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The Extended Block Predictor-Block Corrector Method for Computing Fuzzy Differential Equations

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Abstract: - Over the years, scholars have developed predictor-corrector method to provide estimates for ordinary differential equations (ODEs). Predictor-corrector methods have been reduced to predicting-correcting method with no concern for finding the convergence-criteria for each loop with no suitable vary step size in order to maximize error. This study aim to consider computing fuzzy differential equations employing the extended block predictor-block corrector method (EBP-BCM). The method of interpolation and collocation combined with multinomial power series as the basis function approximation will used. The principal local truncation errors of the block predictor-block corrector method will be utilized to bring forth the convergence criteria to ensure speedy convergence of each iteration thereby maximizing error(s). Thus, these findings will reveal the ability of this technique to speed up the rate of convergence as a result of variegating the step size and to ensure error control. Some examples will solve to showcase the efficiency and accuracy of this technique.

Keywords: fuzzy differential equations; ebp-bcm; principal local truncation errors; converging-criteria; max-error.

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1 Introduction

Fuzzy differential equations are geared towards modelling real life problems with incertitude and fuzziness. For instance, there might be an impossibility to ascertain precisely the initial value and such mapping might comprise of unsettled parametric quantities. This inexactitude extends to the requirement of fuzzy differential equations (FDEs) to surmount the problem. This aspect of fuzzy differential equations takes place in numerous studies like scientific discipline, economic science, psychological science, defense mechanism, human ecology and applied sciences [17], [23], [36] and [39].

Scholars like [9] were the foremost to present fuzzy differential equations. The general idea was unfolded by [11], while authors like [7], [8], [16] and [33] analyze and suggest various essential explanations including the propositions in the fuzzy differential. Others importantly contributed in the areas of H-derivative, Hukuhara differentiable, fuzzy initial value problems (FIVPs) and its generality. See [8], [18], [19] and [34].

Over the years, several research studies on fuzzy differential equations have been executed by scholars adopting numerical methods of single-step methods, multistep methods, predictor-corrector system, Leffler kernel differential operator of the ABC approach, reproducing kernel method (RKM), reproducing kernel Hilbert space method (RKHSM), reproducing kernel theory (RKT) The ideas behind numerical technics are due to the exact results of some FDEs been disadvantage and windy to find out on account of the ramification of the FDEs. Various bookmen talked about and employ the Adams family of predictor-corrector pair for resolving FDEs [4], [5], [7], [12], [13], [14], [17], [20], [36] and [39]. Although, all of these methods resolve to predict and correct the result without a desirable step size to decide the convergence to maximize error with error control. Again, the natures of fuzzy differential equations are exponential in nature with stiff properties. As a result of the incertitude and fuzziness of the differential equations, the numerical technics mentioned earlier lacks the capacity to yield a better result and stabilize the looping via providing a suitable step size to decide the convergence-criteria thereby yielding a better max error. Nevertheless, EBP-BCM has the capacity to vary the step-with same order and utilize a suitable steps size. These ideas used will stabilize the convergence criteria and foster a faster rate of looping with a better max-error. EBP-BCM has an advantage over other numerical methods because of its ability to solve stiff problem using the approach of vary step of the same order and finding a suitable step size. See [1], [2],[22],[23], [24], [27], [28], [29], [30] and [31].

This study is motivated by need to build a block numerical model with high performance ability to solve problem whose analytical solution is exponential in nature with stiffness properties. Thus, EBP-BCM possesses the ability to utilize variable step with same order and suitable step size to resolve exponential solution. This concept will satisfy the convergence criteria and yield lesser max-errors.

The contribution and novelty of this research study will be to introduce the concept of variable step, same order and suitable variable step size as a primal tool to bring about speedy convergence, error control and lesser max-error(s). The convergence of each tolerance is dependent on the ability to determine a suitable step size for each loop process.

The contribution and novelty of the extension of block predictor-block corrector method is the ability to find a suitable step size to satisfy each convergence criteria for every loop of the iteration thereby leading to better efficiency and accuracy. Again, these ideas have been extended to computing the fuzzy differential equations to yield better results compare to others.

This study is coordinated as follows: Introductory explanation and notational systems will be discussed in part 1. Materials and methods of the block extended predictor-block corrector method will be described by fuzzy in part 2. Results and discussion will be established in part 3 and finally, the conclusion will be summarized at the termination of the study in part 4. See [3] and [17].

1.1 Preliminaries

The introductory explanations and notational systems that is very important for this study will be critically appraised in this part. For more action, see [17], [35], [36] and [39].

Definition 1: Consider $\gamma: Rb \to [0,1]$ is a single-valued function number with R as a setup of the entire real numbers. The single-valued function possesses the next attributes:

i. γ is defined as the upper semi continuous,
ii. γ is set as the fuzzy convex, that is $\gamma(\sigma x + (1-\sigma)y) \geq \min\{\gamma(x), \gamma(y)\}, \forall x, y \in R, \gamma \in [0,1]$,
iii. γ is the normal, i. e. there exist $x_0 \in R \ni \gamma(x_0) = 1$,
iv. The stand of γ is sup $\gamma = \{x \in R | \gamma(x) > 0\}$ and its law of closure lcl (supγ) is compact. Assume E is a setup of the entire fuzzy numbers on R See [17], [35], [36] and [39].

Definition 2: Consider a fuzzy number v in parametric pattern is seen as a pair of $(\underline{v}, \overline{v})$ of mappings $\underline{v}(r), \overline{v}(r), 0 \leq r \leq 1$, which meets the next necessities:

i. $\underline{v}(r)$ is a bounded monotonic non-decreasing left uninterrupted mapping.
ii. $\overline{v}(r)$ is a bounded monotonic non increasing left uninterrupted mapping.
iii. $\underline{v}(r) \leq \overline{v}(r), 0 \leq r \leq 1$.

Assume I is a real time interval. A single-valued function $y: I \to E^1$ is denoted as a fuzzy physical process and its $\alpha - level$ set is called by

$$[y(t)]^{\alpha} = \left[\underline{y}(t), \overline{y}(t)\right], \quad t \in I, \alpha \in (0,1]. \quad (1)$$

See [17], [35], [36] and [39].

Definition 3: Consider $B = (a, b, c), a < b < c$ constitute a fuzzy set on $R = (-\infty, \infty)$, since it is referred to triangular fuzzy number assume its membership mapping constituted

$$\rho B(t) = \begin{cases} \frac{t-a}{b-a}, assume\ a \le t \le b \\ \frac{c-t}{c-b}, assume\ b \le t \le c \cdot \\ 0, otherwise. \end{cases}$$
(2)

See [17], [32], [33] and [35].

