

Gas compressibility factor explicit correlations for range of pseudo reduced temperature and pressure

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ABSTRACT

The most essential properties of a natural gas are the thermodynamic property such as Gas compressibility factor (Z), and it is used to quantify the level of deviation of a real gas from an ideal gas at a certain temperature and pressure. Based on the importance of this property, many means have been proposed to derive the Z factor parameter such as through Experimental analysis, Equation of state and Empirical correlations. For correlations, both implicit correlations and explicit correlations have been modelled in order to best measure this deviation. However, the explicit correlation has not considered pseudo reduced temperature of less than 1. This study analyzed previous correlations in order to gain knowledge on their working conditions, limitations, and methods of derivations. A quick and dependable approach in modeling Z factor correlation from the pseudo reduced temperature and pressure was adopted. The study proposed a new and accurate correlation that can be employed in daily calculations that is an extension of Beggs-Brill Correlation (BBC), Azizi-Behbahani-Isazadeh Correlation (ABIC) and Sanjari-Lay Correlation (SLC). The composite correlation technique led to the derivation of 3 new equations for gas compressibility factor. A regression analysis was run to see how far the new correlations deviated from the previous ones and two of the correlations proved to be conforming to the Standing and Katz model as well as the other base correlations used. The result obtained from the 3 new correlations were then validated with field data. The type of natural gas worked with was a binary mixture of methane and decane components. After the evaluation, it was seen that the new correlations worked accurately and should be included in future important calculations.

1. Introduction

Natural gases have certain properties that are very essential in their predictions with some of these properties being gas-oil ratio, viscosity and density as well as flow-rate of gas [1]. These parameters are then functions of relative permeability, capillary pressure, wettability, mobility and mobility ratio, pore volume compressibility, saturation curves, porosity and permeability. In most engineering computations, gas compressibility factor of natural gases is important to gas metering, pressure, planning and arranging of pipelines and surface accommodations [2]. Most common sources of attaining a compressibility factor, usually denoted by Z or referred to as Z - factor, is by laboratory research and testing (and this has proven to be costly and tedious), empirical correlations and equation of state methods.

Attaining fluid properties from gas and oil accumulations has been of great significance to many scholars and petroleum engineers [3]. This knowledge proves to be relevant when dissolved gases, oil and gas reserves capacity, aquifer models depend directly and indirectly on

fluid properties. With regards to this, pressure, volume and temperature (PVT) analysis is vital to the parameters mentioned above. To comprehend and foresee the volumetric conduct of gas accumulations as a composition of pressure, learning of the physical properties of the reservoir fluids is of utmost importance [2]. Following the discovery of the Standing and Katz chart [4], new methods of predicting Z factor are available in the literature. Compressibility factor can be derived using five strategies: laboratory experiments, empirical correlations, corresponding states, Artificial Intelligence (Artificial Neural Network) and Equation of state techniques.

Several well know correlations are used in the petroleum industry to determine values of gas compressibility factor [5]. These investigations of Z -factors for natural gases have demonstrated that their Z -factors can be calculated with sufficient precision for many engineering purposes when expressed in terms of two dimensionless properties;

- Pseudo-reduced pressure P_{pr} , it is the ratios of pressure and specific volume of a real gas to the corresponding critical values.

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- Pseudo-reduced temperature T_{pr} , it is the ratios of temperature and specific volume of a real gas to the corresponding critical values.

These correlations may exist as either explicit correlations or implicit correlations based on some properties. The aim of this study is to provide a new and accurate correlation for a certain range of pseudo reduced temperature and pressure that can be employed in daily calculations that is an extension of Beggs-Brill Correlation (BBC), Azizi-Behbahani-Isazadeh Correlation (ABIC) and Sanjari-Lay Correlation (SLC).

1.1. Review of existing methods

Compressibility factor is extremely fundamental as it is utilized as a part of most petroleum and natural gas calculations (both upstream and downstream). They are necessary for gas metering, gas pressurizing, plan of pipelines and surface facilities [6]. According to Kareem [7], who developed a complex and accurate correlation for assessing Z -factor that can be linearized. His correlation works efficiently within the ranges of $0.2 \leq P_{pr} \leq 15$. This method is characterized to be simple and single-valued. This correlation is continuous and efficient over a wide range of pseudo-reduced pressures (values of 0.2–15). Due to this fact, the use of this correlation is widened and can be applied in the evaluation of natural gas compressibility as the pseudo reduced pressure factor is a function of the gas compressibility factor.

Gas compressibility factor has been efficiently estimated using a detailed method formulated by Fayazi et al. [8]. They presented a modernized improved soft computing approach known as the least square support vector machine (LSSVM). This approach was applied to a wide database of over 2200 samples of both sweet and sour compositions. LSSVM was used to develop a gas compressibility factor as a function of parameters such as the composition of gas molecular weight of C_7+ , pressure and temperature. In designing this model, the parameters were assumed as correlating variables.

Kamari et al. [9] worked on a similar study where they utilized an intelligent method to predict sour and pure gas compressibility factor. The same mathematical based approach was implemented to attain the compressibility factor for both high sulphur and pure natural gases. The approach is the LSSVM model improved with coupled simulated annealing (CSA) optimization tool.

A new correlation was developed by Azizi et al. [10], in Iran. Due to few available experimental data at the required temperature, pressure and composition, an empirical correlation was refined based on the (S–K) chart in order to forecast the gas compressibility factor for sweet gases. This new correlation has two advantages, firstly, it is exceedingly accurate as an examination was completed and the outcomes demonstrated that the new correlation conveys a prevalence over the former. Secondly, the new relationship is explicit thus it does not require iteration to achieve its answer as employed by some other correlations. Their correlation is based on 3038 points from the S–K chart.

Mohamadi-Baghmolaei et al. [3], use of intelligent models such as Adaptive Neuro Fuzzy System (ANFIS) Artificial Neural Network (ANN) and Fuzzy Interface System (FIS) was analyzed in this journal due to the limitations of some other correlations. These intelligent models were tested using 1038 data points and the artificial neural network model which was developed using 263 data points exceeded the performance of the other models in terms of accuracy. The input variables for this model are temperature, pressure and specific gas gravity as a function of gas compressibility factor.

A Society of Petroleum Engineers (SPE) paper on compressibility factor for high molecular weight reservoir gases by Sutton [11], studies the effect of high molecular weight in natural gases. This team introduced a new P_{pc} and T_{pc} property and gas gravity relationship and derived an alternative method for calculating pseudocritical properties. Their correlation is accurate within the range of $0.2 \leq P_{pr} < 30$ and $1.0 < T_{pr} < 3.0$ but cannot be utilized in determining

the P_{pc} and T_{pc} for reservoir gases with specific gravity greater than 0.75.

