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ORIGINAL ARTICLE

Direct current electrical resistivity forward modeling using comsol multiphysics

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Abstract

Forward modeling of direct current (DC) resistivity is very important for the inversion of the resistivity data to obtain the true resistivity of the subsurface. In this study, we demonstrated finite-element forward modeling of DC resistivity method with point electric source using COMSOL Multiphysics. We employed the AC/DC module in COMSOL which often provides comparatively easy implementation of models and permits exterior boundaries to be placed at infinity, a boundary condition often experienced in most geophysical problems. The validity and effectiveness of the results of numerical simulation using COMSOL Multiphysics were evaluated by comparing the output of the numerical simulations with the calculated analytic solutions. The result reveals that the numerical simulation is in agreement with the analytic solution. This study shows that COMSOL Multiphysics can be used to simulate the distribution of electrical potentials of point source in 3D space in real life and the information from this study can be used for further studies, such as DC resistivity inversions.

Keywords Forward modeling · COMSOL · resistivity · Simulation

List of symbols

ρ_a	Apparent resistivity
ΔV	Difference in electrical potential
σ	Electrical conductivity
n	Normal to the surface
σ_2	Anomaly conductivity
V	Electrical potential
E	Error analysis
G	Geometrical factor
I	Current injected
I_o	Current intensity
σ_1	Electric background conductivity
V_{ana}	Analytical potential
r	Radius
x, y and z	Points

Introduction

The geophysical surveys involving the use of DC resistivity methods are used to evaluate the electrical resistivity distribution in the subsurface usually by taking measurements on the ground surface. The measurements of the electrical resistivity could in turn be used to determine the true resistivity of the subsurface. The DC resistivity techniques have been used over the years in groundwater exploration (Gautam and Biswas 2016; Oyeyemi et al. 2018a, b), engineering investigations (Oladunjoye et al. 2017; Oyeyemi et al. 2017, 2020), mineral exploration (Zhang et al. 2015; Sanuade et al. 2018), and environmental studies (Rosales et al. 2012; Akinola et al. 2018; Olaojo et al. 2018; Olaseeni et al. 2018). However, to obtain the electrical resistivity image that would be a representation of the subsurface, the electric potential data that are measured on the field must be inverted, and forward modeling is an important step for any inversion algorithms to be used (Gao et al. 2020).

Forward modeling is very important in electrical prospecting method to understand the subsurface distribution of structures and anomalies (Wang et al. 2011; Butler and Sinha 2012; Song et al. 2017; Udosen and George 2018; Gao et al. 2020). Forward modeling is necessary in geophysics, as it allows model parameters to be adjustable so as to fit observations. This process is part of routine that is usually

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carried out during inversion and physical intuition can be developed due to the variation of model parameters (Butler and Sinha, 2012). The approach of forward modeling in electrical resistivity method can generally be used to solve distribution of electrical field potentials in any given geoelectric model and field source distributions (Gao et al. 2020). This step is very essential for the inversion and interpretation of data in electrical resistivity methods.

The numerical modeling and simulation of DC resistivity can be used to simulate the distribution of electrical potentials of point source in 3D space in real life (Wang et al. 2012). There are basically three methods that can be used to solve the distribution of electrical field potentials: analytic, numerical and physical simulation methods (Wang et al. 2011; Udosen and George 2018). Analytic method is the most commonly used method and its results are often significant. However, analytic solutions can only be used to calculate the electric potential distribution within regular geometries. The physical simulation is often employed to inspect the effectiveness of the results of analytic and numerical methods. The numerical methods that are usually employed for electrical method of prospecting are finite difference (Dey and Morrison 1979; Zhao and Yedlin 1996), finite element (Rucker et al. 2006; Ren and Tang 2010; Wang et al. 2011), finite volume (Pidlisecky and Knight 2008) and integral-equation-based models (Ma 2002). Forward modeling of resistivity method involves solving Laplace's equation where the source terms represent the current electrodes as well as appropriate boundary conditions.