Definition 4: The triangular fuzzy number is a given fuzzy set $v \in E^1$ that portray the character of an ordered three fold $(x_1, x_2, x_3) \in R^3$ with $x_1 \le x_2 \le x_3 \ni [v]^0 = [x_1, x_3]$ and $[v]^1 = \{x_2\}$, where $\alpha - level$ set of a triangular fuzzy number v is established by

$[v]^\propto = [x_2 - (1-\propto)(x_2 - x_1), x_2 + (1-\propto)(x_3 - x_2)]$, for whenver $\propto \in I$. (3)

See [17], [35], [36] and [39].

Remark 1: Consider $F: T \to E^1$ is seen to possess Hukuhara derivative and its Hukuhara differential coefficient F' is integrable [0,1], in that case

$$F(t) = F(t_0) + \int_{t_0}^{t} F'(s)ds, \forall\ 0 \le t_0 \le t \le 1$$
(4)

Definition 5: A single-valued function$I \to E^1$ is referred to as fuzzy physical process. This is indicated as

$$[y(t)]^\alpha = [y_1^\alpha(t), y_2^\alpha(t)], \quad t \in I, \alpha \in [0,1].$$
(5)

The Seikkala differential coefficient $y'(t)$ of a fuzzy physical process y is set by

$$[y'(t)]^\alpha = [(y_1^\alpha)'(t), (y_2^\alpha)'(t)], \quad \alpha \in [0,1],$$
(6)

with the understanding that this par in reality determines a fuzzy number $y'(t) \in E^1$.

Remark 2: Consider: $y: I \to E^1$ is seen as Seikkala derivative and its Seikkala differential coefficient y' is integrable throughout [0,1], so

$$y(t) = y(t_0) + \int_{t_0}^{t} y'(s)ds, \qquad \forall\ t_0, t \in I$$
(7)

See [17], [34], [35], [36] and [39].

1.2 Fuzzy Differential Equations

For this study, we examine the initial value problem

$$g^n(t) = \emptyset\left(t, g, g', \dots, g^{(n-1)}\right),$$
$$g(0) = b_1, \dots, g^{(n-1)}(0) = b_n$$

where $\emptyset$ is a continuous function of $R^+ \times R^n$ into R and b_i, $0 \le i \le n$ represent fuzzy numbers defined in set E. From above, the n^{th} order fuzzy differential equation will be determined by varying quantities

$y_1(t) = g(t), y_2(t) = g'(t), \dots, y_n(t) = g^{(n-1)}(t)$, which translates to the complying fuzzy system

$$\boldsymbol{y_1'(t) = f_1(t, y_1, \dots, y_n),}$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$\boldsymbol{y_n'(t) = f_n(t, y_1, \dots, y_n),}$$
$$\boldsymbol{y_1(0) = y_1^{(0)} = b_1, \dots, y_n(0) = y_n^{(0)} = b_n,}$$

where $f_i, 1 \le i \le n$ define the continuous function of $R^+ \times R^n$ into R and $\boldsymbol{y_1^{(0)}}$ represent fuzzy numbers in set E with $\propto -$ level intervals.

Again, we examine the next fuzzy initial value problem (FIVP) $y'(t) = f(t, y)$, where y is a fuzzy mapping of t, $f(t, y)$ is a fuzzy mapping of the crisp variable t and the fuzzy physical variable y and y' is called the Hukuhara or Seikkala fuzzy differential coefficient of y. Consider the fuzzy Cauchy problem

$$y'(t) = f(t, y), y(t_0) = y_0, \qquad t_0 \le t \le T$$
(8)

See [35].

The state of the existing theorem is discovered for the Cauchy physical problem (8). See [17], [35], [36] and [39].

Consider $[y(t)]^\alpha = \left[\underline{y}^\alpha(t), \overline{y}^\alpha(t)\right]$; assume $y(t)$ is seen as the Hukuhara derivative, so $[y'(t)]^\alpha = \left[\left(\underline{y}^\alpha(t)\right)', \left(\overline{y}^\alpha(t)\right)'\right]$. Then (8) interprets into the next physical system of ordinary differential equations (ODEs):

$$\left(\underline{y}^\alpha(t)\right)' = \underline{f}^\alpha(t, \underline{y}^\alpha(t), \overline{y}^\alpha(t)),$$
$$\left(\overline{y}^\alpha(t)\right)' = \overline{f}^\alpha(t)(t, \underline{y}^\alpha(t), \overline{y}^\alpha(t)),$$
(9)
$$\underline{y}^\alpha(t_0) = \underline{y}_0^\alpha,$$
$$\overline{y}_0^\alpha(t_0) = \overline{y}_0^\alpha$$ See [17], [35], [36] and [39].

Theorem 1: Consider $(t_i, v_i), i = 0,1, \dots, n$ constitute to ascertain the data and presuppose that for each one of the $v_i = (v_i^1, v_i^2, v_i^3) \in E^1$. So for each one $t \in [t_0, t_n], f(t) = (f^1(t), f^2(t), f^3(t))$ be an element in E^1. This shows that

$$f^1(t) = \sum_{l_i(t) \ge 0} l_i(t) v_i^1 + \sum_{l_i(t) < 0} l_i(t) v_i^3,$$
$$f^2(t) = \sum_{i=0}^{n} l_i(t) v_i^2,$$
(10)

$f^3(t) = \sum_{l_i(t)\geq 0} l_i(t)v_i^3 + \sum_{l_i(t)<0} l_i(t)v_i^1$,
where
$l_i(t) = \prod_{i\neq j}((t-t_j)/(t-t_j)), i = 0,1,\dots,n$
See [17], [35], [36], and [39].

2 Formulating the Block Predictor-Block Corrector Pair

This section considers formulating the block predictor-block corrector pair of the fuzzy differential equations. The choice of the block predictor-block corrector originates from [23] which suggest that the key to greater efficiency and accuracy is the implementation of variable step, variable order and variable step size. This study intends to implement variable step, same order and variable step size. This will be clearly defined in the block predictor-block corrector method.

Note: This aspect characterizes Cauchy fuzzy differential equations
$y(t,r) = [\underline{y}(t,r), \bar{y}(t,r)]$,
$F(t, y(t,r)) =$
$[F(t, \underline{y}(t,r), \bar{y}(t,r), G(t, \underline{y}(t,r), \bar{y}(t,r)]$.

