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# Homotopy Analysis Method for Fractional Barrier Option PDE

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# Homotopy Analysis Method for Fractional Barrier Option PDE

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**Abstract**. This paper proposes a fractional numerical study on Time-Fractional Barrier Option Model (T-FBOM) via highly convergence solution series method known as Homotopy Analysis Method (HAM). Furthermore, the pricing formula for T-FBOM was obtained. The performance of the method on some illustrative examples has been considered. The results obtained were compared with some existing approaches.

**Keywords**: Classical barrier option, Caputo sense, fractional order, homotopy analysis method

## 1 Introduction

Most of the practical dynamics phenomena emanated from real life situations were described by fractional calculus, see [1-14], just to mention a few. Highly convergence solution series method under consideration was first applied by [15]. Some researchers have studied the solution of non-integer differential equations, see [16-18]. Fadugba and Edeki [19] and Edeki and Fadugba [20] proposed Laplace-Adomian decoposition method and projected differential transformation method, respectively to obtain the solution of barrier option PDE model It is also asummed in this paper that the underlying asset price is driven by marked point process [21]. Other solution methods that can be adopted for the model include [22-24]. The rest of the paper is structured as follows; The solution of T-FBOM via HAM is presented in Section 2, applications of the method and concluding remarks were presented in Sections 3 and 4, respectively.

## 2. The Solution of T-FBOM via HAM

Consider the time-fractional barrier option model of the form

$$D_{\tau}^{\beta} c = \frac{\partial^2 c}{\partial x^2} + (B-1)\frac{\partial c}{\partial x} - Bc \tag{1}$$

Subject to

$$c(x,0) = (e^x - K), e^x \in (0, e^{B_u}) \tag{2}$$

With

$$\frac{2r}{\sigma^2} = B,\ \ S = e^x,\ \ C(S,t) = c(x,\tau),\ \ t = T - \frac{2\tau}{\sigma^2} \tag{3}$$

In the sense of the Caputo, where $c(x,\tau) = c$ and $\beta$ is the non-integer order. The solution of (1) subject to (2) via is obtained as

$$c(x,\tau) = e^x - K\exp(-B\tau^{\beta}) \tag{4}$$

By means of (3), (4) becomes

$$C(S,t) = S - K\exp\left[-r\left(\frac{\sigma^2}{2}\right)^{\beta-1}(T-t)^{\beta}\right] \tag{5}$$

which is the required solution to TFBOM. Note that the valuation formula for the classical barrier option model is retrievable from (5) for $\beta = 1$ which is the same as the result obtained by [19] and [20]

## 3. Application of the Method

Consider the following parameters [19]: $S$= \$80, σ = 25% and $B_u$= \$85 and by varying the exercise price $K$, maturity date $T$, Risk-neutral interest rate $r$ and fractional / non-integer order $\beta$. The results generated were displayed in Figures 1-5.

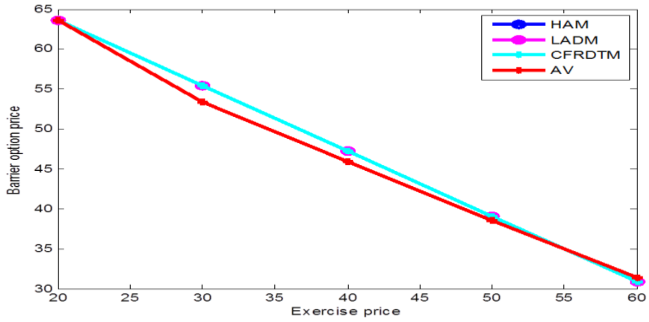


Figure 1: HAM, LADM [19], CFRDTM [17], AV [22] with $T$ = 2, $r$ = 0.1, σ = 25%, $\beta$ = 1, $Bu$ = 85, $S$ =80

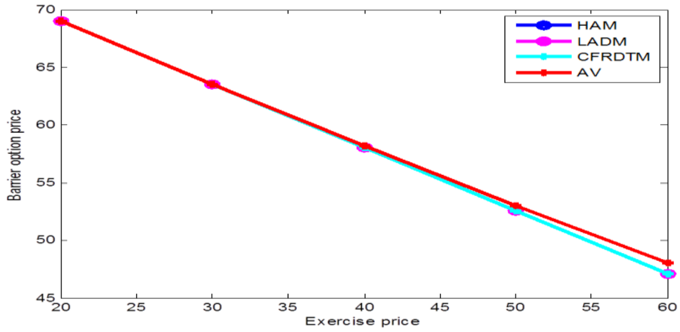


Figure 2: HAM, LADM [19] , CFRDTM [17], AV [22] with $T$ = 6, $r$ = 0.1, σ = 25%, $\beta$ = 1, $Bu$ = 85, $S$ =80