The COMSOL Multiphysics (COMSOL Multiphysics Users' Guide 2017) is a general FE modeling environment software which permits users to build complex numerical models using different geometries and multiple governing equations quickly using a graphical user interface (GUI). The software has several modules including heat transfer, electromagnetic, AC/DC, acoustics, earth science, chemical engineering and structural mechanics modules (Cardiff and Kitanidis 2008; COMSOL Multiphysics Users' Guide 2017).

The most commonly encountered boundary condition in many geophysical problems is the modeling of boundaries at infinite separation from the model domains. However, the AC/DC module of COMSOL Multiphysics has the ability to model these boundaries (Butler and Sinha 2012). Therefore, this study shows forward modeling of DC resistivity method using COMSOL Multiphysics with a point electric source. This study also demonstrates the efficacy of the infinite boundaries in COMSOL Multiphysics. This was achieved by comparing the output of the COMSOL Multiphysics with the results of analytic method.

Several studies have used COMSOL Multiphysics for the forward modeling of geophysical methods. For example. Butler and Sinha (2012) presented forward models of gravity, magnetic, resistivity and induced polarization (IP) geophysical techniques using the finite-element (FE) modeling in COMSOL Multiphysics. In their study, they compared the results of the FE with solutions from the analytical method and the results show that both FE and analytical solutions agreed reasonably well. Duquennoi et al. (2011) simulated the injection of leachates with the use of COMSOL Multiphysics and they obtained the distribution of the effective saturation in the subsurface. Wang et al. (2011) also employed COMSOL Multiphysics to understand the effectiveness of forward modeling in DC resistivity method using point electric source as an example. They compared the output of the FE modeling with the analytical solutions and the results show that the two techniques were in agreement. Park et al. (2010) used COMSOL Multiphysics to develop a 3D FE modeling of controlled-source electromagnetic (CSEM) data and they evaluated the effectiveness of their FE modeling using many examples of CSEM marine data. In addition, Kalavagunta and Weller (2005) employed COMSOL Multiphysics to estimate the geometrical factors for resistivity experiments that were performed in the laboratory. Braun et al. (2005) also employed COMSOL to model the propagation of electromagnetic waves in conducting media with application to magnetic resonance sounding. These studies and many have demonstrated the effectiveness of COMSOL Multiphysics for the numerical simulations of geophysical methods.

We present the procedures involved in the numerical modeling of DC resistivity method, including error and sensitivity analysis. Subsequently, we will compare the result of the numerical simulations with the results of analytical method.

Methodology

Resistivity method

The resistivity method involves the injection of electric current into the ground between one pair of electrodes and measurement of the electric potentials between another electrode pair. Given a particular geometry of an electrode, an ‘‘apparent resistivity,’’ $\rho_{\mathrm{a},}$ in ohm metre (Ωm), of the geometry can be calculated using Eq. 1 (Telford et al. 1990):

$$\rho_{\mathrm{a}} = G\frac{V}{I} \tag{1}$$

where ΔV is the difference in the electric potential between the potential electrodes in volts, I is the current injected into the ground which is measured in amperes (A), and G is the geometrical factor of the array. ρ_{a} is the resistivity that an infinite half-space of constant resistivity must possess to generate similar measured ΔV for a given current injected into the ground. If there is a change in ρ_a due to a change in the lateral array position or as a result of the change in the spacing of array, there would also be a vertical or lateral change in the resistivity of the subsurface (Telford et al. 1990).

An AC/DC conductive-media module in COMSOL Multiphysics software, used in this study, solves Eq. 2 to calculate the electrical potentials (Wang et al. 2012):

$$\sigma V = f \tag{2}$$

where σ (x, y, z) is the electrical conductivity (reciprocal of the resistivity), (x, y, z) is the electrical potential and f is a parameter that is related to the point source.

