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New decomposition method for the solutions of linear Schrödinger equation

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Abstract: The Schrödinger equation serves as the fundamental equation for quantum mechanics. In this article, we consider the theoretical and numerical solutions of Schrödinger's linear equations. This is achieved using the new semi-analytical approach as an alternative to the classical Adomian Decomposition Method (ADM). The new method is referred to as Telescoping Decomposition Method. Some cases are considered; the results obtained display high level of convergence to their exact forms. The improved version is very useful and reliable; it requires less analytical effort, even without giving up precision. Thus, it is also highly recommended for the solution of related linear and nonlinear differential models in the fields of applied research.

Keywords: Differential equations; Modified ADM; Schrödinger Equations; Analytical solutions.

1. Introduction

Differential equations exist in numerous fields such as physical, applied sciences and engineering. Many differential models or equations from real-life systems are nonlinear and or partial forms. This makes it a complicated task for mathematicians in terms of solutions. Schrödinger equation is used in quantum mechanics to describe how the quantum state of a physical system changes. The Schrödinger equation forms partial differential equations (PDEs) used in quantum mechanics to explain a quantum mechanical system's wave function or state function. The most popular form is the time-dependent Schrödinger equation [1, 2]. The Telescoping Decomposition Process (TDM) has been used in [3] to solve Nonlinear Initial Value Problems (IVP) in First Order. This TDM approach gave the numerical and analytical methods an iterative algorithm. The key benefit of the approach to them was to avoid computing the Adomian polynomials and to produce a simple algorithm. Other similar methods of solutions, Fractional Schrödinger equations in include those of [4-18].
The goal of the analysis is to apply a reliable algorithm (Telescoping Decomposition Method) for the solution of the Schrödinger differential equations.

2 Overview of the Method: Telescoping Decomposition Method (TDM)

The proposed technique termed 'Telescoping Decomposition Method' (TDM) is a modern iterative technique that seeks to provide computational and theoretical solutions for linear and nonlinear differential equations [3]. It modifies the popular Adomian Decomposition Method (ADM) by overcoming the calculations of the Adomian polynomials. This is illustrated as follows:
Let a compact subset of $\mathbb{R}$ be defined as $\Xi = [0,T]$, such the following holds:

$$\left.\begin{array}{l} w_t = g\left(t, w, w_t\right),\ t \in \Xi \\ w(0) = w_0 \end{array}\right\}, \tag{2.1}$$

then, in integral form, (3.1) becomes:

$$w(t) = w(0) + \int_0^t g(\tau, w(\tau), w_\tau(\tau))d\tau\,. \tag{2.2}$$

Let the solution be expressed as:

$$w(x,t) = \lim_{h\to\infty}\left(\sum_{n=0}^{h} w_n(t)\right) \tag{2.3}$$

where $w_n(t)$ will be determined iteratively via the following algorithm:

$$\left.\begin{array}{l} w^0 = w(0), \\ w^1 = \int_0^t g(\tau, w^0(\tau), w^0_\tau(\tau))d\tau, \\ \vdots \\ w^n = \int_0^t g\left(\tau, \sum_{k=0}^{n-1} w^k(\tau), \sum_{k=0}^{n-1} w^k_\tau(\tau)\right)d\tau - \int_0^t g\left(\tau, \sum_{k=0}^{n-2} w^k(\tau), \sum_{k=0}^{n-2} w^k_\tau(\tau)\right)d\tau,\ n \ge 1. \end{array}\right\} \tag{2.4}$$

3.0 Illustrative Cases

Here, the proposed method (TDM) is applied for the solutions of a linear Schrödinger equation for $\xi = -1,\ h(x) = 0,\ and\ \lambda = 0$ in the following generalized form of Schrödinger equation is of the form:

$$\left.\begin{array}{l} i\dfrac{\partial w}{\partial t} + \xi\dfrac{\partial^2 w}{\partial x^2} + h\left(x\right)w + \lambda\left|w\right|^2 w = 0 \\ \\ f\left(x\right) = w\left(x,0\right),\quad i^2 = -1 \end{array}\right\} \tag{3.1}$$

Case 3.1: Consider the following linear Schrödinger Equation [18] :

$$\begin{cases} \dfrac{\partial w}{\partial t} + i\dfrac{\partial^2 w}{\partial x^2} = 0, \quad i = \sqrt{-1}, \quad w = w(x,t) \\ \\ w(x,0) = e^{3ix} \end{cases} \tag{3.2}$$

whose exact solution is given as:

$$m(x,t) = e^{3i(x+3t)} \tag{3.3}$$

Solution to problem 3.1:

In integral form, (4.1) is given as:

$$w(x,t) = e^{3ix} - i\int_0^t \left(\frac{\partial^2 w}{\partial x^2}\right) dt \tag{3.4}$$

By the TDM, the following relation is obtained:

$$\begin{cases} w_0 = e^{3ix} \\ w_n = -i\left[\int_0^t \left(\sum_{k=0}^{n-1} w_k''\right) dt - \int_0^t \left(\sum_{k=0}^{n-2} w_k''\right) dt\right] \end{cases}. \tag{3.5}$$

Thus, for $n = 1, 2, 3, \cdots$, the following are obtained:

$$\begin{cases} w_1 = (9it)e^{3ix} \\ w_2 = \dfrac{(9it)^2 e^{3ix}}{2!} \\ w_3 = \dfrac{(9it)^3 e^{3ix}}{3!} \\ w_4 = \dfrac{(9it)^4 e^{3ix}}{4!} \\ \vdots \\ w_p = \dfrac{(9it)^p e^{3ix}}{p!}, \; p = 0, 1, 2, \cdots \end{cases} \tag{3.6}$$

$$
\therefore \quad
\begin{aligned}
w(x,t) &= \lim_{h\to\infty}\left(\sum_{n=0}^{h} w_n(t)\right) \\
&= e^{3ix} + (9it)e^{3ix} + \frac{(9it)^2 e^{3ix}}{2!} + \frac{(9it)^3 e^{3ix}}{3!} + \frac{(9it)^4 e^{3ix}}{4!} + \cdots + \frac{(9it)^p e^{3ix}}{p!} + \cdots \\
&= e^{3ix}\underbrace{\left(1 + (9it) + \frac{(9it)^2}{2!} + \frac{(9it)^3}{3!} + \frac{(9it)^4}{4!} + \cdots + \frac{(9it)^p}{p!} + \cdots\right)}_{e^{9it}} \\
&= e^{3i(x+3t)}.
\end{aligned}
\tag{3.7}
$$

This coincides with the exact solution as contained in [18].

4.0 Concluding Remarks

This paper has provided the theoretical and exact solutions to the linear Schrödinger equations by a new semi-analytical approach (Telescoping Decomposition Method). The TDM is a modified version of the classical ADM. Some illustrative cases are cases; the results obtained coincide with the exact form. On this this, the modified version is shown to be very effective and accurate. There is less analytical work involved, even without giving up precision, and there is no linearization, interference, or control involved. Therefore the approach is proposed for the solution of linear, nonlinear, stiff, and delayed partial differential equations with applications in related fields.

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