Obuba et al. [12], presented a new natural gas compressibility factor correlation for the Niger Delta fields utilizing 22 laboratory gas PVT reports from Niger Delta fields. The new correlation was used to determine Z factors for four natural gas reservoirs: dry gas, solution gas, rich CO_2 gas and rich condensate gas and the results were then compared with other correlations. From this paper, it was concluded that their correlation and the Papay correlation are most appropriate for Niger Delta gas fields. For this research, Wichert–Aziz correlation was not considered as Nigerian gas is sweet.

Okoro et al. [2], focused on evaluating the best Z factor method from the Hall and Yarborough, Dranchuk, Abu and Kassem and Dranchuk, Purvis and Robinson methods for Niger Delta fields using Gas Well Inflow. According to their research, Hall and Yarborough is ranked first as the best gas deviation model for Niger Delta fields. Their study was proven based on production data from about four gas fields.

The need to understand and forecast the gas deviation factor at low to moderate pressures (less than 8000 psia) and temperatures (less than 212 °F) or at HPHT has become important with regards exploration and production activities. The Azubuike et al. [5], paper showcases the laboratory measurement of gas compressibility factors at HPHT. Samples of gas mixtures were retrieved from high pressured gas reservoirs in the Niger Delta region and were compared to some other correlations used in the petroleum engineering sector. As mentioned above, Hall and Yarborough performed better than other existing correlation at 270 °F and at 370 °F Beggs and Brill was predicted to be better than other correlations.

1.1.1. Gaps

1. Beggs and Brill Correlation [13]: This technique is not recommended to be utilized for reduced temperature (T_{pr}) values less than 0.92. This correlation is of moderately low accuracy, except at moderate pressures and temperatures. Based on previous research, it was seen that the Beggs and Brill Correlation is accurate within $1.2 \leq T_{pr} \leq 2.4$ and $0.0 \leq P_{pr} \leq 10$.
2. Heidarayan et al. Correlation [14]: They established a new explicit correlation using regression analysis but only applied it in the analysis of the Z factor experimental value for reduced pseudo-pressure of less than or equal to 3. Thus, for this correlation, there is a discontinuity at $P_{pr} = 3$.
3. Hall and Yarborough Correlation [15]: It has been recorded to be very efficient and accurate but requires iteration using the Newton Raphson procedure and thus requires several steps of iterations which is time-consuming.
4. Dranchuk et al. Correlation [16]: Their correlation is valid within the following ranges $1.05 < T_{pr} < 3.0$ and $0.2 < P_{pr} < 3.0$. Thus, it can be said that there is accuracy only within a short range of P_{pr} values
5. Burnett Correlation [17]: This non-iterative calculation can be used within the ranges of $1.3 \leq T_{pr} \leq 3.0$ and $0.2 \leq P_{pr} \leq 4.0$. He specified that the accuracy of the correlation diminishes for T_r below 1.3 and above 1.85.

2. Methodology

This method adopted in this study is of five major steps, and they are planning, evaluation, conception, execution and analysis as seen in Fig. 1.

Data relevant to this study were extracted from materials such as research papers, journals, textbooks, library and reports got from the internet. The program that was adopted for this methodology is Excel. Microsoft Excel is a spreadsheet that involves the use of calculations, graphing tools and user-defined functions. This report employed the graph plotting features for the derivations and evaluations the gas

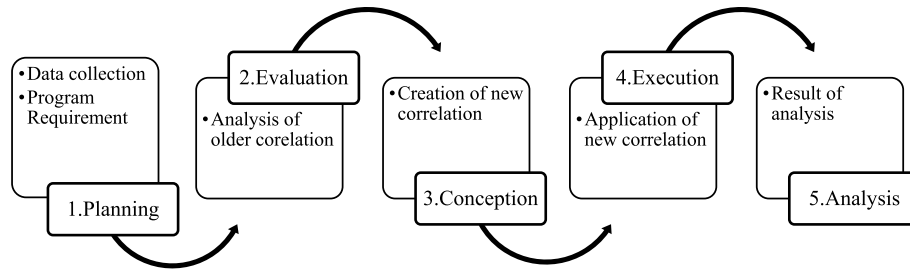


Fig. 1. Methodology Flow chart.

compressibility factor correlations.

2.1. Evaluation

This section of this chapter is based on the step by step analysis on explicit correlations that have been derived from the Standing and Katz model. For the S-K model, pseudo-critical temperature and pressure are first obtained via the Sutton correlation below:

$$T_{pc} = 169.2 + 349.5\gamma_g - 74\gamma_g^2 \quad (1)$$

$$P_{pc} = 756.8 + 131.07\gamma_g - 3.6\gamma_g^2 \quad (2)$$

before they are computed alongside the pressure and temperature to derive the pseudo reduced temperature and pressure of the gas blend, where γ_g is gas specific gravity.

As mentioned earlier, pseudo reduced pressures and temperature were included for this research not original pressures and temperature of the gases because they provide a means to create unexpected values for not just one gas but for a combination of several gases irrespective of their c + component.

Beggs and Brill [13] correlation (BBC) evaluation

A step by step approach shall be used to evaluate BBC. Below is the compressibility factor of BBC:

$$z = A(1 - A)\exp(-B) + CP_{pr}^D \quad (3)$$

Where

$$A = 1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101$$

$$B = (0.62 - 0.23T_{pr})P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037\right)P_{pr}^2 + \left(\frac{0.32}{10^{(9(T_{pr}-1))}}\right)P_{pr}^6$$

$$C = 0.132 - 0.32 \log(T_{pr})$$

$$D = 10^{(0.3106 - 0.49T_{pr} + 0.1824T_{pr}^2)}$$

Once the values were computed for the Beggs and Brill correlation, it was noted that T_{pr} values ranged between > 0.92 – 2.4 . Values for $T_{pr} < 0.92$ and > 2.4 provided Z factor values that were not viable thus they had to be quality checked (Fig. 2).