In a DC resistivity survey that involves the use of symmetrical four electrode configurations, two current electrodes will be used for the injection of current into the ground while two electrodes are used as potential electrodes for the measurement of the resulting potential difference. The intensity of current of the two current sources is equal but opposite in direction; therefore, f in Eq. 2 is given as:

$$f = I_o[\delta(x - x_1)(y - y_1)(z - z_1) - \delta(x - x_2)(y - y_2)(z - z_2) \tag{3}$$

where I_o represent the current intensity, while (x_1, y_1, z_1) and (x_2, y_2, z_2) are the locations of the positive current source, and negative current source, respectively.

However, away from the source, Eq. 2 reduces to Laplace's equation as given by Eq. 4:

$$\nabla^2 V = 0 \tag{4}$$

Resistivity model using COMSOL Multiphysics

COMSOL Multiphysics is a FE analysis, solver and simulation software that has found applications in numerous engineering and physics studies. The software is often used for coupled phenomena, or Multiphysics which is widely used in many science and engineering studies (Li et al. 2012). COMSOL Multiphysics allows users to enter coupled systems of partial differential equations in addition to the conventional physics-based user interfaces. The partial differential equations can either be entered directly or using the so-called weak form.

In this study, an electrically homogeneous half-space is modeled as a 3D point source with the dimension of 0.5 m × 0.5 m × 0.5 m as shown in Fig. 1. The remaining parts of the top surface boundaries were made electrically insulating, that is, $\nabla V \cdot \mathbf{n} = 0$; where $\mathbf{n}$ is the normal to the surface (Wang et al. 2012). The background electrical conductivity (σ_1) of the model is set to $\sigma_1 = 0.01$ S/m (corresponding to a resistivity of 100 Ωm) while the anomaly conductivity, σ_2 is set to be 0.1 S/m (corresponding to a resistivity of 10 Ωm; Fig. 2). An electric current, I, of 0.1 A was injected into the subsurface through the point source.

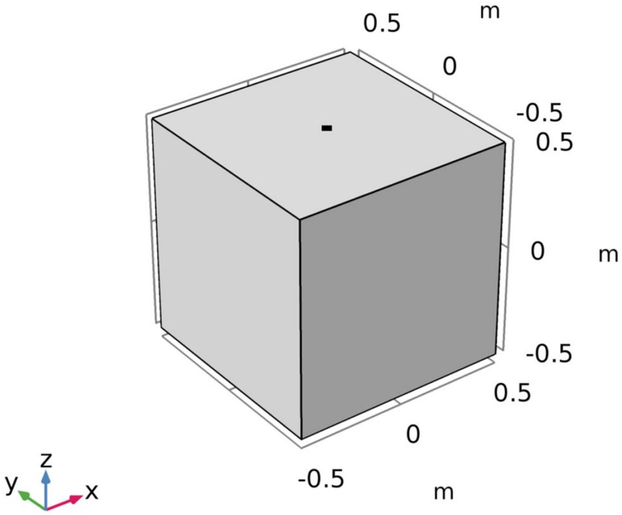

Fig. 1 3D geoelectrical modeling as a block

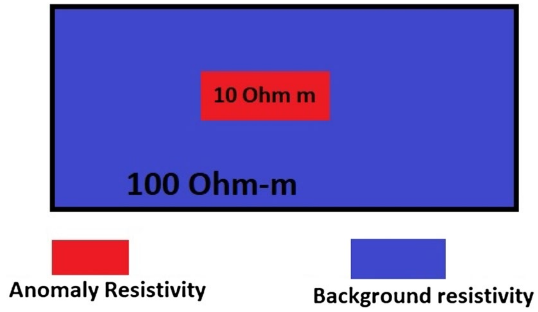

Fig. 2 Sketch of two-layer geoelectric section used for the model building

The point current source was injected at the top of the block (Fig. 1). Dirichlet boundary conditions were defined on all faces except at the top face where Neumann boundary conditions were set (Chave and Jones 2012; Schaa et al. 2016). The procedure employed for the forward modeling in COMSOL is shown in Fig. 3. Generally, COMSOL Multiphysics has three sections: pre-process, solution and post-process (COMSOL Multiphysics Users' Guide 2017). Pre-processing involves finite-element model and parameters setting, while the solution process involves the generation of mesh and solving of equations. The post-processing involves visualization and analysis of results.

