{\left(j+0.5\right)\pi}\right]z_j \tag{3.10}$$

such that

$$dB(t) = \sqrt{2}\sum_{j=0}^{\infty}\left(\cos\left[\left(j+0.5\right)\pi t\right]\right)z_j\ . \tag{3.11}$$

Replacing the finite terms of K-L Expansion in the SDE (1.2), we obtain:

$$\begin{cases} dY\left(t\right) = a\left(Y,t\right)dt + \sigma(Y,t)d\left(\sum_{j=0}^{L} z_j\Phi_j(t)\right), \\ Y(0) = Y_0. \end{cases} \tag{3.12}$$

For the purpose of this work, we choose $L = 5$ (which represents the number of functions of K-L Expansion) and z_j is generated in MATLAB software using normarnd.

4. NUMERICAL EXAMPLES

In what follows, two kinds of approximate solutions viz: Φ_3 and Φ_4 will be considered. Hence,

$$\begin{cases} \Phi_3 = \sum_{i=0}^{3} Y_i, \\ \Phi_4 = \sum_{i=0}^{4} Y_i, \end{cases} \tag{4.1}$$

such that $Y(t)$ represents the exact solution.

Example 4.1 Consider the Stratonovich Stochastic Differential Equation [4]

$$\begin{cases} dY(t) = \beta Y(t) \circ dW(t), \\ Y(0) = 1. \end{cases} \tag{4.2}$$

The exact solution of (4.2) is

$$Y(t) = Y(0)\exp\left(\beta W(t)\right),\ t \in [0,T]\ .$$

In integral form, (4.1) becomes:

$$\begin{aligned} Y(t) &= 1 + \beta \int_0^t \left(Y(s)\, dW\right) dt \\ &= b + \bar{N}[Y]. \end{aligned} \tag{4.3}$$

Thus,

$$\begin{cases} b = 1, \\ \bar{N}[Y] = \beta \int_0^t \left(Y(s)\, dW(s)\right) ds. \end{cases}$$

By applying the DJM to (4.3), we get the following:

$$Y_0 = 1,$$

$$\begin{aligned} Y_1 = \bar{N}(Y_0) = \beta \int_0^t Y_0\, dW(s) \\ &= \beta \int_0^t \left(Y_0 \left[\sqrt{2} \sum_{j=0}^{5} \left(\cos\left[(j+0.5)\pi t \right] \right) z_j \right] \right) dt \quad , \\ &= \int_0^t \left(\left[\sqrt{2} \sum_{j=0}^{5} \left(\cos[(j+0.5)\pi t] \right) z_j \right] \right) dt,\ \beta = 1 \end{aligned}$$

$$\begin{aligned} Y_2 &= \bar{N}(1+Y_1) - \bar{N}(Y_0) \\ &= \int_0^t \left((1+Y_1)\, dW \right) dt - \int_0^t \left(Y_0\, dW \right) dt \\ &= \int_0^t \left((1+Y_1) \left[\sqrt{2} \sum_{j=0}^{5} \cos\left[(j+0.5)\pi t \right] \right] z_j \right) - \{Y_1\}, \end{aligned}$$

$$\begin{aligned} Y_3 = &\left\{ \int_0^t \left((1+Y_1+Y_2) \left[\sqrt{2} \sum_{j=0}^{5} \cos\left[(j+0.5)\pi t \right] \right] z_j \right) \right\} dt \\ &- \left\{ \int_0^t \left((1+Y)_1 \left[\sqrt{2} \sum_{j=0}^{5} \cos\left[(j+0.5)\pi t \right] \right] z_j \right) \right\} dt, \end{aligned}$$

$$Y_4 = \left\{ \int_0^t \left(\left(1+Y_1+Y_2+Y_3\right) \left[\sqrt{2} \sum_{j=0}^{5} \cos\left[\left(j+0.5 \right) \pi t \right] \right] z_j \right) \right\} dt$$
$$- \left\{ \int_0^t \left(\left(1+Y_1+Y_2\right) \left[\sqrt{2} \sum_{j=0}^{5} \cos\left[\left(j+0.5 \right) \pi t \right] \right] z_j \right) \right\} dt,$$
$$\vdots$$

The computed results and errors analysis are shown in the table 1 and two graphs are plotted for $Y(t)$ against Φ_3 and $Y(t)$ against Φ_4.