Shell oil company [18] correlation (SOC) evaluation

A step by step approach shall be used to evaluate SOC. Below is the compressibility factor of SOC:

$$Z = A + BP_{pr} + (1 - A)\exp(-C) - D\left(\frac{P_{pr}}{10}\right)^4 \quad (4)$$

where

$$A = -0.101 - 0.36T_{pr} + 1.3868\sqrt{T_{pr} - 0.919}$$

$$B = 0.021 + \frac{0.04275}{T_{pr}}$$

$$C = P_{pr}(E + FP_{pr} + GP_{pr}^4)$$

$$D = 0.122\exp(-11.3(T_{pr} - 1))$$

$$E = 0.6222 - 0.224T_{pr}$$

$$F = \frac{0.0657}{T_{pr} - 0.85} - 0.037$$

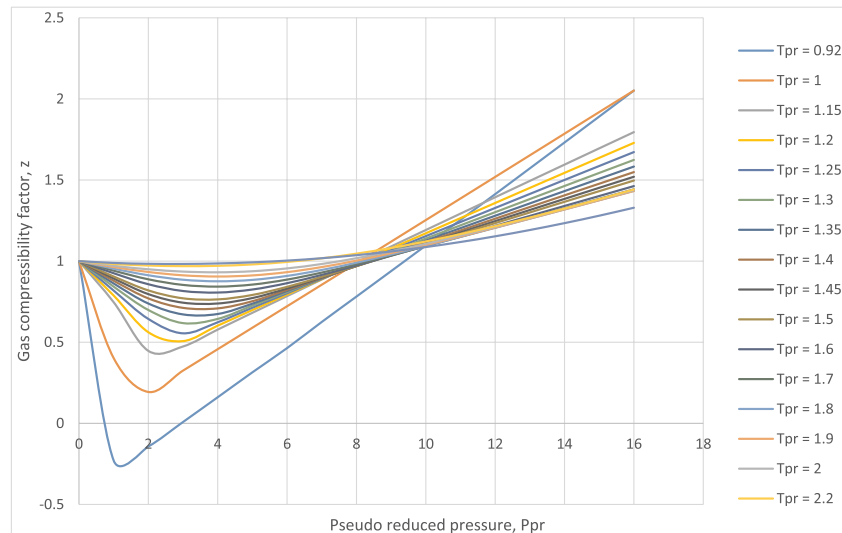


Fig. 2. Chart representing Beggs and Brill correlation (BBC).

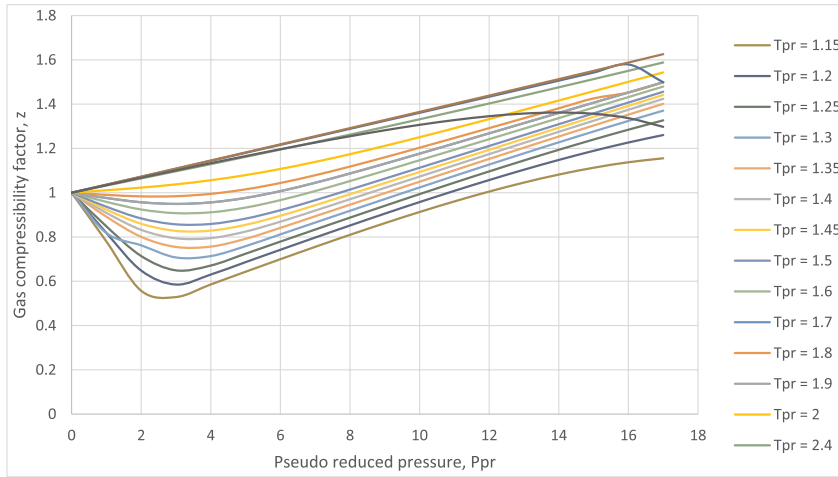


Fig. 3. Chart representing Shell Oil Company correlation.

Heidarayan et al. [14], correlation

$$G = 0.32 \exp(-19.53(T_{pr} - 1))$$

For the Shell Oil Company correlation, their range is larger than that of Beggs and Brill [13]. This correlation has 7 inputs ranging from A - G that have to be input to calculate a compressibility factor (Fig. 3).

A step by step approach is used to evaluate the Heidarayan correlation. The compressibility factor of the Heidarayan correlation is:

$$Z = \ln \left(\frac{A_1 + A_3 \ln(P_{pr}) + \frac{A_5}{T_{pr}} + A_7 (\ln P_{pr})^2 + \frac{A_9}{T_{pr}^2} + \frac{A_{11}}{T_{pr}} \ln(P_{pr})}{1 + A_2 \ln(P_{pr}) + \frac{A_4}{T_{pr}} + A_6 (\ln P_{pr})^2 + \frac{A_8}{T_{pr}^2} + \frac{A_{10}}{T_{pr}} \ln(P_{pr})} \right) \quad (5)$$

Whilst the Heidarayan et al., correlation is accurate for $P_{pr} \leq 3$, there is a line of discontinuity for values of P_{pr} greater than this (Fig. 4). The T_{pr} values cover a range of $1.15 \leq T_{pr} \leq 3$.

A step by step approach is used to evaluate the Azizi correlation. The compressibility factor of the Azizi correlation is:

$$Z = A + \frac{B + C}{D + E} \quad (6)$$

Where

$$A = aT_{pr}^{2.16} + bP_{pr}^{1.028} + cP_{pr}^{1.58}T_{pr}^{-2.1} + d \ln(T_{pr})^{-0.5}$$

$$B = e + fT_{pr}^{-2.4} + gP_{pr}^{1.56} + hP_{pr}^{0.124}T_{pr}^{3.033}$$

$$C = i \ln(T_{pr})^{-1.28} + j \ln(T_{pr})^{1.37} + k \ln(P_{pr}) + l \ln(P_{pr})^2 + m \ln(P_{pr}) \ln(T_{pr})$$

$$D = 1 + nT_{pr}^{5.55} + oP_{pr}^{0.68}T_{pr}^{0.33}$$

$$E = p \ln(T_{pr}^{1.18}) + q \ln(T_{pr}^{2.1}) + r \ln(P_{pr}) + s \ln(P_{pr}^2) + t \ln(P_{pr}) \ln(T_{pr})$$

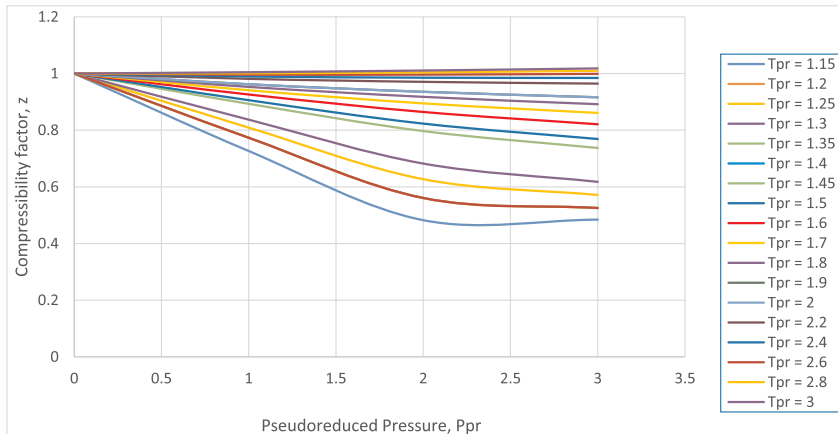


Fig. 4. Chart representing Heidarayan Correlation.