Suppose $y(t,r)$ has differentiability then in that case $y'(t,r) = [\underline{y}'(t,r), \bar{y}'(t,r)]$. Thus, it follows right away
$\underline{y}'(t,r) = \underline{f}(t, y(t), r) = F(t, \underline{y}(t,r), \bar{y}(t,r)$,
$\bar{y}'(t,r) = \bar{f}(t, y(t), r) = G(t, \underline{y}(t,r), \bar{y}(t,r)$,
$\underline{y}(t_0,r) = \underline{y_0}(r), \bar{y}(t_0,r) = \overline{y_0}(r)$.

Again, suppose $y(t,r)$ possess differentiability and then $y'(t,r) = [\underline{y'}(t,r), \bar{y}'(t,r)]$. Thusly, it agrees at once
$\underline{y'}(t,r) = \bar{f}(t, y(t), r) = G(t, \underline{y}(t,r), \bar{y}(t,r)$,
$\bar{y}'(t,r) = \underline{f}(t, y(t), r) = F(t, \underline{y}(t,r), \bar{y}(t,r)$,
$\underline{y}(t_0,r) = \underline{y_0}(r), \bar{y}(t_0,r) = \overline{y_0}(r)$.

Writing the block predictor method in the form of (10) as shown below. Setting y_n as a point of interpolation and f_n, f_{n-1}, f_{n-2} as the given point of collocations. The fuzzy initial values are constituted as y_n, f_n, f_{n-1}, f_{n-2}. The block predictor method is constructed utilizing (x_n, y_n) as interpolation, while $(t_n, f_n), (t_{n-1}, f_{n-1}), (t_{n-2}, f_{n-2})$ represent points of collocation. The evaluated values are (t_{n+1}, y_{n+1}), (t_{n+2}, y_{n+2}), (t_{n+3}, y_{n+3}). Likewise, the block corrector method is developed using $(t_{n-1}, y_{n-1}), (t_{n+1}, f_{n+1}), (t_{n+2}, f_{n+2})$, (t_{n+3}, f_{n+3}) to constitute collocation points. The estimated values are (t_{n+1}, y_{n+1}), (t_{n+2}, y_{n+2}), (t_{n+3}, y_{n+3}). This unification will form

$$y(t) = \sum_{i=1}^{1} \alpha_i y_{m+1-i} + h\sum_{i=1}^{k} \beta_i f_{m+1-i},$$
$$y(t) = \sum_{i=0}^{1} \alpha_i y_{m-1-i} + h_1\sum_{i=0}^{k} \beta_i f_{m+1+i}. \qquad (11)$$

Exploring (10) to interpolation and collocation results to $\bar{y}(t_n), \bar{f}(t_n, y_n), \bar{f}(t_{n-1}, y_{n-1})$, $\bar{f}(x_{n-2}, y_{n-2})$for block predictor method and $\bar{y}(t_{n-1}), \bar{f}(t_{n+1}, y_{n+1}), \bar{f}(t_{n+2}, y_{n+2})$, $\bar{f}(x_{n+3}, y_{n+3})$ for block corrector method. See [17], [26], [27], [28], [29], [30], [31], [34], [35], [36] and [39].

From (10), the fuzzy combination of (11) gives
$f^l(h, t, y(t)) =$
$\sum_{\alpha_j(t)\geq 0}^{1} \alpha_j(t) f^l(t_{m+1-j}, y(t_{m+1-j}))+$
$h\sum_{\beta_j(t)<0}^{k} \beta_j(t) f^r(t_{m+1-j}, y(t_{m+1-j})$
$f^c(t) = \sum_{j=i}^{1} \alpha_j(t) f^c(t_{m+1-j}, y(t_{m+1-j}))$,
$f^r(h, t, y(t)) =$
$\sum_{\alpha_j(t)\geq 0}^{1} \alpha_j(t) f^r(t_{m+1-j}, y(t_{m+1-j}))+$
$h\sum_{\beta_j(t)<0}^{k} \beta_j(t) f^r(t_{m+1-j}, y(t_{m+1-j})$.

Given the basis function approximation
$y(t) = \sum_{i=0}^{k} a_i \left(\frac{t-t_n}{h}\right) \geq 0$,
for $t_{i-1} \leq t \leq t_i$ of the block predictor method, the point of interpolation and collocations are derived as:

$f_{i-2}(t) = \frac{a_1}{h} - \frac{4a_2}{h} + \frac{12a_3}{h} \leq 0$,
$f_{i-1}(t) = \frac{a_1}{h} - \frac{2a_2}{h} + \frac{3a_3}{h} \geq 0$,
$f_i(t) = \frac{a_0}{h} \geq 0$,
$y_i(t) = a_0 \geq 0$.

Similarly, for $t_i \leq t \leq t_{i+1}$ of block corrector method, the point of interpolation and collocations are generated as:

$f_{i+3}(t) = \frac{a_1}{h} - \frac{4a_2}{h} + \frac{12a_3}{h} \leq 0$,
$f_{i+2}(t) = \frac{a_1}{h} - \frac{2a_2}{h} + \frac{3a_3}{h} \geq 0$,
$f_{i+1}(t) = \frac{a_0}{h} \geq 0$,
$y_{i-1}(t) = a_0 \leq 0$.
Consequently, conforming to the above interpolated and collocated results will bring forth the block predictor- block corrector method as

$f^l(t,y(t)) = f_{i-2}(t)f^l(t_{i-2},y(t_{i-2})) + f_{i-1}(t)f^l(t_{i-1},y(t_{i-1})) + f_i(t)f^l(t_i,y(t_i)) + y_i(t)f^l(t_i,y(t_i))$,
$f^c(t,y(t)) = f_{i-2}(t)f^c(t_{i-2},y(t_{i-2})) + f_{i-1}(t)f^c(t_{i-1},y(t_{i-1})) + f_i(t)f^c(t_i,y(t_i)) + y_i(t)f^c(t_i,y(t_i))$,
$f^r(t,y(t)) = f_{i-2}(t)f^r(t_{i-2},y(t_{i-2})) + f_{i-1}(t)f^r(t_{i-1},y(t_{i-1})) + f_i(t)f^r(t_i,y(t_i)) + y_i(t)f^r(t_i,y(t_i))$.
$f^l(t,y(t)) = f_{i+3}(t)f^l(t_{i+3},y(t_{i+3})) + f_{i+2}(t)f^l(t_{i+2},y(t_{i+2})) + f_{i+1}(t)f^l(t_{i+1},y(t_{i+1})) + y_{i-1}(t)f^l(t_{i-1},y(t_{i-1}))$,
$f^c(t,y(t)) = f_{i+3}(t)f^c(t_{i+3},y(t_{i+3})) + f_{i+2}(t)f^c(t_{i+2},y(t_{i+2})) + f_{i+1}(t)f^c(t_{i+1},y(t_{i+1})) + y_{i-1}(t)f^c(t_{i-1},y(t_{i-1}))$,
$f^r(t,y(t)) = f_{i+3}(t)f^r(t_{i+3},y(t_{i+3})) + f_{i+2}(t)f^r(t_{i+2},y(t_{i+2})) + f_{i+1}(t)f^r(t_{i+1},y(t_{i+1})) + y_{i-1}(t)f^r(t_{i-1},y(t_{i-1}))$.