Table 1: Solutions and absolute errors via DJM: $Y(t)$, $\Phi_3(t)$, $\Phi_4(t)$ for Example 4.1

t	$Y(t)$	$\Phi_3(t)$	$\Phi_4(t)$	$\lvert Y(t)-\Phi_3(t)\rvert$	$\lvert Y(t)-\Phi_4(t)\rvert$	$\lvert \Phi_4(t)-\Phi_3(t)\rvert$
0.00	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.05	1.2716	1.2714	1.2716	0.0002	0.0000	0.0002
0.10	1.5981	1.5959	1.5979	0.0022	0.0002	0.0020
0.15	1.9399	1.9307	1.9387	0.0092	0.0012	0.0080
0.20	2.2045	2.1853	2.2015	0.0192	0.0030	0.0162
0.25	2.3007	2.2768	2.2968	0.0239	0.0039	0.0200
0.30	2.2390	2.2181	2.2357	0.0209	0.0033	0.0176
0.35	2.1463	2.1296	2.1438	0.0167	0.0025	0.0142
0.40	2.1791	2.1610	2.1763	0.0181	0.0028	0.0153
0.45	2.4635	2.4302	2.4577	0.0333	0.0058	0.0275
0.50	3.0838	2.9983	3.0653	0.0855	0.0185	0.0670