Azizi et al., [10] correlation

This correlation has 20 constants and requires 6 inputs and the correlation is graphically seen in Fig. 5. Upon review, there seems to be a consistency in the relationship between pseudo reduced pressure and gas compressibility factor.

A step by step approach shall be used to evaluate Sanjari and Lay correlation. Below is the compressibility factor of SAL correlation:

$$Z = 1 + A_1 P_{pr} + A_2 P_{pr}^2 + \frac{A_3 P_{pr}^{A_4}}{T_{pr}^{A_5}} + \frac{A_6 P_{pr}^{(A_4+1)}}{T_{pr}^{A_7}} + \frac{A_8 P_{pr}^{(A_4+2)}}{T_{pr}^{(A_7+1)}} \quad (7)$$

Upon looking at the Sanjari and Lay Correlation Layout, it can be seen that it has a similarity to the Standing and Katz chart. The percentage of accuracy in relation to the Standing and Katz chart is very high and this correlation is feasible over a broad range of T_{pr} and P_{pr} values (Fig. 6).

2.2. Conception

For the creation of the model that will work efficiently to solve the problem statement, 3 correlations were chosen. The three correlations are the Beggs and Brill correlation [13], the Sanjari and Lay correlation [19] and the Azizi et al. [9], correlation, based on the following reasons and similarities listed below.

2.2.1. Similarities in these correlations

1. All correlations are explicit.
2. All the three correlations have constants that have been experimentally tested in the laboratory for accuracy.

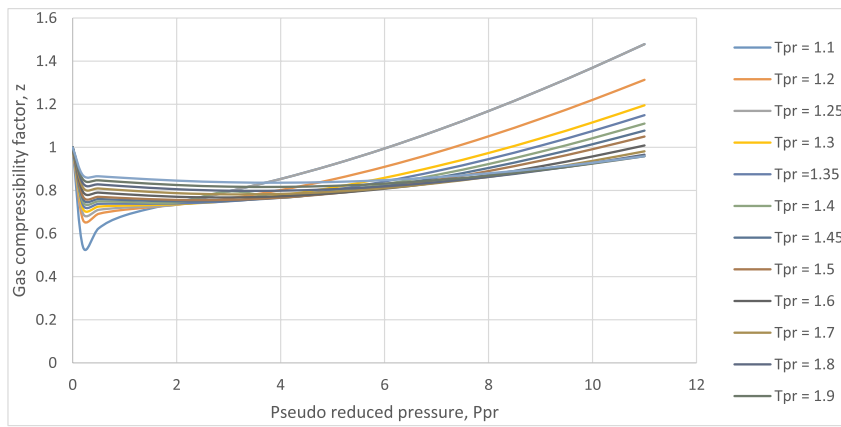


Fig. 5. Chart representing Azizi et al., Correlation.

Sanjari and Lay [19] correlation (SAL) evaluation

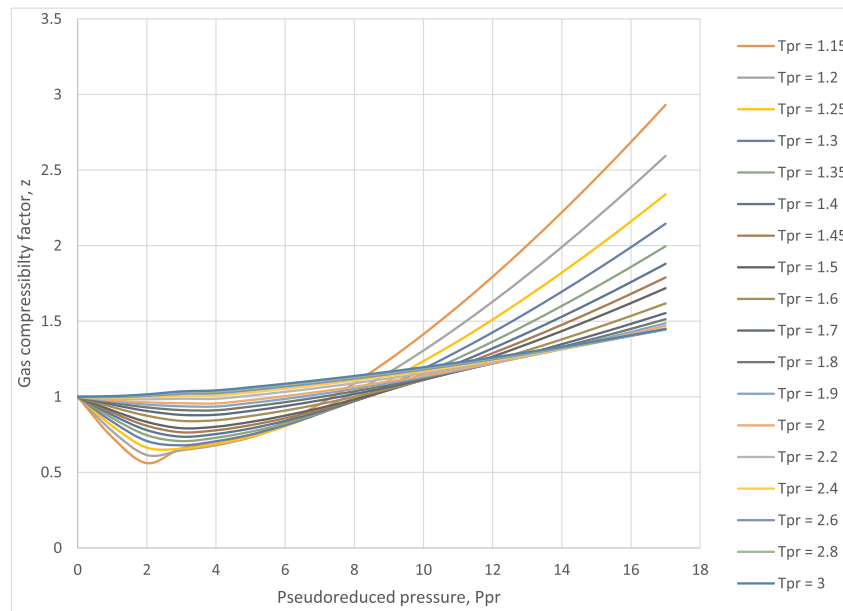


Fig. 6. Chart representing Sanjari and Lay correlation.

3. The three correlations have good agreement in terms of the Z factor values derived. It is stated that this is only feasible within a short range of T_{pr} and P_{pr} values. However, the Beggs and Brill correlation covers $1.2 \leq T_{pr} \leq 2.4$ and $0.0 \leq P_{pr} \leq 10$, the Sanjari and Lay correlation covers $1.01 \leq T_{pr} \leq 3$ and $0.01 \leq P_{pr} \leq 15$, and the Azizi et al., correlation covers $0.2 \leq P_{pr} \leq 11$ and $1.1 \leq T_{pr} \leq 2$

It should be noted that the agreement is based on comparison with the Standing and Katz model.

If we recall the problem statement where no explicit gas compressibility factor has been accurately created to function under the conditions of $0.92 \leq T_{pr} \leq 2.2$ and $0.2 \leq P_{pr} \leq 15$, the three correlations have been considered based on:

- P_{pr} value range
- T_{pr} value range
- The accuracy of their Z values obtained within these ranges
- Level of use in the industry

Previous common methods of deriving or modifying a correlation.

- Computer application (Fortran, Matlab)
- Use of tabulated values and interpolation techniques
- Iteration – derivation of the implicit correlation

Other explicit correlations were looked into as well, such as those by the Shell Oil Company [18], Hankinson, Thomas and Phillips [20] and Heidaryan et al., [14] correlations but due to some shortcomings, such as significant deviations from reference values for gas compressibility factor thus implying a reduced level of accuracy, long derivation equations and inconsistency of some Z values after a particular range of T_{pr} and P_{pr} values, the correlations listed above best fit. The study proposed a gas compressibility factor as a function of pseudo reduced properties, based on the above mentioned equations.

The method adopted is called the correlation composition technique.

This new correlation was modified to accommodate pure natural gases and not sour or sweet gases or liquids as this modified correlation does not take into consideration the impurities.

2.3. Explanation of the terms in the correlation

Note: input + constant → variable → combined variables → Equation → Z Correlation.