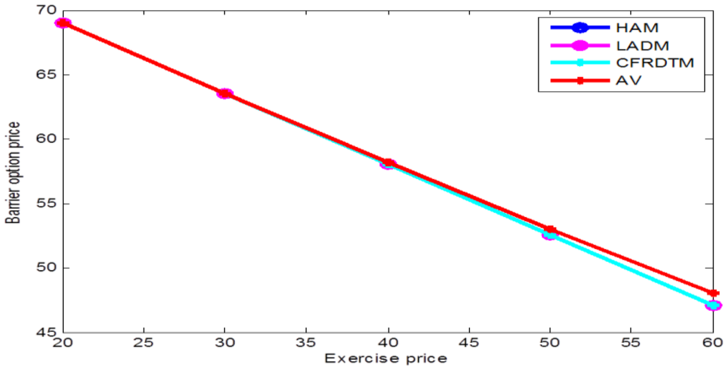


Figure 3: HAM, LADM [19], CFRDTM [17] and AV with $T$ = 10, $r$ = 0.1, σ = 25%, $\beta$ = 1, $Bu$ = 85, $S$ =80

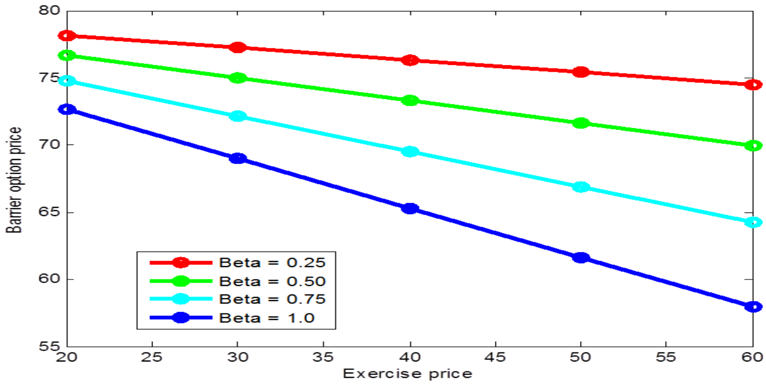


Figure 4: The effect of the non-integer order on T-FBOM prices generated via HAM with σ = 25%, $r$ = 20%, $S$ = $80, $T$ =10.

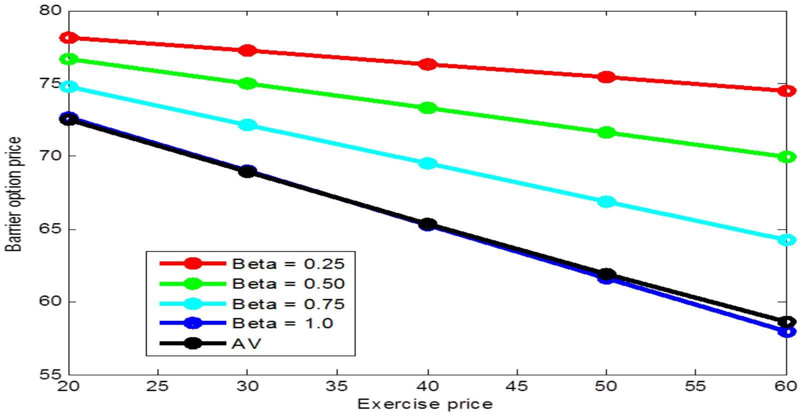


Figure 5: HAM versus AV with different non-integer order

## 4. Concluding Remarks

In this paper, fractional numerical study on T-FBOM has been considered in the sense of Caputo. Furthermore, the result of HAM coincides with that of LADM, CFRDTM and also in agreement with AV as this is evident in Figures 1-3. Figure 4 shows the effect of the non-integer order on T-FBOM prices obtained via HAM. It is observed Figure 5 that when $\beta = 0.25, 0.50$, T-FBOM is overpriced and when $\beta = 1$, T-FBOM has the reduced price in maturity time. Decrease in $\beta$ leads to increase in the payoff of T-FBOM. Moreover, the results obtained showed the suitability and effectiveness of HAM. Hence, the methodology can be extended for the valuation of financial derivatives with different flavours.

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