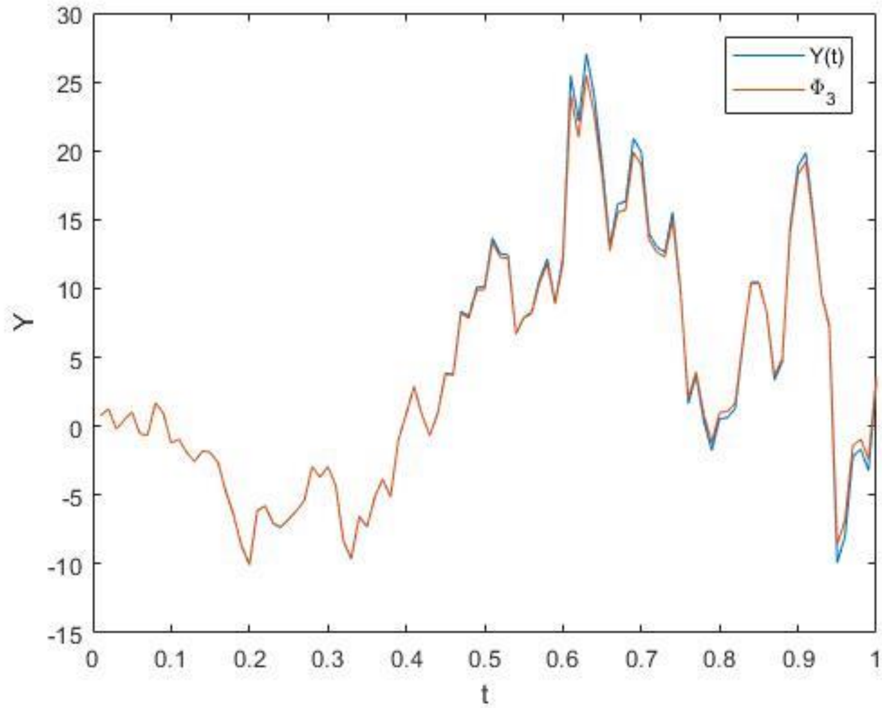

Figure 1: $Y(t)$ and Φ_3(t) via DJM for Example 4.1 $(N = 100, L = 5)$

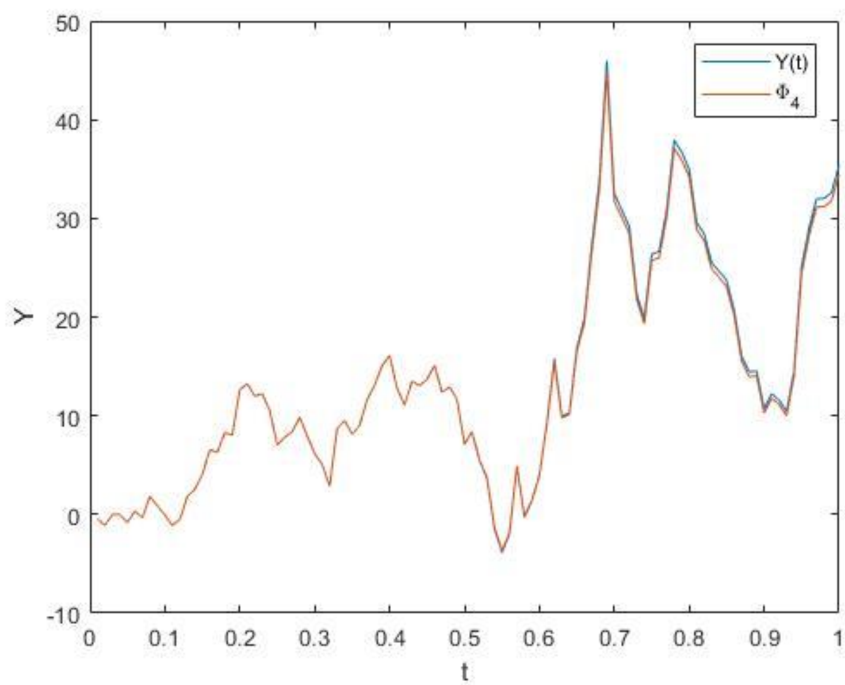

Figure 2: $Y(t)$ and Φ_4(t) via DJM for Example 4.1 $(N = 100, L = 5)$

Example 4.2: Consider the Stochastic Differential Equation [23]:

$$\begin{cases} dY(t) = \frac{1}{2}Y(t)dt + Y(t)\circ dB(t), \\ Y(0) = 1. \end{cases} \tag{4.4}$$

The Stratonovich exact solution of (4.4) is given as

$$Y(t) = \exp\left(\frac{t}{2} + B(t)\right). \tag{4.5}$$

In integral form, we have:

$$Y(t) = 1 + \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\,dB(s). \tag{4.6}$$

Therefore, with the following definitions:

$$\bar{N}(Y(t)) = \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\,dB(s), \tag{4.7}$$

$$dB(s) = \sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j \quad , \tag{4.8}$$

and

$$\bar{N}(Y(t)) = \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right), \tag{4.9}$$

we obtain the iterative scheme via DJM as follows:

$$\begin{aligned}
Y_0 &= 1, \\
Y_1 &= \bar{N}(Y_0) \\
&= \frac{1}{2}\int_0^t Y_0\,ds + \int_0^t \left(Y_0\,dB(s)\right)ds \\
&= \frac{1}{2}\int_0^t (Y_0)\,ds + \int_0^t \left(Y_0\left[\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right]\right)ds, \\
Y_2 &= \bar{N}(Y_0+Y_1) - \bar{N}(Y_0) \\
&= \left\{\frac{1}{2}\int_0^t (Y_0+Y_1)\,ds + \int_0^t \left((Y_0+Y_1)\,dB(s)\right)ds\right\} - \left\{\frac{1}{2}\int_0^t (Y_0)\,ds + \int_0^t \left(Y_0\,dB(s)\right)ds\right\} \\
&= \left\{\frac{1}{2}\int_0^t \left((Y_0+Y_1)\right)ds + \int_0^t \left((Y_0+Y_1)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\} - \{Y_1\},
\end{aligned}$$

$$Y_3 = \bar{N}\left(Y_0+Y_1+Y_2\right)-\bar{N}\left(Y_0+Y_1\right)$$
$$=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)dB(s)\right)ds\right\}$$
$$-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1\right)ds+\int_0^t\left(\left(Y_0+Y_1\right)dB(s)\right)ds\right\}$$
$$=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\}$$
$$-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1\right)ds+\int_0^t\left(\left(Y_0+Y_1\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\},$$

$$Y_4 = \bar{N}\left(Y_0+Y_1+Y_2+Y_3\right)-\bar{N}\left(Y_0+Y_1+Y_2\right)$$
$$=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2+Y_3\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2+Y_3\right)dB(s)\right)ds\right\}$$
$$-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)dB(s)\right)ds\right\}$$
$$=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2+Y_3\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2+Y_3\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\}$$
$$-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\},$$
$$\vdots$$

The computed results and errors analysis are shown in the table 2; two graphs are plotted for $Y(t)$ against Φ_3 and $Y(t)$ against Φ_4.

Table 2: Solutions and absolute errors via DJM: $Y(t)$, $\Phi_3(t)$, $\Phi_4(t)$ for Example 4.2

t	$Y(t)$	$\Phi_3(t)$	$\Phi_4(t)$	$\lvert Y(t)-\Phi_3(t)\rvert$	$\lvert Y(t)-\Phi_4(t)\rvert$	$\lvert \Phi_4(t)-\Phi_3(t)\rvert$
0.00	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.05	1.3038	1.3036	1.3038	0.0002	0.0000	0.0002
0.10	1.6801	1.6767	1.6797	0.0034	0.0004	0.0030
0.15	2.0910	2.0766	2.0889	0.0144	0.0021	0.0123
0.20	2.4363	2.4047	2.4309	0.0316	0.0054	0.0262
0.25	2.6071	2.5640	2.5991	0.0431	0.0080	0.0351
0.30	2.6013	2.5586	2.5934	0.0427	0.0079	0.0348
0.35	2.5567	2.5172	2.5496	0.0395	0.0071	0.0324
0.40	2.6615	2.6143	2.6526	0.0472	0.0089	0.0383
0.45	3.0851	2.9995	3.0666	0.0856	0.0185	0.0671
0.50	3.9596	3.7574	3.9068	0.2022	0.0528	0.1494