2.3.1. Derivation of equation

$$Z = A(1 - A)\exp(-B) + CP_{pr}^D \quad (8)$$

Table 1
 P_{pr} values for the new correlation [19].

Constants	$P_{pr} \leq 3$	$P_{pr} > 3$
E	0.007698	0.015642
F	0.003839	0.000701
G	−0.46721	2.341511
H	1.018801	−0.6579
I	3.805723	8.902112
J	−0.08736	−1.136
K	7.138305	3.543614
L	0.08344	0.134041

Table 2
 Constants ranging from 1 to 20 for new correlation [10].

Constant	Value	Constant	Value
A_1	0.0373142485385592	A_{11}	−24449114791.1531
A_2	−0.0140807151485369	A_{12}	19357955749.3274
A_3	0.016363245387186	A_{13}	−126354717916.607
A_4	−0.0307776478819813	A_{14}	623705678.385784
A_5	13843575480.943800	A_{15}	17997651104.3330
A_6	−16799138540.763700	A_{16}	151211393445.064
A_7	1624178942.6497600	A_{17}	139474437997.172
A_8	13702270281.086900	A_{18}	−2423301984.0950
A_9	−41645509.896474600	A_{19}	1893804797.5205
A_{10}	237249967625.01300	A_{20}	−141401620722.689

$$Z = 1 + A_1 P_{pr} + A_2 P_{pr}^2 + \frac{A_3 P_{pr}^{A_4}}{T_{pr}^{A_5}} + \frac{A_6 P_{pr}^{(A_4+1)}}{T_{pr}^{A_7}} + \frac{A_8 P_{pr}^{(A_4+2)}}{T_{pr}^{(A_7+1)}} \quad (9)$$

$$Z = A + \frac{B + C}{D + E} \quad (10)$$

The new correlation is based on the three correlations above.

Renaming the variables in equation (9)

$$Z = 1 + EP_{pr} + FP_{pr}^2 + \frac{GP_{pr}^H}{T_{pr}^I} + \frac{JP_{pr}^{(H+1)}}{T_{pr}^K} + \frac{LP_{pr}^{(H+2)}}{T_{pr}^{(K+1)}} \quad (11)$$

Renaming the variables in equation (10)

$$Z = M + \frac{N + O}{P + Q} \quad (12)$$

From equations (8), (11) and (12), the variable A-D and M-Q denote inputs which contain broader formulas that will be seen further into the derivation. Whereby, the variables E-L denote constants which can be seen in Table 1.

Equation (12) contains 5 variables and 20 constants ranging from A_1 to A_{20} . These constants remain the same for P_{pr} values ranging from 0.2 to 15 unlike the constants from equation (11). The constants are stated in Table 2.

Writing out all the inputs within the variables.

$$A = 1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101 \quad (13)$$

$$B = (0.62 - 0.23T_{pr})P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037 \right) P_{pr}^2 + \left(\frac{0.32}{10^{(9(T_{pr}-1))}} \right) P_{pr}^6 \quad (14)$$

$$C = 0.132 - 0.32 \log(T_{pr}) \quad (15)$$

$$D = 10^{(0.3106 - 0.49T_{pr} + 0.1824T_{pr}^2)} \quad (16)$$

$$M = A_1 T_{pr}^{2.16} + A_2 P_{pr}^{1.028} + A_3 P_{pr}^{1.58} T_{pr}^{-2.1} + A_4 \ln(T_{pr})^{-0.5} \quad (17)$$

$$N = A_5 + A_6 T_{pr}^{-2.4} + A_7 P_{pr}^{1.56} + A_8 P_{pr}^{0.124} T_{pr}^{3.033} \quad (18)$$

$$O = A_9 \ln(T_{pr})^{-1.28} + A_{10} \ln(T_{pr})^{1.37} + A_{11} \ln(P_{pr}) + A_{12} \ln(P_{pr})^2 + A_{13} \ln(P_{pr}) \ln(T_{pr}) \quad (19)$$

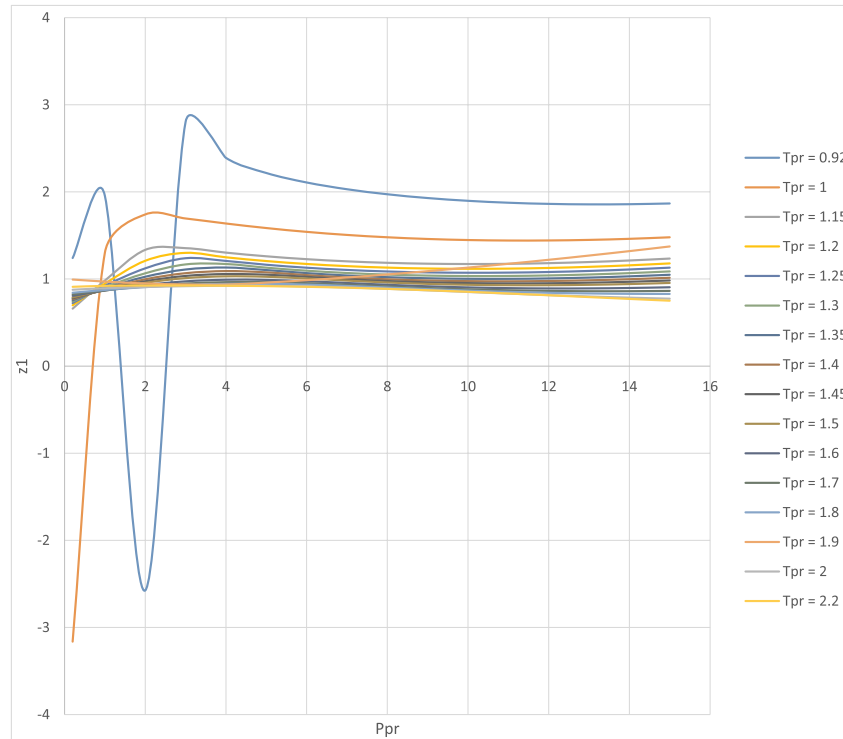


Fig. 7. Correlation 1 showing variation in P_{pr} with T_{pr} .