The mesh generated around the electrodes for the 3D resistivity model is shown in Fig. 4. The complete mesh consists of 100,938 domain elements, 4694 boundary elements, and 216 edge elements with degrees of freedom of 139,262, and the solution time was 11 s.

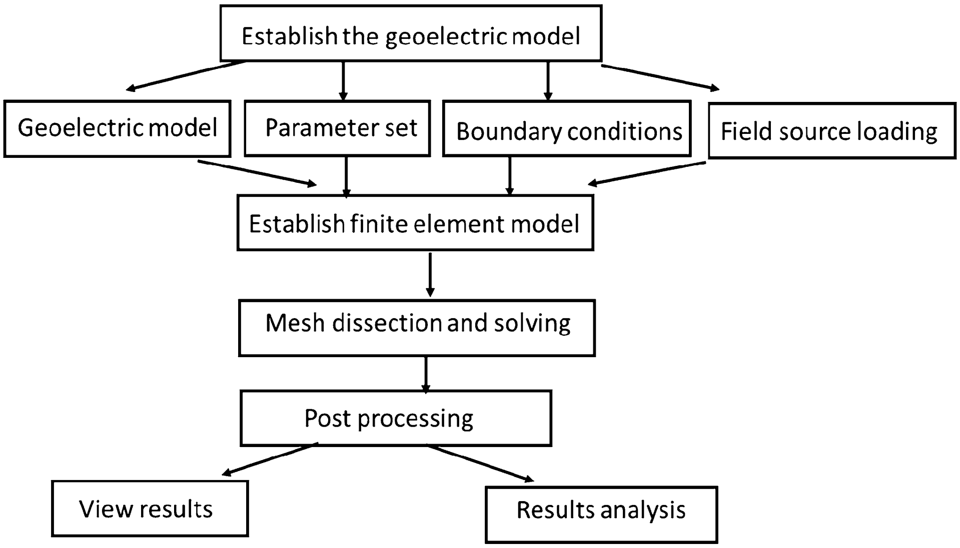

Fig. 3 Chart of COMSOL Multiphysics analysis of geophysical field

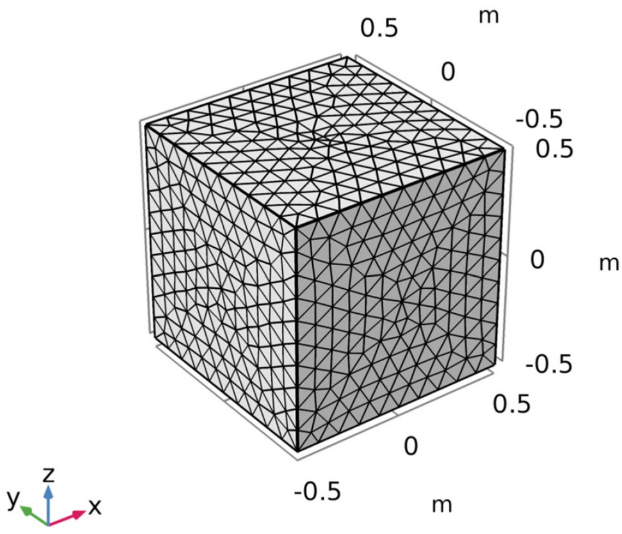

Fig. 4 Mesh generated around the electrodes from the 3D resistivity model

Sensitivity and error analysis

To examine the accuracy of the numerical forward modeling, we compared the electric potentials computed from the numerical modeling with that of the analytic solutions. The analytic expression (V_{ana}) for an infinite half-space of constant conductivity (σ), and one current source of strength, I, at points x, y and z is calculated using Eq. 5:

$$V_{\text{ana}} = \frac{I}{2\pi\sigma r} \tag{5}$$

where r is calculated as given in Eq. 6:

$$r = \sqrt{x^2 + y^2 + (z - 0.5)^2} \tag{6}$$

Error analysis was then performed using Eq. 7:

$$E(\%) = \frac{\left|V - V_{\text{ana}}\right|}{V_{\text{ana}}} \times 100 \tag{7}$$