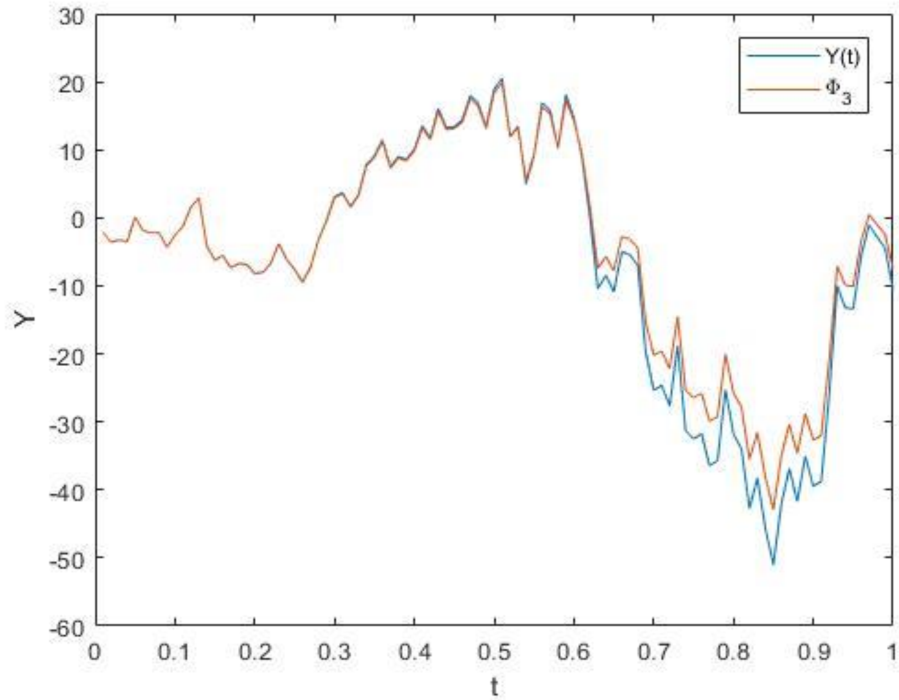

Figure 3: $Y(t)$ and $\Phi_3(\mathrm{t})$ via DJM for Example 4.2 $(N = 100,\ L = 5)$

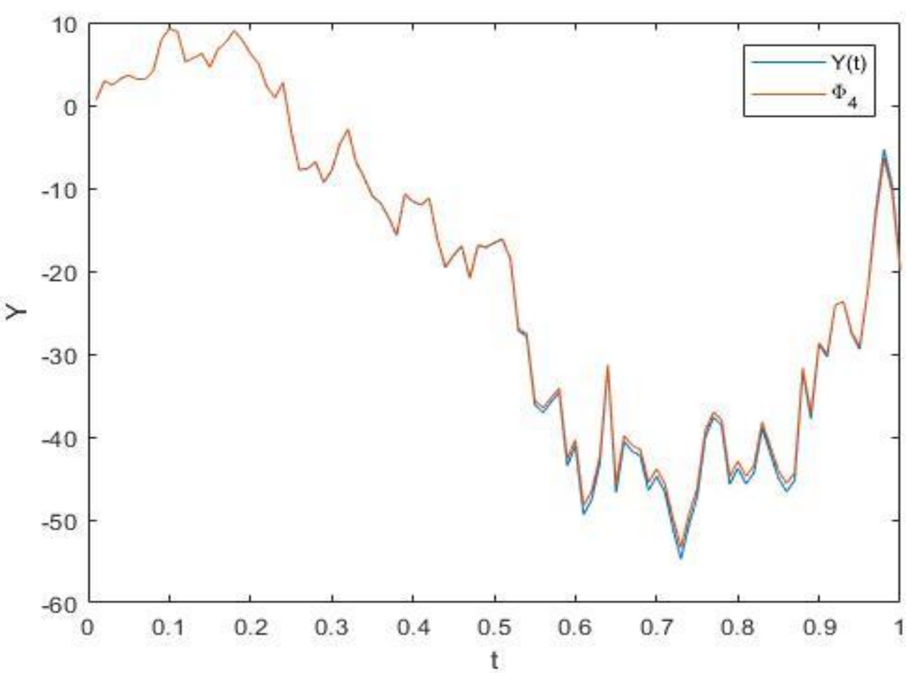

Figure 4: $Y(t)$ and $\Phi_4(\mathrm{t})$ via DJM for Example 4.2 $(N = 100,\ L = 5)$

