

## **Determination of the Optimal Manpower Size Using Linear Programming Model**

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**Abstract:** There would be no meaningful development until manpower that involves in the transformation of production facilities into useful goods and services is well trained and planned. Recent advances in mathematical programming methodology have included: development of interior methods, competing with the simplex method, improved simplex codes, vastly improved performance for mixed-integer programming using strong linear programming formulations and a renewed interest in decomposition. Application areas have been expanding from the traditional refinery planning and distribution models to include finance, scheduling, manufacturing, manpower planning and many others. We see the acceleration of better methods and improved codes moving together with faster, lower-cost and more interesting hardware into a variety of application areas, thereby opening up new demands for greater function of optimization codes. This study applies Linear Programming (LP) model based on integer programming to the determination of effective size of manpower to be engaged. The study also incorporates global constraints such as production capacity/demand rate and allowable time of operation into the model to reflect the reality activities in production organizations in developing countries. The results obtained show that the model is more efficient than the existing model for effective manpower determination.

**Key words:** Manpower planning, Linear/integer programming, methodology, production capacity, optimizations, models

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### **INTRODUCTION**

Manpower is a cardinal factor that determines the level at which production facilities are being used to achieve desired set of organizational objectives. The speed at which these facilities are being transformed to goods and services varies depending on capacity, relationship and achievement in terms of experience of the engaged manpower. Resource availability also influences the categories (low performance and high performance) of manpower necessary at a given time. Sometimes combination of these categories may be required. In this sense, low performance manpower means a competent manpower that can operate at low speed as a result of some factors such as old age, ability, capacity, health condition, etc; while high performance manpower can operate with high speed.

However, one is expected to pay more when high performance staff is engaged than low performance staff and at the same time, more is expected in terms of transformation of resources into useful products from high performance staff than low performance staff. In developing countries, as a result of low capital outlay, there is need to keep the cost of production of goods and services to a minimum possible so that production can continue. This can be possible by employing appropriate quantity of competent manpower when needed based on the available production capacity and time of operation. Competent manpower can be achieved through vocational training and re-training in house and abroad.

Manpower was defined by Armstrong (1996) as human resource used in carrying out jobs on any organization while in Fajana (2002) it was defined as process intended to assure an organization that it would have the proper number of properly qualified and motivated employees in its workforce at some specific future time to carry on the work that would then have to be done.

The formal definition did not state the nature of human resources really needed to carry out desired activity, while the latter specified the capacity, relationship and achievement of the desired manpower. Variation in capacity, relationship and achievement of manpower would definitely influence the cost of manpower.

However, many quantitative techniques have been used to determine effective size of competent manpower to be engaged at a particular time. Among them are: Workload method (Omolayole, 1996; Banjoko, 1996; Folayan, 1998; Fajana, 2002; Fapetu and Kareem, 2003), Maintenance times analysis (Sodiki, 2001), Dynamic programming (Aderoba, 2000), Multiple Activities Process Chart (Lindley, 1988), Queuing System (Aderoba and Kareem, 1997; Kareem and Aderoba, 2000, 2004; Taha, 1987; Rao and Rao, 1993; Shaw, 1976; Aust, 1976; Angel, 1986; Axsater, 1987; Ibaraki, 1978; Fite, 1953). In the above models, equal ability of manpower was assumed under infinite production capacity and time of operation. This study utilizes linear programming model to take care of identified two categories of manpower: low performance and high performance, with production facilities/demand capacity and time of operation constraints are included to satisfy the nature of production facilities in developing countries. Besides, incorporation of integer programming into the LP model ensures holistic determination of manpower size. This eliminates likely errors from round up to the whole number as suggested in the past works enumerated above. The new approach aimed at determining the effective size of manpower to be employed to ensure maximum profit, taking into consideration, production/demand capacity and time of operation.

### **MODEL FORMULATION**

The objective, variables and constraints of the LP model are formulated thus:

The main objective of manpower planning in an organization is to minimize cost or maximize profit subject to available production/demand capacity and time of operation.