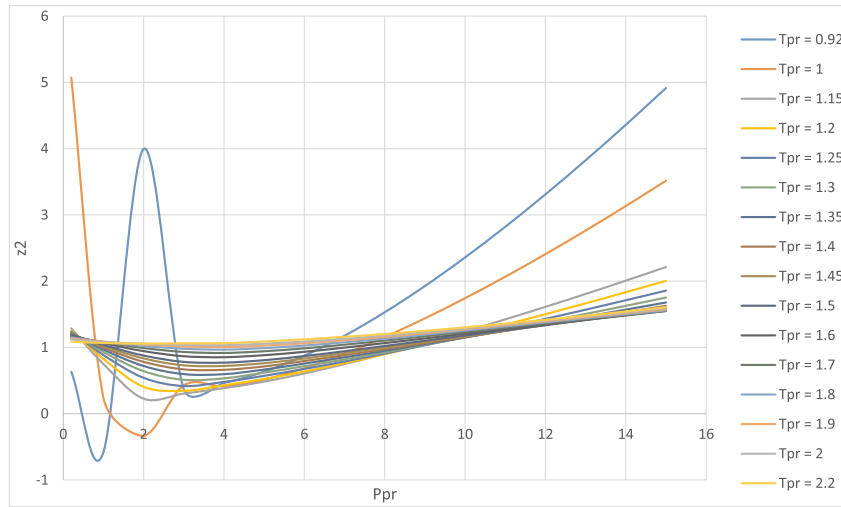


Fig. 8. Correlation 2 plot showing variation in P_{pr} with T_{pr} .

$$P = 1 + nA_{14} + A_{15}P_{pr}^{0.68}T_{pr}^{0.33} \quad (20)$$

$$Q = A_{16}\ln(T_{pr}^{1.18}) + A_{17}\ln(T_{pr}^{2.1}) + A_{18}\ln(P_{pr}) + A_{19}\ln(P_{pr}^2) + A_{20}\ln(P_{pr})\ln(T_{pr}) \quad (21)$$

Thus, Equation (8)

$$Z = A(1 - A)\exp(-B) + CP_{pr}^D$$

$$Z = (1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \left[1 - (1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \exp \left[- \left((0.62 - 0.23T_{pr})P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037 \right) P_{pr}^2 + \left(\frac{0.32}{10^{9(T_{pr}-1)}} \right) P_{pr}^6 \right) \right] \right] + \left[(0.132 - 0.32 \log(T_{pr})) * P_{pr}^{10^{(0.3106-0.49T_{pr}+0.1824T_{pr}^2)}} \right] \quad (22)$$

Equation (11) does not have any inputs but only constants which has been stated above.

Repeating the same thing for Equation (12),

$$Z = M + \frac{N + O}{P + Q} \quad Z = (A_1 T_{pr}^{2.16} + A_2 P_{pr}^{1.028} + A_3 P_{pr}^{1.58} T_{pr}^{-2.1} + A_4 \ln(T_{pr})^{-0.5}) + \left[(A_5 + A_6 T_{pr}^{-2.4} + A_7 P_{pr}^{1.56} + A_8 P_{pr}^{0.124} T_{pr}^{3.033}) + A_9 \ln(T_{pr})^{-1.28} + A_{10} \ln(T_{pr})^{1.37} + A_{11} \ln(P_{pr}) + A_{12} \ln(P_{pr})^2 + A_{13} \ln(P_{pr})\ln(T_{pr}) \right] / \left[(1 + nA_{14} + A_{15}P_{pr}^{0.68}T_{pr}^{0.33}) + (A_{16}\ln(T_{pr}^{1.18}) + A_{17}\ln(T_{pr}^{2.1}) + A_{18}\ln(P_{pr}) + A_{19}\ln(P_{pr}^2) + A_{20}\ln(P_{pr})\ln(T_{pr})) \right] \quad (23)$$

Merging the (3) equations for the gas Compressibility factor and representing the three new modelled correlations by Z_1 , Z_2 and Z_3 .

$$Z = 1 + EP_{pr} + FP_{pr}^2 + \frac{GP_{pr}^H}{T_{pr}^I} + \frac{JP_{pr}^{(H+1)}}{T_{pr}^K} + \frac{LP_{pr}^{(H+2)}}{T_{pr}^{(K+1)}} \quad (24)$$

Correlation 1

$$Z_1 = \left[(1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \left(1 - (1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \exp \left[- \left((0.62 - 0.23T_{pr})P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037 \right) P_{pr}^2 + \left(\frac{0.32}{10^{9(T_{pr}-1)}} \right) P_{pr}^6 \right) \right] \right) \right] + \left[(0.132 - 0.32 \log(T_{pr})) * P_{pr}^{10^{(0.3106-0.49T_{pr}+0.1824T_{pr}^2)}} \right] - \left[1 + EP_{pr} + FP_{pr}^2 + \frac{GP_{pr}^H}{T_{pr}^I} + \frac{JP_{pr}^{(H+1)}}{T_{pr}^K} + \frac{LP_{pr}^{(H+2)}}{T_{pr}^{(K+1)}} \right] - [(A_1 T_{pr}^{2.16} + A_2 P_{pr}^{1.028} + A_3 P_{pr}^{1.58} T_{pr}^{-2.1} + A_4 \ln(T_{pr})^{-0.5}) + (A_5 + A_6 T_{pr}^{-2.4} + A_7 P_{pr}^{1.56} + A_8 P_{pr}^{0.124} T_{pr}^{3.033}) + A_9 \ln(T_{pr})^{-1.28} + A_{10} \ln(T_{pr})^{1.37} + A_{11} \ln(P_{pr}) + A_{12} \ln(P_{pr})^2 + A_{13} \ln(P_{pr})\ln(T_{pr})] / [(1 + nA_{14} + A_{15}P_{pr}^{0.68}T_{pr}^{0.33}) + (A_{16}\ln(T_{pr}^{1.18}) + A_{17}\ln(T_{pr}^{2.1}) + A_{18}\ln(P_{pr}) + A_{19}\ln(P_{pr}^2) + A_{20}\ln(P_{pr})\ln(T_{pr}))] \quad (25)$$

Correlation 2

$$Z_2 = \left[1 + EP_{pr} + FP_{pr}^2 + \frac{GP_{pr}^H}{T_{pr}^I} + \frac{JP_{pr}^{(H+1)}}{T_{pr}^K} + \frac{LP_{pr}^{(H+2)}}{T_{pr}^{(K+1)}} \right] - \left[(1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \left(1 - (1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101) * \exp \left[- \left((0.62 - 0.23T_{pr})P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037 \right) P_{pr}^2 + \left(\frac{0.32}{10^{9(T_{pr}-1)}} \right) P_{pr}^6 \right) \right] \right) \right] + \left[(0.132 - 0.32 \log(T_{pr})) * P_{pr}^{10^{(0.3106-0.49T_{pr}+0.1824T_{pr}^2)}} \right] - (A_1 T_{pr}^{2.16} + A_2 P_{pr}^{1.028} + A_3 P_{pr}^{1.58} T_{pr}^{-2.1} + A_4 \ln(T_{pr})^{-0.5}) + [(A_5 + A_6 T_{pr}^{-2.4} + A_7 P_{pr}^{1.56} + A_8 P_{pr}^{0.124} T_{pr}^{3.033}) + A_9 \ln(T_{pr})^{-1.28} + A_{10} \ln(T_{pr})^{1.37} + A_{11} \ln(P_{pr}) + A_{12} \ln(P_{pr})^2 + A_{13} \ln(P_{pr})\ln(T_{pr})] / [(1 + nA_{14} + A_{15}P_{pr}^{0.68}T_{pr}^{0.33}) + (A_{16}\ln(T_{pr}^{1.18}) + A_{17}\ln(T_{pr}^{2.1}) + A_{18}\ln(P_{pr}) + A_{19}\ln(P_{pr}^2) + A_{20}\ln(P_{pr})\ln(T_{pr}))] \quad (26)$$