Combining the points of interpolation and collocation derived above will give rise to form $AX = B$. Noting that the block predictor method is $y(t_{n-i}) \approx y_{n-i}$, $y'(t_{n-i}) \approx f_{n-i}$, $i = 0,1,2$ while the block corrector method is $y(t_{n-i}) \approx y_{n-i}$, $f(t_{n+i}) \approx f_{n+i}$, $i = 1,2,3$. See [17], [26], [27], [28], [29], [30], [31], [34], [35], [36] and [39] for more details.

Recalling fuzzy process and Seikkala derivative $y'(t)$ of the fuzzy process is outlined as
$[y(t)]^\propto = [y_1(t), y_2(t)]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t)]^\propto = [(y_1^\propto)'(t), (y_2^\propto)'(t)]$, $t\epsilon I, 0 \leq\propto\leq 1$.
From the above, these results are true for block predictor method:
$[y(t_n)]^\propto = [y_1(t_n), y_2(t_n)]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_n)]^\propto = [(y_1^\propto)'(t_n), (y_2^\propto)'(t_n)]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_{n-1})]^\propto = [(y_1^\propto)'(t_{n-1}), (y_2^\propto)'(t_{n-1})]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_{n-2})]^\propto = [(y_1^\propto)'(t_{n-2}), (y_2^\propto)'(t_{n-2})]$, $t\epsilon I, 0 \leq\propto\leq 1$,
and block corrector method:
$[y(t_{n-1})]^\propto = [y_1(t_{n-1}), y_2(t_{n-1})]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_{n+1})]^\propto = [(y_1^\propto)'(t_{n+1}), (y_2^\propto)'(t_{n+1})]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_{n+2})]^\propto = [(y_1^\propto)'(t_{n+2}), (y_2^\propto)'(t_{n+2})]$, $t\epsilon I, 0 \leq\propto\leq 1$,
$[y'(t_{n+3})]^\propto = [(y_1^\propto)'(t_{n+3}), (y_2^\propto)'(t_{n+3})]$, $t\epsilon I, 0 \leq\propto\leq 1$.

Applying the Hukuhara differentiability of $[y(t)]^\propto = [\underline{y}^\propto(t), \overline{y}^\propto(t)]$ then $[y'(t)]^\propto = [\underline{y}^\propto(t)', (\overline{y}^\propto(t))']$, the following results are acceptable as
$(\underline{y}^\propto(t_n))^\propto = [\underline{y}^\propto(t_n), \overline{y}^\propto(t_n)]$,
$(\underline{y}^\propto(t_n))' = \underline{f}^\propto(t_n, \underline{y}^\propto(t_n), \overline{y}^\propto(t_n))$,
$(\overline{y}^\propto(t_n))' = \overline{f}^\propto(t_n, \underline{y}^\propto(t_n), \overline{y}^\propto(t_n))$,
$(\underline{y}^\propto(t_{n-1}))' = \underline{f}^\propto(t_{n-1}, \underline{y}^\propto(t_{n-1}), \overline{y}^\propto(t_{n-1}))$,
$(\overline{y}^\propto(t_{n-1}))' = \overline{f}^\propto(t_{n-1}, \underline{y}^\propto(t_{n-1}), \overline{y}^\propto(t_{n-1}))$,
$(\underline{y}^\propto(t_{n-2}))' = \underline{f}^\propto(t_{n-2}, \underline{y}^\propto(t_{n-2}), \overline{y}^\propto(t_{n-2}))$,
$(\overline{y}^\propto(t_{n-2}))' = \overline{f}^\propto(t_{n-2}, \underline{y}^\propto(t_{n-2}), \overline{y}^\propto(t_{n-2}))$,
and block corrector method:
$(\underline{y}^\propto(t_{n-1}))^\propto = [\underline{y}^\propto(t_{n-1}), \overline{y}^\propto(t_{n-1})]$,
$(\underline{y}^\propto(t_{n+1}))' = \underline{f}^\propto(t_{n+1}, \underline{y}^\propto(t_{n+1}), \overline{y}^\propto(t_{n+1}))$,
$(\overline{y}^\propto(t_{n+1}))' = \overline{f}^\propto(t_{n+1}, \underline{y}^\propto(t_{n+1}), \overline{y}^\propto(t_{n+1}))$,
$(\underline{y}^\propto(t_{n+2}))' = \underline{f}^\propto(t_{n+2}, \underline{y}^\propto(t_{n+2}), \overline{y}^\propto(t_{n+2}))$,
$(\overline{y}^\propto(t_{n+2}))' = \overline{f}^\propto(t_{n+2}, \underline{y}^\propto(t_{n+2}), \overline{y}^\propto(t_{n+2}))$,
$(\underline{y}^\propto(t_{n+3}))' = \underline{f}^\propto(t_{n+3}, \underline{y}^\propto(t_{n+3}), \overline{y}^\propto(t_{n+3}))$,
$(\overline{y}^\propto(t_{n+3}))' = \overline{f}^\propto(t_{n+3}, \underline{y}^\propto(t_{n+3}), \overline{y}^\propto(t_{n+3}))$. See [17], [24], [25], [26], [27], [28], [29], [31], [32], [33] and [35] for more particulars.