Example 4.3: Consider the Stochastic Differential Equation [43]:

$$\begin{cases} dY(t) = \frac{1}{2}Y(t)dt + Y(t)\circ dB(t), \\ Y(0) = \mathrm{e}^{-1}. \end{cases} \tag{4.10}$$

The Stratonovich exact solution of (4.10) is given as:

$$Y(t) = \exp\left(\frac{t}{2} + B(t) - 1\right) \quad . \tag{4.11}$$

In integral form, we have:

$$Y(t) = e^{-1} + \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\,dB(s). \tag{4.12}$$

Therefore, with the following definitions:

$$\bar{N}(Y(t)) = \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\,dB(s), \tag{4.13}$$

$$dB(s) = \sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j \quad , \tag{4.14}$$

and

$$\bar{N}(Y(t)) = \frac{1}{2}\int_0^t Y(s)\,ds + \int_0^t Y(s)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right), \tag{4.15}$$

we obtain the iterative scheme via DJM as follows:

$$Y_0 = e^{-1},$$

$$\begin{aligned} Y_1 &= \bar{N}(Y_0) \\ &= \frac{1}{2}\int_0^t Y_0\,ds + \int_0^t \left(Y_0\,dB(s)\right)ds \\ &= \frac{1}{2}\int_0^t (Y_0)\,ds + \int_0^t \left(Y_0\left[\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right]\right)ds, \end{aligned}$$

$$\begin{aligned} Y_2 &= \bar{N}(Y_0+Y_1) - \bar{N}(Y_0) \\ &= \left\{\frac{1}{2}\int_0^t (Y_0+Y_1)\,ds + \int_0^t \left((Y_0+Y_1)\,dB(s)\right)ds\right\} - \left\{\frac{1}{2}\int_0^t (Y_0)\,ds + \int_0^t \left(Y_0\,dB(s)\right)ds\right\} \\ &= \left\{\frac{1}{2}\int_0^t \left((Y_0+Y_1)\right)ds + \int_0^t \left((Y_0+Y_1)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\} - \{Y_1\}, \end{aligned}$$

$$
\begin{aligned}
Y_3 &= \bar{N}\left(Y_0+Y_1+Y_2\right)-\bar{N}\left(Y_0+Y_1\right)\\
&=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)dB(s)\right)ds\right\}\\
&\quad-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1\right)ds+\int_0^t\left(\left(Y_0+Y_1\right)dB(s)\right)ds\right\}\\
&=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\}\\
&\quad-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1\right)ds+\int_0^t\left(\left(Y_0+Y_1\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\},
\end{aligned}
$$

$$
\begin{aligned}
Y_4 &= \bar{N}\left(Y_0+Y_1+Y_2+Y_3\right)-\bar{N}\left(Y_0+Y_1+Y_2\right)\\
&=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2+Y_3\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2+Y_3\right)dB(s)\right)ds\right\}\\
&\quad-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)dB(s)\right)ds\right\}\\
&=\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2+Y_3\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2+Y_3\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\}\\
&\quad-\left\{\frac{1}{2}\int_0^t\left(Y_0+Y_1+Y_2\right)ds+\int_0^t\left(\left(Y_0+Y_1+Y_2\right)\left(\sqrt{2}\sum_{j=0}^{5}\left(\cos\left[(j+0.5)\pi t\right]\right)z_j\right)\right)ds\right\},\\
&\vdots
\end{aligned}
$$

The computed results and errors analysis are shown in the table 3 and two graphs are plotted for $Y(t)$ against Φ_3 and $Y(t)$ against Φ_4.

Table 3: Solutions and absolute errors via DJM: $Y(t)$, $\Phi_3(t)$, $\Phi_4(t)$ for Example 4.3

t	$Y(t)$	$\Phi_3(t)$	$\Phi_4(t)$	$\lvert Y(t)-\Phi_3(t)\rvert$	$\lvert Y(t)-\Phi_4(t)\rvert$	$\lvert \Phi_4(t)-\Phi_3(t)\rvert$
0.00	0.3679	0.3679	0.3679	0.0000	0.0000	0.0000
0.05	0.4796	0.4796	0.4798	0.0000	0.0002	0.0002
0.10	0.6181	0.6168	0.6198	0.0013	0.0017	0.0030
0.15	0.7692	0.7639	0.7763	0.0053	0.0071	0.0124
0.20	0.8963	0.8846	0.9108	0.0117	0.0145	0.0262
0.25	0.9591	0.9432	0.9784	0.0159	0.0193	0.0352
0.30	0.9570	0.9413	0.9761	0.0157	0.0191	0.0348
0.35	0.9406	0.9260	0.9584	0.0146	0.0178	0.0324
0.40	0.9791	0.9618	1.0000	0.0173	0.0209	0.0382
0.45	1.1350	1.1035	1.1706	0.0315	0.0356	0.0671
0.50	1.4567	1.3823	1.5317	0.0744	0.0750	0.1494