Results and discussion

By applying COMSOL Multiphysics to the resistivity model shown in Fig. 1 for the 3D forward modeling numerical simulation, the distribution of electric potentials obtained is shown in Fig. 5. It was observed that the potentials (in volts, V) are uniformly distributed in the subsurface as seen in Fig. 5. The values of electrical potentials close to the source vary considerably while at the boundary, the potentials are 0. However, as the current travels to the subsurface, the amount of potentials decreased (Fig. 5). The error analysis in Fig. 6 shows that there are relatively small percentage errors near the source point (0–10%), while the percentage errors became very large near the boundaries (70–90%).

To reduce the percentage errors near the boundaries, we imposed mixed-boundary conditions (Dey and Morrison 1979; Li and Spitzer 2002) on the 3D resistivity model in Fig. 1. After imposing the mixed-boundary conditions, it was observed that the percentage errors near the boundaries were reduced substantially, but the percentage errors near the source point are still very large. To further reduce the percentage errors near the source point, a finer mesh (Fig. 7) was used for the 3D resistivity model (Pain et al. 2002; Rucker et al. 2006). The results of the 3D electric potential in volts (V) and its relative errors in percentage

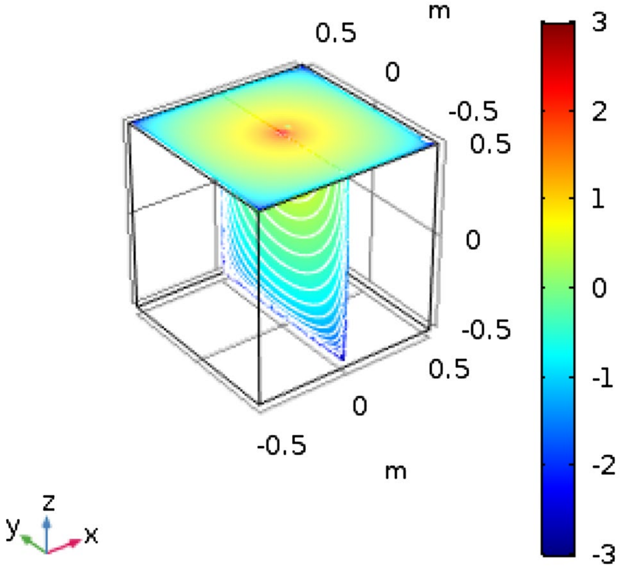

Fig. 5 Electric field distribution of 3D geoelectric model

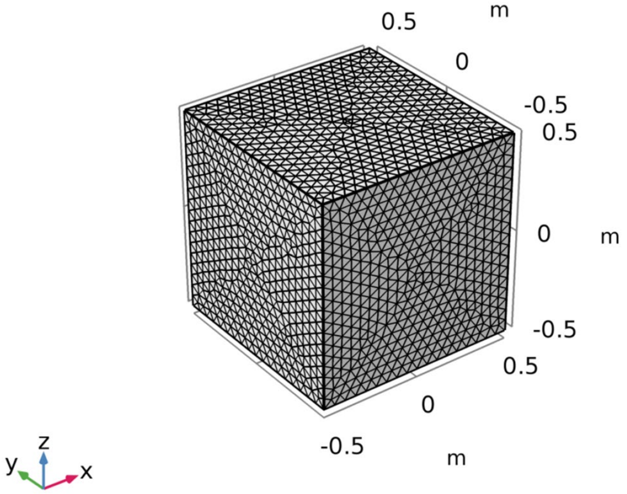

Fig. 7 Finer mesh generated around the electrodes to reduce errors near the point source