Computing the generated results of Seikkala/Hukuhara differentiability and substituting into the basis function will result to the extended block predictor-block method. By adopting fuzzy Hukuhara differentiability process, the block predictor method for computing fuzzy differential equations is formed as:

$$\begin{aligned}
\underline{y}^\propto(t_{n+1}) &= \underline{y}^\propto(t_n) + \tfrac{h}{12}\Big[23\overline{f}^\propto(t_n, y(t_n)) - 16\overline{f}^\propto(t_{n-1}, y(t_{n-1})) + 5\overline{f}^\propto(t_{n-2}, y(t_{n-2}))\Big],\\
\overline{y}^\propto(t_{n+1}) &= \overline{y}^\propto(t_n) + \tfrac{h}{12}\Big[23\underline{f}^\propto(t_n, y(t_n)) - 16\underline{f}^\propto(t_{n-1}, y(t_{n-1})) + 5\underline{f}^\propto(t_{n-2}, y(t_{n-2}))\Big],\\
\underline{y}^\propto(t_{n+2}) &= \underline{y}^\propto(t_n) + \tfrac{h}{3}\Big[19\overline{f}^\propto(t_n, y(t_n)) - 20\overline{f}^\propto(t_{n-1}, y(t_{n-1})) + 7\overline{f}^\propto(t_{n-2}, y(t_{n-2}))\Big],\\
\overline{y}^\propto(t_{n+2}) &= \overline{y}^\propto(t_n) + \tfrac{h}{3}\Big[19\underline{f}^\propto(t_n, y(t_n)) - 20\underline{f}^\propto(t_{n-1}, y(t_{n-1})) + 7\underline{f}^\propto(t_{n-2}, y(t_{n-2}))\Big],
\end{aligned} \tag{12}$$

$$\underline{y}^\propto(t_{n+3}) = \underline{y}^\propto(t_n) + \tfrac{h}{4}\Big[57\overline{f}^\propto(t_n, y(t_n)) - 72\overline{f}^\propto(t_{n-1}, y(t_{n-1})) + 27\overline{f}^\propto(t_{n-2}, y(t_{n-2}))\Big],$$

$\overline{y}^{\propto}(t_{n+3}) = \overline{y}^{\propto}(t_n) + \frac{h}{12}\Big[57\underline{f}^{\propto}(t_n, y(t_n)) - 72\underline{f}^{\propto}(t_{n-1}, y(t_{n-1})) + 27\underline{f}^{\propto}(t_{n-2}, y(t_{n-2}))\Big].$

In the same manner, the block corrector method is generated as

$$\underline{y}^{\propto}(t_{n+1}) = \underline{y}^{\propto}(t_{n-1}) + \frac{h}{3}\Big[19\overline{f}^{\propto}(t_{n+1}, y(t_{n+1})) - 20\overline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 7\overline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big],$$
$$\overline{y}^{\propto}(t_{n+1}) = \overline{y}^{\propto}(t_{n-1}) + \frac{h}{3}\Big[19\underline{f}^{\propto}(t_{n+1}, y(t_{n+2})) - 20\underline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 7\underline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big],$$
$$\underline{y}^{\propto}(t_{n+2}) = \underline{y}^{\propto}(t_{n-1}) + \frac{h}{4}\Big[27\overline{f}^{\propto}(t_{n+1}, y(t_{n+1})) - 24\overline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 9\overline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big], \quad (13)$$
$$\overline{y}^{\propto}(t_{n+2}) = \overline{y}^{\propto}(t_{n-1}) + \frac{h}{4}\Big[27\underline{f}^{\propto}(t_{n+1}, y(t_{n+2})) - 24\underline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 9\underline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big],$$
$$\underline{y}^{\propto}(t_{n+3}) = \underline{y}^{\propto}(t_{n-1}) + \frac{h}{3}\Big[20\overline{f}^{\propto}(t_{n+1}, y(t_{n+1})) - 16\overline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 8\overline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big],$$
$$\overline{y}^{\propto}(t_{n+3}) = \overline{y}^{\propto}(t_{n-1}) + \frac{h}{3}\Big[20\underline{f}^{\propto}(t_{n+1}, y(t_{n+1})) - 16\underline{f}^{\propto}(t_{n+2}, y(t_{n+2})) + 8\underline{f}^{\propto}(t_{n+3}, y(t_{n+3}))\Big].$$

Equations (12) and (13) are called the extended block predictor- block corrector method of the fuzzy differential equations. See [17], [26], [27], [28], [29], [30], [31], [34], [35], [36] and [39] for more reference.

2.1 Implementing the Convergence-Criteria of the Extended Block Predictor-Block Corrector Method

To drive this process to a logical conclusion, the w-step block predictor method and w-1-step block corrector method must have similar order. See [6], [10], [15], [22], [23], [26], [27], [28], [29], [30] and [31] for more. Linking [6], [10], [15], [22], [23], [26], [27], [28], [29], [30] and [31]. It is executable to determine an approximative principal-local-truncation-error of the extended block predictor-block corrector method voiding loftier derivatives, $y(t)$. Nevertheless, for $pr = cr$, wherepr and crdefines the order of the extended block predictor-block corrector method. Quickly, for method of order $p_r$defines the order and analyzing the block predictor method of $w - step$to generate the principal-local-truncation-errors as

$$\underline{P}^{[1]}_{\underline{pr}+1}(t_{n+1}) - \underline{y}^{\underline{p}_{r+1}}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+1}) - \underline{y}^{[p_1]}_{n+1} + \frac{3}{8} + O(h^{\underline{p}_r+2}),$$
$$\overline{P}^{[1]}_{pr+1}(t_{n+1}) - \overline{y}^{\overline{p}_{r+1}}(t_{n+1}) \approx \overline{y}(\overline{t}_{n+1}) - \overline{y}^{[p_1]}_{n+1} + \frac{3}{8} + O(h^{\overline{p}_r+2}),$$
$$\underline{P}^{[2]}_{\underline{pr}+1}(t_{n+1}) - \underline{y}^{\underline{p}_{r+1}}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+2}) - \underline{y}^{[p_2]}_{n+2} + \frac{8}{3} + O(h^{\underline{p}_r+2}),$$
$$\overline{P}^{[2]}_{pr+4}(t_{n+4}) - \overline{y}^{\overline{p}_{r+4}}(t_{n+4}) \approx \overline{y}(\overline{t}_{n+2}) - \overline{y}^{[p_2]}_{n+2} + \frac{8}{3} + O(h^{\overline{p}_r+5}), \quad (14)$$
$$\underline{P}^{[3]}_{\underline{pr}+1}(t_{n+1}) - \underline{y}^{\underline{p}_{r+1}}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+3}) - \underline{y}^{[p_3]}_{n+3} + \frac{75}{8} + O(h^{\underline{p}_r+2}),$$
$$\overline{P}^{[3]}_{pr+1}(t_{n+1}) - \overline{y}^{\overline{p}_{r+1}}(t_{n+1}) \approx \overline{y}(\overline{t}_{n+3}) - \overline{y}^{[p_3]}_{n+3} + \frac{75}{8} + O(h^{\overline{p}_r+1}).$$

Equation (14) complies with the fuzzy process of Hukuhara differentiability derivatives.