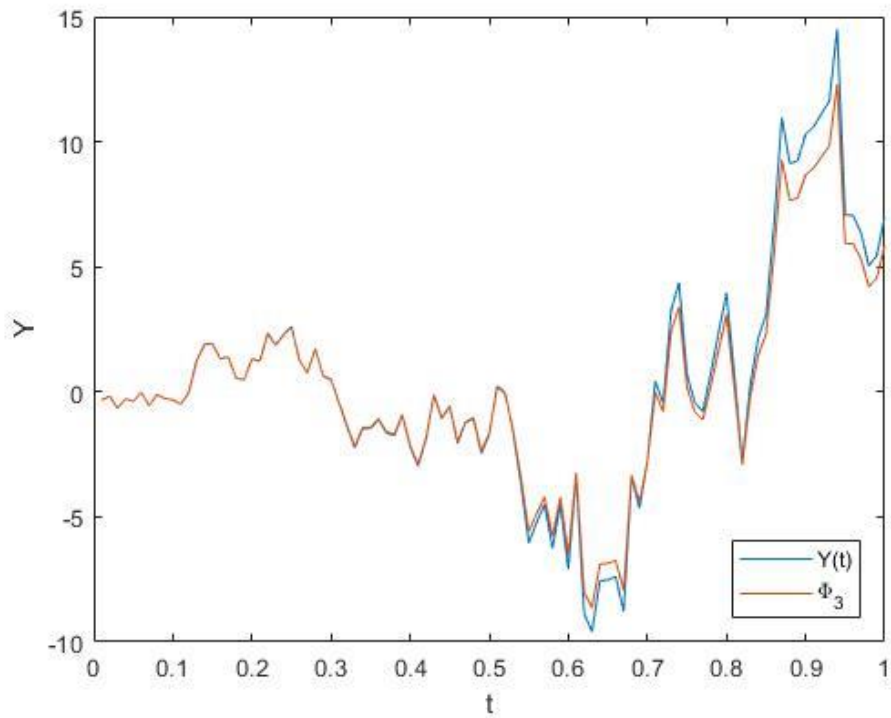

Figure 5: $Y(t)$ and $\Phi_3(\mathrm{t})$ via DJM for Example 4.3 $\left(N=100,\ \mathrm{L}=5\right)$

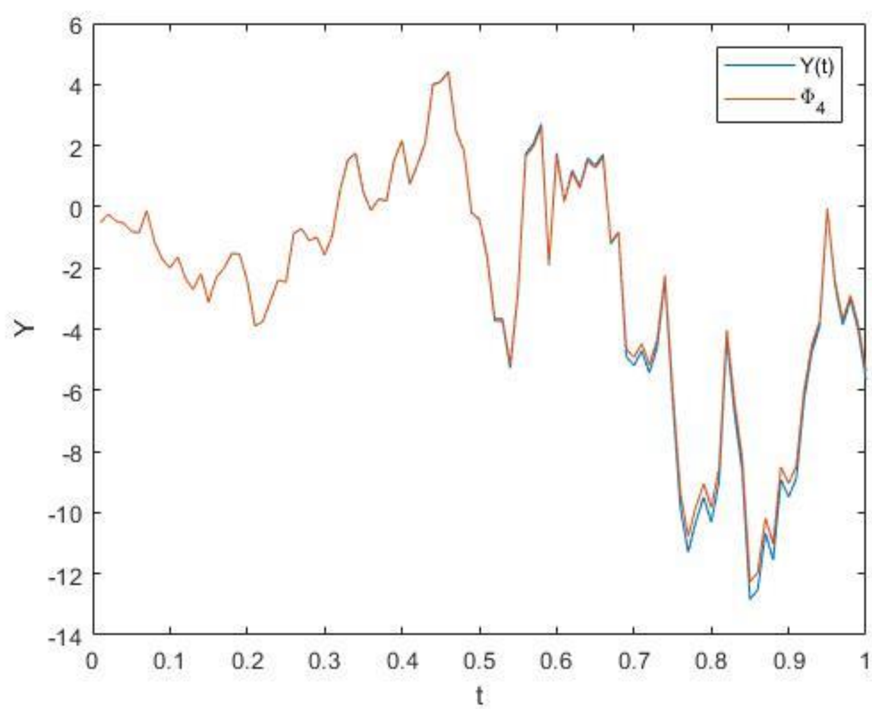

Figure 6: $Y(t)$ and $\Phi_4(t)$ via DJM for Example 4.3 $\left(N=100,\ L=5\right)$