Two salient variables are considered: Low performance manpower and high performance manpower. Low performance manpower needed when the production capacity/demand rate is high. Based on this, high performance manpower, termed category-A, would likely get more pay than low performance, termed category-B. This, in turn, would affect their expected revenue.

Let variable  $X_1$  and  $X_2$  be the sizes of the categories-A and B manpower required respectively to carry out a specific task in a given time of operation, while the cost of living a category-A manpower is  $C_1$  and that of category-B is  $C_2$ , with the expected revenues,  $r_1$  and  $r_2$ , respectively, then the objective function can be written as:

$$\begin{aligned} \text{Maximize Profit [P]} &= \text{Total revenue} * \text{cost of operation} \\ &= (R_1 X_1 + r_2 X_2) * (C_1 X_1 + C_2 X_2) \\ &= (R_1 C_1) X_1 + (r_2 - C_2) X_2 \end{aligned} \tag{1}$$

The organization always-engage appropriate size of manpower such that the revenue exceeds cost of manpower.

The two main constraints that are paramount in developing countries are: production capacity and operation duration.

This constraint is based on maximum available time for production activities. If maximum available time for (production activities is denoted as  $b_1$  and the category-A manpower can take  $a_1$  period to flush a task, category-B can take  $a_2$  period to finish a similar task, then, constraint equation becomes:

$$a_1X_1 + a_2X_2 \leq b_1 \quad (2)$$

### **PRODUCTION/DEMAND CAPACITY**

This could be measured based on the maximum quantity the production facilities can produce or maximum demand that can be catered for by the available production facilities  $q_1$  at an available time of operation. If manpower of category-A can produce quantity  $d_1$  while that of category-B can produce  $d_2$  of similar product in operation times  $a_1$  and  $a_2$ , respectively, then the constraint equation will be:

$$D_1X_1 + d_2X_2 \leq q_1 \quad (3)$$

### **NON-NEGATIVE CONSTRAINT**

There is need to satisfy non- negativity condition of LP model

$$X_1, X_2 \geq 0 \quad (4)$$

The formulated LP model could be solved using simplex or graphical methods of solution. LP solution technique like 'branch' and 'bound' method becomes necessary to give holistic solution. This would safeguard unrealistic approximation.

### **SUMMARY OF MATHEMATICAL MODEL**

$$\text{Max. } p = (r_1C_1)X_1 + (r_2C_2)X_2 \quad (5)$$

Subject to:

$$A_1X_1 + a_2X_2 \leq b_1 \quad (6)$$

$$D_1X_1 + d_2X_2 \leq q_1 \quad (7)$$

$$X_1, X_2, 0 \text{ and integer} \quad (8)$$

The steps of solving the above-formulated LP model are as follows:

- Solve the LP model using simplex or graphical method.
- If the values of  $X_1$  and  $X_2$  are whole numbers, go to 8 else 3.
- Set the bound for objective function: upper bound for maximization problem.
- Create sub-problems or subsets by partitioning (branching) the feasible region.
- Solve sub-problems and fathom subset that yields inferior objective function value.
- Partition/branch again selected unfathomed subset into sub-problems.
- If all the nodes have been satisfied, then 8 else 3.
- Stop.

**VALUATION OF THE MODEL**

The model was validated with a maintenance job shop operating in Lagos, Nigeria. The job shop engaged in maintenance of parts of industrial machinery like shafts, engine blocks, injectors, gear systems, etc. The maintenance personnel were grouped into high performance (category-A) and low performance (category-B). Both categories were found to be highly competent in diagnosing various defective parts brought. The data related to production capacity per day, time (h) of operation per day, costs and revenues are stated in Table 1-3.

Using the algorithm above, the LP model can be formulated thus:

**Step 1**

Let  $X_1$  be category-A staff and  $X_2$  be category-B staff.