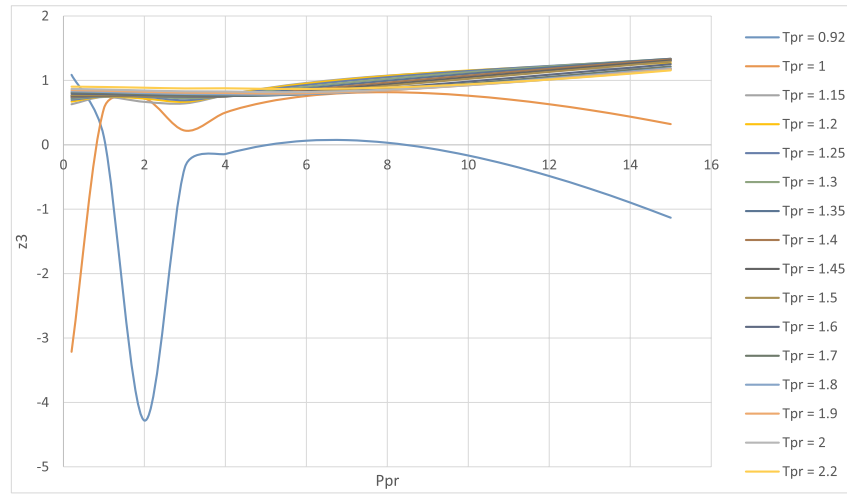


Fig. 9. Correlation 3 plot showing variation in P_{pr} with T_{pr} .

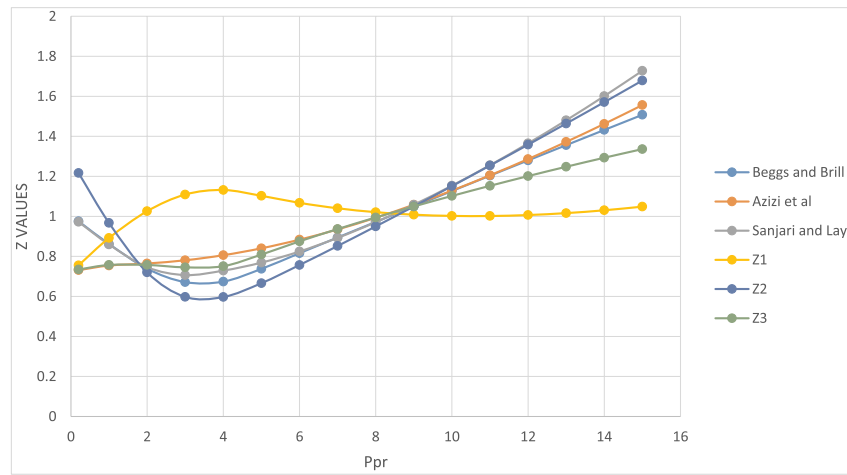


Fig. 10. Chart showing similarities and dissimilarities between z factors of reference and new correlations.

Correlation 3

$$\begin{aligned}
 Z_3 = & (A_1 T_{pr}^{2.16} + A_2 P_{pr}^{1.028} + A_3 P_{pr}^{1.58} T_{pr}^{-2.1} + A_4 \ln(T_{pr})^{-0.5}) \\
 & + [(A_5 + A_6 T_{pr}^{-2.4} + A_7 P_{pr}^{1.56} + A_8 P_{pr}^{0.124} T_{pr}^{3.033}) + A_9 \\
 & \ln(T_{pr})^{-1.28} + A_{10} \ln(T_{pr})^{1.37} + A_{11} \ln(P_{pr}) + A_{12} \ln(P_{pr})^2 + A_{13} \\
 & \ln(P_{pr}) \ln(T_{pr})] / [(1 + n A_{14} + A_{15} P_{pr}^{0.68} T_{pr}^{0.33}) + (A_{16} \ln(T_{pr}^{1.18}) + A_{17} \\
 & \ln(T_{pr}^{2.1}) + A_{18} \ln(P_{pr}) + A_{19} \ln(P_{pr}^2) + A_{20} \ln(P_{pr}) \ln(T_{pr}))] \\
 & - \left[(1.39(T_{pr} - 0.92)^{0.5} - 0.36 T_{pr} - 0.101) * \right. \\
 & \left(1 - (1.39(T_{pr} - 0.92)^{0.5} - 0.36 T_{pr} - 0.101) * \right. \\
 & \exp \left[- \left((0.62 - 0.23 T_{pr}) P_{pr} + \left(\frac{0.066}{T_{pr} - 0.86} - 0.037 \right) P_{pr}^2 + \left(\frac{0.32}{10^{(9(T_{pr}-1))}} \right) \right. \right. \\
 & \left. \left. P_{pr}^6 \right) \right] + \left[(0.132 - 0.32 \log(T_{pr})) * P_{pr}^{10^{(0.3106 - 0.49 T_{pr} + 0.1824 T_{pr}^2)}} \right] \left. \right] \\
 & - \left[1 + E P_{pr} + F P_{pr}^2 + \frac{G P_{pr}^H}{T_{pr}^I} + \frac{J P_{pr}^{(H+1)}}{T_{pr}^K} + \frac{L P_{pr}^{(H+2)}}{T_{pr}^{(K+1)}} \right]
 \end{aligned} \quad (27)$$

3. Results and discussion

The three correlations were then inputted into the application and it was used in the evaluation of the reference correlations. The three new correlations were modified for the gas compressibility factor of natural gas reserves as a function of pseudo reduced temperature and pressure.

Three charts were plotted with modified Z values on the y-axis and pseudo reduced pressure values on the x-axis where each isotherm represents pseudo reduced temperature. These results are presented in Figs. 7–9.

A similarity trend that was observed upon evaluation of the curves for the following charts is that the isotherm $T_{pr} > 1.15$ plot follow a trend but there is an obvious diversion in the isotherms $T_{pr} = 0.92$ and $T_{pr} = 1$.