4. CONCLUDING REMARKS

This paper considered the application of a new iterative integral method referred to as Daftardar-Jafari Integral Method (DJM) for approximate analytical solution of linear Stratonovich Stochastic Differential Equations. The concerned white noise was handled using Karhunen-Loéve Expansion in a finite series form, unlike other approaches whose solutions were given in terms of Brownian Motion. Some of the primary advantages of the DJM include its ease of application, direct nature, and less computational activities. Illustrative test cases were considered for testing the effectiveness and efficiency of the method, and the results converged faster to the exact solutions when compared. Based on these, higher order and nonlinear models of Ito or Stratonovich Stochastic Differential Equations (SDEs) can be considered via this approach, and the finite series expansion definition.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] A. Napoli, On a class of stochastic Runge Kutta method, Int. J. Contemp. Math. Sci. 7 (36) (2012), 1757-1769.

[2] M. Bayram, Automatic analysis of the control of metabolic networks, Comput. Biol. Med. 26 (5) (1996), 401-408.

[3] N. Guzel, M. Bayram, Numerical solution of differential-algebraic equations with index-2. Appl. Math. Comput. 174 (2) (2006), 1279-1289.

[4] J. G. Gaines, T. J. Lyons, Variable step size control in the numerical solution of stochastic differential equations, SIAM J. Appl. Math. 57 (5) (1997), 1455-1484.

[5] T. Sauer Numerical solution of stochastic differential equations in finance. In: Duan J-C, Hardle W, Gentle J,

eds. Handbook of Computational Finance. Springer, Berlin-Heidelberg; (2012), 529–550.

[6] M. Bayram, T. Partal, G. Orucova Buyukoz, Numerical methods for simulation of stochastic differential equations, Adv. Differ. Equ. 2018 (2018), 17.

[7] M. Asgari, E. Hashemizadeh, M. Khodabin, K. Maleknejad, Numerical solution of nonlinear stochastic integral equation by stochastic operational matrix based on Bernstein polynomials. Bull. Math. Soc. Sci. Math. Roum. 57 (105) (2014), 3–12.

[8] J.C. Cortes, L. Jodar, L. Villafuerte, Mean square numerical solution of random differential equations: facts and possibilities, Comput. Math. Appl. 53 (2007), 1098-1106.

[9] S. Jankovic, D. Ilic, One linear analytic approximation for stochastic integro-differential equations, Acta Math. Sci. 30 (2010), 1073-1085.

[10] F. Mohammadi, A wavelet-based computational method for solving stochastic Ito-Volterra equations, J. Comput. Math. 298 (2015), 254-265.

[11] F. Mohammadi, Second kind Chebyshev wavelet Galerkin method for stochastic Itô-Volterra integral equations, Mediterr. J. Math. 13 (2016), 2613-2631.

[12] M. G. Murge, B. G. Pachpatte, Successive approximations for solutions of second order stochastic integro-differential equations of Ito type, Indian J. Pure Appl. Math. 21 (1990), 260-274.

[13] M. H. Heydari, M. R. Hooshmandasl, F. M. Maalek, C. Cattani, A computational method for solving stochastic Ito-Volterra integral equations based on stochastic operational matrix for generalized hat basis functions, J. Comput. Phys. 270 (2014), 402-415.

[14] K. Nouri, Study on efficiency of Adomian decomposition method for stochastic differential equations, Int. J. Nonlinear Anal. Appl. 8 (2017), 61-68.

[15] V. Daftardar-Gejji, H. Jafari, An iterative method for solving nonlinear functional equations, J. Math. Anal. 316 (2006), 753–763.

[16] S. Bhalekar, V. Daftardar-Gejji, Solving a system of nonlinear functional equations using revised new iterative method, World Academy of Science, Eng. Technol. 6 (2012), 968-972.

[17] S. Bhalekar, V. Daftardar-Gejji, New iterative method: application to partial differential equations, Appl. Math. Comput. 203 (2008), 778-783.

[18] V. Daftardar-Gejji, S. Bhalekar, Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method, Comput. Math. Appl. 59 (2010), 1801-1809.

[19] O. Gonzalez-Gaxiola, J. Ruiz de Chavez, S. O. Edeki, Iterative method for constructing analytical solutions to the Harry-DYM initial Value Problem, Int. J. Appl. Math. 31 (4),(2018), 627-640.