Objective function is to maximize profit:

$$\begin{aligned} \text{Max. } P &= (5000-3000) X_1 + (3000-2000) X_2 \\ &= 2000X_1 + 1000X_2 \end{aligned} \tag{9}$$

Subject to:

Constraint on operation duration

$$2X_1 + 3X_2 \leq 8 \tag{10}$$

Constraint on production capacity

$$2X_1 + X_2 \leq 5 \tag{11}$$

Non-negativity constraint

$$X_1 \geq 0, X_2 \geq 0 \text{ and integer} \tag{12}$$

**Step 2**

Solve the model as LP using simplex graphical method

$$X_1 = 1.75, X_2 = 1.5, P^* = 5$$

Table 1: Operation time data

Staff category	Operation time (h)
Category-A staff	2
Category-B staff	3
Maximum operation Time (h) day	8

Table 2: Production capacity data

Staff category	Quantity produced/time/staff
Category-A staff	2
Category-B staff	1

Table 3: Costs and revenue of staff

Staff category	Cost/staff	Revenue
Category-A	3000	5000
Category-B	2000	3000

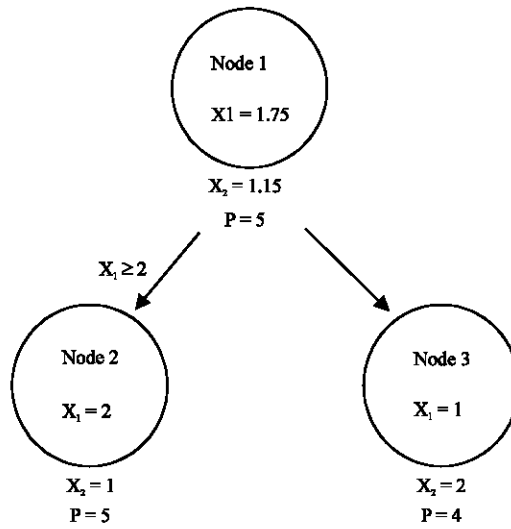


Fig. 1: Branching and Bounding

Since values of  $X_1$  and  $X_2$  are not whole numbers, LP solution is necessary.

**Step 3**

Create subsets or subsystems as follows:

$$X_1 \geq 2 \text{ (Fig. 1)}$$

**Step 4**

Substituting  $X_1 = 2$  into Eq. 2 and 3, it is observed that

Only  $X_2 = 1$  with optimal profit  $P^* = 5$  will satisfy the inequality/boundary condition (node 2).

**Step 5**

Similarly, put  $X_1 = 1$  into Eq. 10 and 11, it can be seen that

Only  $X_2 = 2$  with optimal profit  $P^* = 4$  will satisfy the boundary condition (node 3).

**Step 6**

Search for fractional values. It can be observed that the Optimal point is reached since the values are integers.

**Step 7**

Search for optimal solution. Since the aim is to maximize Profit, values in node 2 (upper bound) seem optimal.

$$X_1^* = 2, X_2^* = 1 \text{ and } P^* = 5$$

**Step 8: Stop**

Table 4: Comparison of results with existing models

Models	Category-A	Category-B	Total optimal staff	Total optimal revenue
LP model	1.75	1.5	2.25	5
IP model	2	1	3	5
Other models	2	2	4	6+

Table 4 shows the comparison of the results obtained using LP method of solution with the existing models for each category of staff. It can be seen in Table 4 that the use of existing models will lead to employment of one additional staff of category-B that would add to the cost of operation. The profit of #6000 obtained through existing models seems unrealistic as constraints on production capacity and operation time are violated.

### CONCLUSION

The study has given insight into how best manpower can be effectively planned so that maximum profit could be realized with available production capacity and operation time. The study is versatile and it can be adapted to all facts of economy such as manufacturing, design, banking, refineries, insurance, etc, to effectively plan for manpower based on available capacity and operation time. The study can be enlarged to take care of more than two constraints. With this enlargement, the model will be more robust and the solution can be got through several iterations using simplex tableau.

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