A comparison was taken between the 3 correlations referenced and the 3 new correlations by plotting all 6 of them on the same chart at the same T_{pr} value (1.35), ranging P_{pr} values and different Z values (Fig. 10).

From Fig. 10, it can be deduced that the new correlations follow a trend with the referenced correlations. It can also be observed that correlation 1 shows a higher level of deviation from P_{pr} values ranging from 2 to 8. Since the standard chart usually referred to in the oil and gas industry is the Standing and Katz chart, Fig. 11 was drawn to

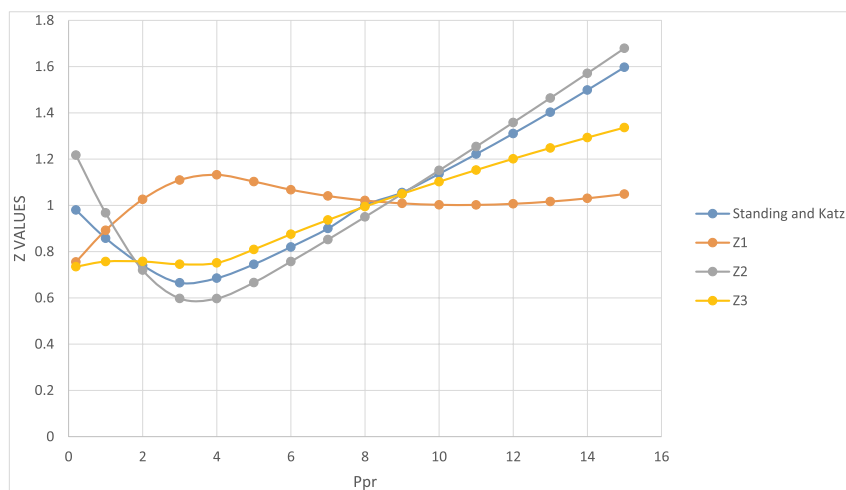


Fig. 11. Standing & Katz and new correlations.

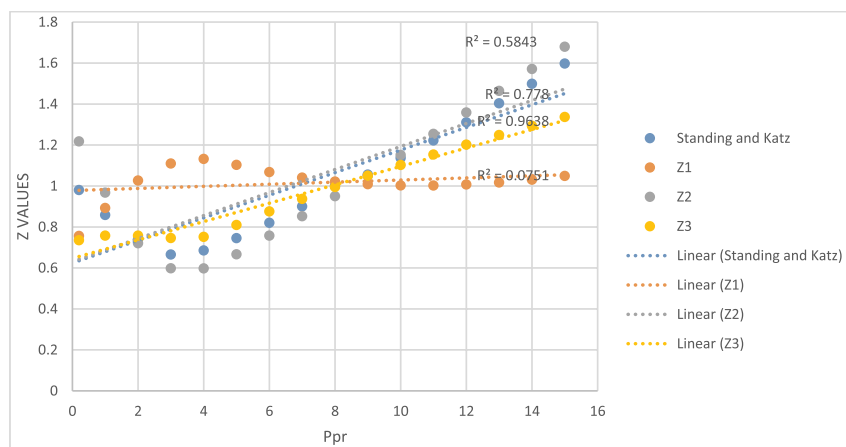


Fig. 12. Charts showing the accuracy of the new correlations in relation to base correlations.

Table 3Correlations values at different T_{pr} and P_{pr} values.

T_{pr}	P_{pr}	Beggs and Brill	Azizi et al.	Sanjari and Lay	Standing and Katz	Z_1	Z_2	Z_3
1.35	0.2	0.976	0.731	0.972	0.980	0.755	1.217	0.735
1	1	0.862	0.754	0.859	0.858	0.892	0.967	0.757
1.15	2	0.739	0.765	0.746	0.740	1.026	0.720	0.757
1.2	3	0.671	0.781	0.707	0.665	1.109	0.598	0.745
1.25	4	0.674	0.806	0.729	0.685	1.132	0.597	0.751
1.3	5	0.738	0.840	0.769	0.745	1.103	0.666	0.809
1.35	6	0.816	0.884	0.825	0.820	1.068	0.757	0.875
1.4	7	0.894	0.935	0.893	0.900	1.040	0.852	0.937
1.45	8	0.972	0.993	0.971	1.000	1.021	0.950	0.995
1.5	9	1.050	1.058	1.058	1.055	1.009	1.050	1.0496
1.6	10	1.127	1.129	1.153	1.136	1.002	1.151	1.102
1.7	11	1.203	1.205	1.256	1.222	1.002	1.254	1.153
1.8	12	1.280	1.287	1.365	1.310	1.007	1.358	1.201
1.9	13	1.356	1.372	1.480	1.403	1.016	1.464	1.248
2	14	1.432	1.462	1.601	1.499	1.030	1.571	1.293
2.2	15	1.508	1.556	1.728	1.597	1.049	1.679	1.336

Table 4
Regression Analysis Table for the comparison.

Regression Statistics						
Multiple R		0.998				
R Square		0.995				how far it deviated from the y-axis
Adjusted R Square		0.994				
Standard Error		0.380				average error of prediction
Observations		16				
ANOVA						
	df	SS	MS	F	Significance F	
Regression	4	335.448	83.862	580.240	1.038E-12	
Residual	11	1.590	0.145			
Total	15	337.038				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	−18.467	4.163	−4.436	0.001	−27.628	−9.305
Standing and Katz	8.205	12.152	0.675	0.513	−18.540	34.950
Z1	4.830	4.039	1.196	0.257	−4.060	13.720
Z2	−6.669	7.969	−0.837	0.420	−24.208	10.871
Z3	19.890	5.009	3.971	0.002	8.866	8.866

compare the new correlations with Standing and Katz chart.

Fig. 12 shows a simple linear regression analysis, using the Standing and Katz model as a reference for the new correlations. For correlation 1, it can be seen from all the charts that the curve always moves in the opposite direction of the other two correlations and the base correlations. Tabulated comparison of these correlations at different T_{pr} and P_{pr} values are represented in Table 3.

From the regression analysis carried out, it was seen that correlations 2 and 3 conform to the Standing and Katz model and it can be seen in Table 3.

4. Conclusion

Accurate knowledge of correlations for gas compressibility factor is critical to many aspects of the petroleum sector. From the results obtained, it can be concluded that;

1. The new correlations derived were explicit correlations that now exist for reduced values less than 1.
2. The evaluation of other correlation helped to establish their actual range.
3. Upon acquiring the result, it could be seen that for correlation 1, there was a variation in the parameters as opposed to what we have in the other two correlations.

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