[20] H. Jafari, S. J. Johnston, S. M. Sani, D. Baleanu, A decomposition method for solving q-difference equations, Appl. Math. Inform. Sci. 9 (2015), 2917-2920.

[21] H. Jafari, S. Seifi, A. Alipoor, M. Zabihi, An iterative method for solving linear and nonlinear fractional diffusion-wave equation, Int. e-J. Numer. Anal. Related Topics. 3 (2009), 20-32.

[22] M. Khodabin, K. Maleknejad, F. Hosseini Shekarabi, Application of triangular functions to numerical solution of stochastic Volterra integral equations, IAENG Int. J. Appl. Math. 43 (2011), 1-9.

[23] H. Deilami Azodi, Application of DJ method to Ito stochastic differential equations, J. Linear Topol. Algebra. 8 (3) (2019), 183- 189.

[24] L. Arnold, Stochastic differential equations, theory and applications, Wiley-Interscience, New York, 1974.

[25] S. Bhalekar, V. Daftardar-Gejji, Convergence of the new iterative method, Int. J. Differ. Equ. 2011 (2011), 989065.

[26] S. Bhalekar, V. Daftardar-Gejji, Solving evolution equations using a new iterative method, Numer. Methods Partial Differ. Equations. 26 (3) (2010), 906-916.

[27] G.O. Akinlabi, R.B. Adeniyi, Sixth-order and fourth-order hybrid boundary value methods for systems of boundary value problems,WSEAS Trans. Math. 17 (2018), 258-264.

[28] A. Hasan, A study on approximate solutions and accuracy of parallel algorithms for solving system of odes, J. Math. Comput. Sci. 10 (1) (2020), 110-135.

[29] S.O. Edeki, O.O. Ugbebor, E.A. Owoloko, He’s Polynomials for Analytical Solutions of the Black-Scholes Pricing Model for Stock Option Valuation, Proc. World Congr. Eng. 2(2016), 1-3.

[30] G.O. Akinlabi, R.B. Adeniyi, E.A. Owoloko, The solution of boundary value problems with mixed boundary conditions via boundary value methods, Int. J. Circ. Syst. Sign. Processi. 12 (2018), 1-6.

[31] A. Bibi, F. Merahi, Adomian decomposition method applied to linear stochastic differential equations, Int. J. Pure Appl. Math. 118 (3) (2018), 501-510.

[32] S.O. Edeki, E.A. Owoloko, O.O. Ugbebor, The Modified Black-Scholes Model via Constant Elasticity of Variance for Stock Options Valuation, AIP Conf. Proc. 1705 (2016), 020041.

[33] K.M. Owolabi, K.C. Patidar, A. Shikongo, A fitted operator method for a model arising in vascular tumor dynamics, Commun. Math. Biol. Neurosci. 2020 (2020), 4.

[34] G.O Akinlabi, S.O Edeki, The Solution of Initial-value Wave-like Models via Perturbation Iteration Transform Method, Proc. Int. Multi Conf. Eng. Comput. Sci. 2(2017), 1-4.

[35] O.A. Adesoji, A new monotonically stable discrete model for the solution of differential equations emanating from the evaporating raindrop, J. Math. Comput. Sci. 10 (1) (2020), 40-50.

[36] S.O. Edeki, G.O. Akinlabi, Zhou Method for the Solutions of System of Proportional Delay Differential Equations, MATEC Web of Conferences, 125(2017), 02001.

[37] D. Lesnic, The decomposition method for initial value problems, Appl. Math. Comput.181 (2006), 206-213.

[38] M.B. Matadi, Lie symmetry analysis of stochastic SIRS model, Commun. Math. Biol. Neurosci. 2019 (2019), 23.

[39] S.O. Edeki, G.O. Akinlabi, Coupled Method for Solving Time-Fractional Navier-Stokes Equation, Int. J. Circ. Syst. Sign. Process. 12 (2018), 27-34.

[40] W.A. Robin, Solving differential equations using modified Picard iteration, Int. J. Math. Educ. Sci. Technol. 41 (5) (2010), 649-665.

[41] M.I. Stefano, Simulation and Inference for Stochastic Differential Equations, Springer, New York, 2008.

[42] A. Fakharzadeh, E. Hesamaeddini and M. Soleimanivareki, Multi-step stochastic differential transformation method for solving some class of random differential equations, Appl. Math. Engi. Manage. Technol. 3(2015), 115-123.

[43] R. Farnoosh, H. Rezazadeh, A. Sobhani, M. Behboudi, Analytical solutions for stochastic differential equations via Martingale processes, Math. Sci. 9 (2015), 87–92.