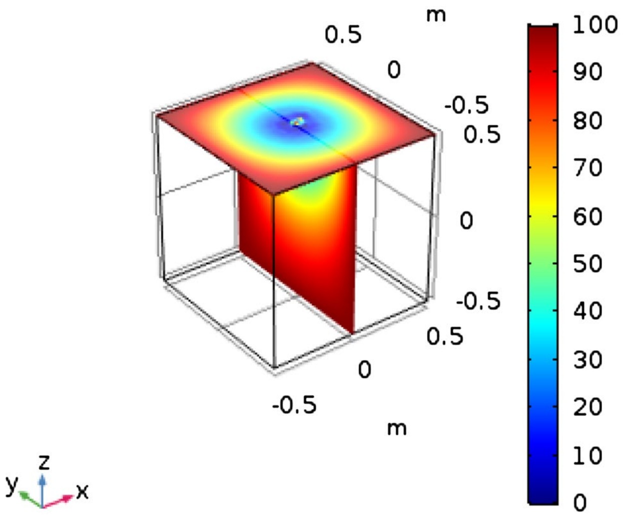

Fig. 6 Error analysis

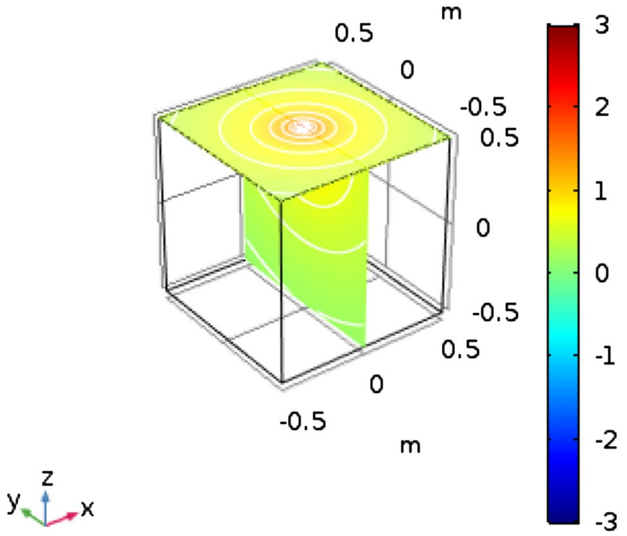

Fig. 8 3D potentials using finer mesh

after error reduction are shown in Figs. 8 and 9, respectively. The potential at the source ranges from 0 to 1.5 V while at the boundary, the potentials are between 0 and 1 V. We observed that the errors at the source and near the boundaries have reduced to about 0–10%.

Sensitivity analysis

To demonstrate the accuracy of the numerical modeling, we compared its output with the calculated analytical solution. The result of the sensitivity analysis is shown in Fig. 10. The electrical potentials from numerical simulation and the analytic solution are almost the same as observed in the figure. This means that both methods are in agreement and this demonstrates the effectiveness of COMSOL for numerical modeling and simulations of electrical potentials in the subsurface.

We estimated the error analysis of this comparison using Eq. 7, and the result is displayed in Fig. 11. The error analysis shows the difference between the numerical modeling and the analytic solutions. The figure reveals that the difference between the two methods is almost 0% at nearly all the locations except near the point source where the error is about 8%. However, at the point source, the error is about 40%. The high percentage of errors close to the point source is acceptable because the potential