In similar manner, examining the block corrector method of $w - 1 - step$ brings about the principal-local-truncation-error as

$$\underline{C}^{[1]}_{\underline{cr}+1}(t_{n+1}) - \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+1}) - \underline{y}^{[c_1]}_{n+1} - \frac{8}{3} + O(h^{\underline{c}_r+1}),$$
$$\overline{C}^{[1]}_{\overline{cr}+1}(t_{n+1}) - \overline{y}^{\overline{c}_{r+1}[1]}(t_{n+1}) \approx \overline{y}(\overline{t}_{n+1}) - \overline{y}^{[c_1]}_{n+1} - \frac{8}{3} + O(h^{\overline{c}_r+2}),$$
$$\underline{C}^{[2]}_{\underline{cr}+1}(t_{n+1}) - \underline{y}^{\underline{c}_{r+1}[2]}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+2}) - \underline{y}^{[c_2]}_{n+2} - \frac{21}{8} + O(h^{\underline{c}_r+2}), \quad (15)$$
$$\overline{C}^{[2]}_{\overline{cr}+1}(t_{n+1}) - \overline{y}^{\overline{c}_{r+1}[2]}(t_{n+1}) \approx \overline{y}(\overline{t}_{n+2}) - \overline{y}^{[c_2]}_{n+2} - \frac{21}{8} + O(h^{\overline{c}_r+2}),$$
$$\underline{C}^{[3]}_{\underline{cr}+1}(t_{n+1}) - \underline{y}^{\underline{c}_{r+1}[3]}(\underline{t}_{n+1}) \approx \underline{y}(\underline{t}_{n+3}) - \underline{y}^{[c_3]}_{n+1} - \frac{8}{3} + O(h^{\underline{c}_r+2}),$$

$$\overline{C}_{\overline{cr}+1}^{[3]}(t_{n+1}) - \overline{y}^{\overline{c}_{r+1}[3]}(t_{n+1}) \approx \overline{y}(\overline{t}_{n+3}) - \overline{y}_{n+3}^{[c_3]} - \frac{8}{3} + O(h^{\overline{c}_r+2}),$$

where $\underline{P}_{\underline{pr}+1}^{[1]}(t_{n+1}), \overline{P}_{pr+1}^{[1]}(t_{n+1}), \underline{P}_{\underline{pr}+1}^{[2]}(t_{n+1}), \overline{P}_{pr+1}^{[2]}(t_{n+1}), \underline{P}_{\underline{pr}+1}^{[3]}(t_{n+1}), \overline{P}_{pr+1}^{[3]}(t_{n+1}), \underline{C}_{\underline{cr}+1}^{[1]}(t_{n+1}), \overline{C}_{\overline{cr}+1}^{[1]}(t_{n+1}), \underline{C}_{\underline{cr}+1}^{[2]}(t_{n+1}), \overline{C}_{\overline{cr}+1}^{[2]}$ and $\overline{C}_{\overline{cr}+1}^{[3]}(t_{n+1})$ proceeds as the sorted physical quantities of h and $\underline{y}^{\underline{c}_{r+1}}(\underline{t}_{n+1}), \overline{y}^{\overline{c}_{r+1}}(t_{n+1})$ work as the analytical results of the loftier derivatives which function in conformity to the initial assumptions $\underline{y}(t_n) \approx \underline{y}_n,\ \overline{y}(t_n) \approx \overline{y}_n$. See [6], [10], [15], [22], [23], [24], [25], [26], [27], [28], [29], [31], [32], [33], [34] and [35].

In moving forward, for small presumption values of h is ascertained as

$$\underline{y}(t_n) \approx \underline{y}_n, \overline{y}(t_n) \approx \overline{y}_n,$$

where the strength of the extended block predictor-block corrector counts on this presumptuousness submitted before. Further step-down of the principal-local-truncation-errors of (14) and (15) above likewise voiding preconditions of order $O(h^{\underline{p}_r+2})$, $O(h^{\overline{p}_r+2})$. Therefore, this gives no challenge reaching the computational output of the principal-local-truncation errors of the extended block predictor-block corrector method

$$\underline{C}_{\underline{cr}+1}^{[1]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1}) \approx \frac{64}{73}\left[\underline{y}_{n+1}^{[p_1]} - \underline{y}_{n+1}^{[c_1]}\right] \underline{\tau_1}$$

$$\overline{C}_{\overline{cr}+}^{[1]} h^{\overline{cr}+5} \overline{y}^{\overline{cr}+1[1]}(t_{n+1}) \approx \frac{64}{73}\left[\overline{y}_{n+1}^{[p_1]} - \overline{y}_{n+1}^{[c_1]}\right] < \overline{\tau_1}$$

$$\underline{C}_{\underline{cr}+1}^{[2]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1}) \approx \frac{63}{127}\left[\underline{y}_{n+2}^{[p_2]} - \underline{y}_{n+2}^{[c_2]}\right] < \underline{\tau_2} \qquad (16)$$

$$\overline{C}_{\overline{cr}+1}^{[2]} h^{\overline{cr}+1} \overline{y}^{\overline{cr}+1[1]}(t_{n+1}) \approx \frac{63}{127}\left[\overline{y}_{n+2}^{[p_2]} - \overline{y}_{n+2}^{[c_2]}\right] < \overline{\tau_2}$$

$$\underline{C}_{\underline{cr}+1}^{[3]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1}) \approx \frac{64}{289}\left[\underline{y}_{n+3}^{[p_3]} - \underline{y}_{n+3}^{[c_3]}\right] < \underline{\tau_3}$$

$$\overline{C}_{\overline{cr}+1}^{[3]} h^{\overline{cr}+1} \overline{y}^{\overline{cr}+1[1]}(t_{n+1}) \approx \frac{64}{289}\left[\overline{y}_{n+3}^{[p_3]} - \overline{y}_{n+3}^{[c_3]}\right] < \overline{\tau_3}.$$