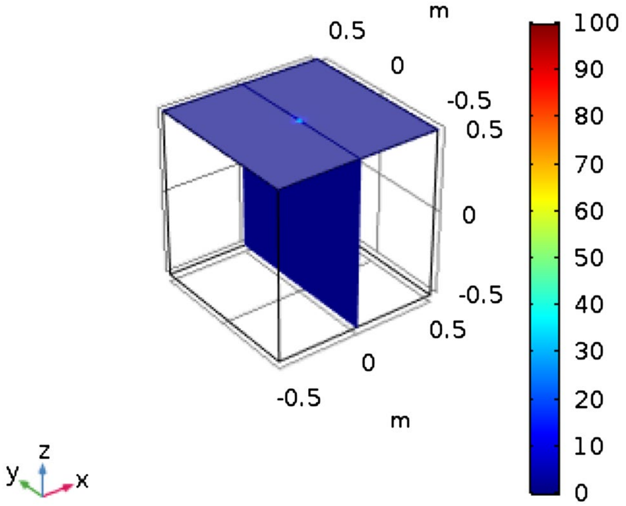

Fig. 9 Relative errors with finer mesh. The error at the source and near the boundaries have substantially reduced

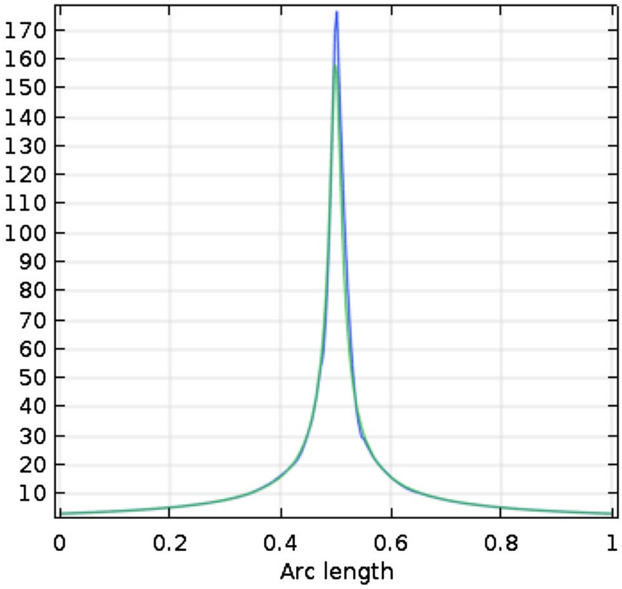

Fig. 10 Comparison of electric potentials (blue color) and the analytic solutions (green colour)

electrodes are not often located close to the point source (Wang et al. 2012).

Conclusion

The results of numerical simulation using COMSOL Multiphysics show the reliability and effectiveness of investigating the DC resistivity method forward modeling. One of the very interesting features of COMSOL is that it is a powerful tool for numerical computation and has the ability to visualize features that are in post-processing stage, which makes forward calculations simple and more informative. Therefore, COMSOL Multiphysics can be used to simulate several DC method forward modeling problems. The ability to ensure boundaries can be placed at infinity using infinite elements in COMSOL is very important in modeling geophysical problems. This study is an initial approach that can be used to enhance the interpretation of electrical resistivity data and can provide information for further studies, such as DC resistivity inversions and sensitivity analyses.

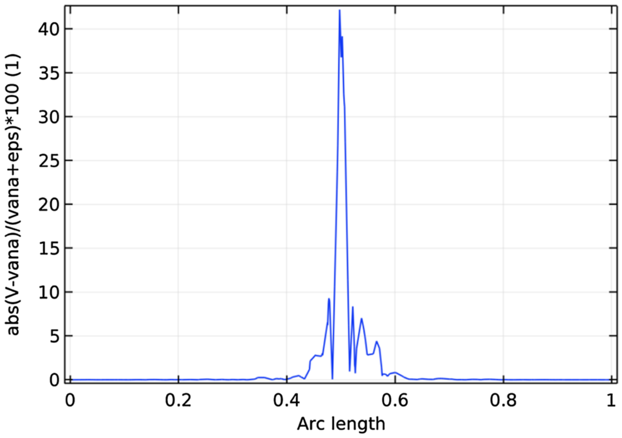

Fig. 11 1D relative errors of the comparison of numerical modeling and analytical solution

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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