Concern asserting the existence of the truth that $\underline{y}_{n+1}^{[p_1]} \neq \underline{y}_{n+1}^{[c_1]},\ \underline{y}_{n+2}^{[p_2]} \neq \underline{y}_{n+2}^{[c_2]},\ \underline{y}_{n+3}^{[p_3]} \neq \underline{y}_{n+3}^{[c_3]},\ \overline{y}_{n+1}^{[p_1]} \neq \overline{y}_{n+1}^{[c_1]},\ \overline{y}_{n+2}^{[p_1]} \neq \overline{y}_{n+2}^{[c_1]},\ \overline{y}_{n+3}^{[p_1]} \neq \overline{y}_{n+3}^{[c_1]}$ defines the predicting and correcting approximations instituted via the extended block predictor-block corrector method of order cr, despite $\underline{C}_{\underline{cr}+1}^{[1]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1})$, $\overline{C}_{\overline{cr}+1}^{[1]} h^{\overline{cr}+1} \overline{y}^{\overline{cr}+1[1]}(t_{n+1})$, $\underline{C}_{\underline{cr}+1}^{[2]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1})$, $\overline{C}_{\overline{cr}+1}^{[2]} h^{\overline{cr}+1} \overline{y}^{\overline{cr}+1[1]}(t_{n+1})$, $\underline{C}_{\underline{cr}+1}^{[3]} h^{\underline{c}_r+1} \underline{y}^{\underline{c}_{r+1}[1]}(\underline{t}_{n+1})$ and $\overline{C}_{\overline{cr}+1}^{[3]} h^{\overline{cr}+1} \overline{y}^{\overline{cr}+1[1]}(t_{n+1})$ are distinctively referred to as principal-local-truncation-errors. $\underline{\tau_1}, \overline{\tau_1},\ \underline{\tau_2}, \overline{\tau_2},\ \underline{\tau_3}, \overline{\tau_3}$ are the convergence-criteria of the extended block predictor-block corrector method.

Setting ahead, these approximative of the principal-local-truncation-errors (16) are used to make vital conclusion on the iteration results to either accept or remodel the looping with a smaller step size. The acceptability of the step size is established on an experiment placed by (16). See [6], [10], [15], [22], [23], [26], [27], [28], [29], [30], [31], [34], [35], [36], [38] and [39] for more items.

The principal-local-truncation-errors of the converging-criteria is distinctively the extended block predictor-corrector method aligning to convergence which acts as the stabilizer of the computed results.

3 Numerical Problems of Fuzzy Differential Equations

Two fuzzy differential equations were solved using EBP-BCM. The two problems of fuzzy differential equations are exponential in nature with stiff properties. The stiffness properties of the fuzzy differential equations engenders the use of vary step with same order and suitable variable step size approach. The convergence-criteria of 10^{-3}, 10^{-4}, 10^{-5}, $10^{-6}$and 10^{-7} were utilized. See [16] and [31]. A written computer-code in Mathematica format is carried-out on the platform of Mathematica 9 kernel. See [1], [2], [17] and [36].

Example 1

Study the linear fuzzy differential equations:
$\overline{g}'(t) = -\overline{g}(t) + t + 1, \overline{g}(0) = (0.96,1,1.01), \quad t \geq 0.$

Exact solutions

$\underline{G}(t) = t - 0.025e^{t} + 0.985e^{-t}$,
$\overline{g}^{1}(t) = t + 1.0e^{t}$,
$\overline{G}(t) = t + 0.025e^{t} + 0.985e^{-t}$.

Example 2

Consider un-dimensional fuzzy differential equations:
$g'(t) = g(t), \overline{g}(0) = [0.75 + 0.25r, 1.125 - 0.125r], t, r \in [0,1]$.
Exact solutions
$\underline{G}(t) = t - 0.025e^{t} + 0.985e^{-t}$,
$\overline{g}^{1}(t) = t + 1.0e^{t}$,
$\overline{G}(t) = t + 0.025e^{t} + 0.985e^{-t}$.

4 Results and Discussion

Table I and Table II displays the computed results carried out on two fuzzy differential equations. The implementation is executed on Mathematica 9 Kernel to quicken and easy computation burden in order to demonstrate the potency and proficiency of EBP-BCM. The implementation of the EBP-BCM starts by combining the block predictor-block corrector method together with the converging criteria to find a worthy step size that will bring about speedy convergence and maximize error. Again, to achieve faster convergence with better efficiency and accuracy, a suitable step is found for each EBP-BCM to satisfy the convergence criteria. This process is carried out repeatedly until the convergence criteria are met. Fuzzy differential equations have analytical solutions that are exponential in nature and as such EBP-BCM is required for better results. See [1], [2], [17] and [36]. Table 1 and Table 2 presents the summary as follows

Table 1. Numerical Results for Example 1

M_{utized}	**$Max_{errs}(\underline{Y}(t,r))$**	**C_c**
PC 10^{-3}	0.001112770464	
PC	0.001229801555	
PC	0.001359140914	
EBP-BCM 10^{-3}	0.0000046511	
EBP-BCM	0.00000465854	
EBP-BCM	0.00000466296	
M_{utzed}	***$Max_{errs}(\overline{Y}(t,r))$***	***C_c***
PC 10^{-3}	0.001112770464	
PC	0.001229801555	
PC	0.001359140914	
EBP-BCM 10^{-3}	0.00000425994	
EBP-BCM	0.00000430304	
EBP-BCM	0.00000434646	
M_{utzed}	**$Max_{errs}(Y^{I}(t,r))$**	C_c
PC 10^{-4}	0.000898657928	
PC	0.000813139319	
PC	0.000735758882	
EBP-BCM 10^{-4}	0.000000036403	
EBP-BCM	0.0000000512305	
EBP-BCM	0.0000000701155	
M_{utized}	**$Max_{errs}(\underline{Y}(t,r))$**	**C_c**
PC 10^{-4}	$0.222554092849e-003$	
PC	$0.245960311115e-003$	
PC	$0.271828182845e-003$	
EBP-BCM 10^{-4}	$0.489698e-007$	
EBP-BCM	$0.669954e-007$	
EBP-BCM	$0.895054e-007$	
M_{utzed}	**$Max_{errs}(\overline{Y}(t,r))$**	C_c
PC 10^{-4}	$0.222554092849e-003$	
PC	$0.245960311115e-003$	
PC	$0.271828182845e-003$	
EBP-BCM 10^{-4}	$0.519542e-007$	
EBP-BCM	$0.711323e-007$	
EBP-BCM	$0.951053e-007$	
M_{utzed}	**$Max_{errs}(Y^{I}(t,r))$**	C_c
IPC 10^{-4}	$0.449328964117e-003$	
IPC	$0.406569659740e-003$	
IPC	$0.367879441171e-003$	
EBP-BCM 10^{-4}	$0.93711e-007$	
EBP-BCM	$0.12271e-007$	
EBP-BCM	$0.157846e-007$	

Table 2. Numerical Results for Example 2

M_{utzed}	**$Max_{errs}(\underline{Y}(t,r))$**	C_c
2PDO4 10^{-7}	$9.008840(-7)$	
2PDO4	$9.245915(-7)$	
2PDO4	$9.482990(-7)$	
EBP-BCM	$6.81677(-14)$	10^{-7}
EBP-BCM	$7.57172(-14)$	
EBP-BCM	$8.32667(-14)$	
ERK4[1]	$2.462607(-6)$	10^{-6}
ERK4[1]	$2.345461(-6)$	
ERK4[1]	$1.751478(-6)$	
RK4[1]	$1.985770(-6)$	10^{-6}
RK4[1]	$2.107043(-6)$	
RK4[1]	$1.989897(-6)$	
EBP-BCM	$1.36298(-11)$	10^{-6}
EBP-BCM	$1.39975(-11)$	
EBP-BCM	$1.43732(-11)$	
ANN[1]	$9.726296(-5)$	10^{-5}
ANN[1]	$1.421725(-5)$	
ANN[1]	$5.582846(-5)$	
EBP-BCM	$4.09186(-10)$	10^{-5}
EBP-BCM	$4.13908(-10)$	
EBP-BCM	$4.18671(-10)$	
M_{utzed}	**$Max_{errs}(\overline{Y}(t,r))$**	C_c
2PDO4	$9.601527(-7)$	10^{-7}
2PDO4	$9.601527(-7)$	
2PDO4	$9.482990(-7)$	
EBP-BCM	$6.81677(-14)$	10^{-7}
EBP-BCM	$7.57172(-14)$	
EBP-BCM	$8.32667(-14)$	
ERK4[1]	$9.190771(-7)$	10^{-6}
ERK4[1]	$2.050534(-6)$	
ERK4[1]	$1.751478(-6)$	
RK4[1]	$1.872751(-6)$	10^{-6}
RK4[1]	$2.050534(-6)$	
RK4[1]	$1.989897(-6)$	
EBP-BCM	$6.56208(-12)$	10^{-6}
EBP-BCM	$6.58296(-12)$	
EBP-BCM	$6.60494(-12)$	
ANN[1]	$3.012583(-5)$	10^{-5}
ANN[1]	$1.543513(-5)$	
ANN[1]	$2.817154(-5)$	
EBP-BCM	$4.23477(-10)$	10^{-5}
EBP-BCM	$4.28323(-10)$	
EBP-BCM	$4.33211(-10)$	

4.1 Nomenclature

The word form stated on Table I and Table 2 are defined as follows:

EBP-BCM: calculated-max-errors in EBP- BCM (extended block predictor-block corrector method) for example 1 and 2.

r : represents fuzzy counts having restricted $r-level\ off\ time\ intervals$

M_{utzed} : method put to use.

$Max_{errs}(\underline{Y}(t,r))$: mag. of lower calculated-max-errors in EBP-BCM.

$Max_{errs}(Y^I(t,r))$: mag. of average calculated-max-errors in EBP-BCM.

$Max_{errs}(\overline{Y}(t,r))$: mag. of upper calculated-max-errors in EBP-BCM.

C_c: converging-criteria.

PC (($\underline{Y}(t,r)$): mag. of lower calculated max-errors in PC (predictor-corrector algorithm of 10^{-3},10^{-4}) for example 1. See [36].

PC ($Y^I(t,r)$): mag. of average calculated max-errors in PC (predictor-corrector algorithm of 10^{-3} ,10^{-4}) for example 1. See [36].

PC ($\overline{Y}(t,r)$): mag. of lower calculated max-errors in PC (predictor-corrector algorithm of 10^{-3} ,10^{-4}) for example 1 See [36].

IPC (($\underline{Y}(t,r)$): mag. of lower calculated max-errors in PC (predictor-corrector algorithm of 10^{-3} ,10^{-4}) for example 1. See [36].

IPC ($Y^I(t,r)$): mag. of average calculated max-errors in PC (predictor-corrector algorithm of 10^{-3} ,10^{-4}) for example 1. See [36].

IPC ($\overline{Y}(t,r)$): mag. of lower calculated max-errors in PC (predictor-corrector algorithm of 10^{-3} ,10^{-4}) for example 1. See [36].

ANN(($\underline{Y}(t,r)$): mag. of lower calculated max-errors in

ERK4 (artificial neural network approach of 10^{-5}) for
example 2. See [17].
ANN ($\overline{Y}(t,r)$): mag. of upper calculated max-errors in ERK4
(artificial neural network approach of 10^{-5})for example 2.
See [17].
ERK4(($\underline{Y}(t,r)$): mag. of lower calculated max-errors in
ERK4 (extended Runge-Kutta formulae of order 4 of 10^{-6})
for example 2. See [17].
ERK4 ($\overline{Y}(t,r)$): mag. of upper calculated max-errors in
ERK4 (extended Runge-Kutta formulae oforder 4 of 10^{-6})
for example 2. See [17].
RK4(($\underline{Y}(t,r)$): mag. of lower calculated max-errors in RK4
(Runge-Kutta formulae of order 4 of 10^{-6}) forexample 2.
See [17].
RK4 ($\overline{Y}(t,r)$): mag. of upper calculated max-errors in RK4
(Runge-Kutta formulae of order 4of 10^{-6}) for example 2.
See [17].
2PDO4(($\underline{Y}(t,r)$): mag. of lower calculated max-errors in
2PDO4 (2-point diagonally implicit multistep method of
order four of 10^{-7}) for example 2. See [17].
2PDO4 ($\overline{Y}(t,r)$): mag. of upper calculated max-errors in
2PDO4 (2-point diagonally implicit multistep method of
order four of 10^{-7}) for example 2. See [17].

5 Conclusion

The result presented that EBP-BCM is achievable applying the convergence-criteria. This convergence-criterion resolves the toleration or non-toleration of the loop with suited/variegating step size. The final result shows the functioning and accuracy in terms of the magnitude define as ($\underline{Y}(t,r)$), ($Y^I(t,r)$) and ($\overline{Y}(t,r)$) (lower, average and upper max errors at all levels of 10^{-3} , 10^{-4} , 10^{-5} , 10^{-6} and 10^{-7}. The possibility of this research study is done via working-out a worthier step size that will satisfy the convergence-criteria which will bring about the needed results. Thusly, the EBP-BCM is seen to do better than PC, IPC ANN, ERK4, RK4 and 2PDO compare to others without these vantages stated above. Lastly, this procedure of utilizing the EBP-BCM generates a faster method of looping and stability towards a better accuracy. The Mathematica Kernel is utilized to preserve the time of computation, space and memory.

Future Work:
The future work will be to develop a block predictor-block corrector method of variable step-variable order-suitable variable step size to compute fuzzy differential equations with exponential and trigonometry solutions.

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The authors reiterate that there are no conflicts of interest regarding the publication of this article and among the authors.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Jimevwo Godwin Oghonyon propounded the concept, methodology and executed the problem using Mathematica.

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