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A Lotka-Volterra Non-linear Differential Equation Model for Evaluating Tick Parasitism in Canine Populations

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ABSTRACT

This research employs a modified version of the Lotka-Volterra non-linear first-order ordinary differential equations to model and analyze the parasitic impact of ticks on dogs. The analysis reveals that fluctuations in pesticide effects significantly influence tick populations and the size of the canine host. The study also uncovers that alterations in the size of the interacting species can lead to both stable and unstable states. Interestingly, in a pesticide-free environment, a decline in the inter-competition coefficient catalyzes an increase in the sizes of both interacting species. This increase, although marginal for the tick population, contributes to overall system stability. The findings underscore the utility of the Lotka-Volterra non-linear first-order ordinary differential equations in modeling the parasitic effect of ticks on dogs. To protect pets, particularly dogs, from the harmful effects of tick infestation, this study recommends the appropriate and regular application of disinfectants.

1. INTRODUCTION

A mathematical model serves as a system representation, employing a set of variables, parameters, and equations to delineate relationships between the system components. This approach translates issues from an application area into a comprehensible mathematical formulation, facilitating system explanation, component effect studies, and behavior prediction within the system. Numerous mathematical model types, such as Partial Differential Equations (PDE), Integral Equations, dynamical models, statistical models, ordering differential equations, and functional differential equations, can be used to examine the stability of interactions between ticks and dogs. These models enable the identification, characterization, and comparison of dynamic structures within various natural and artificial systems, seeking to elucidate system behavior [1-3].

Parasitism is a relationship in which an organism, the parasite, survives at the expense of another organism, the host. Parasitism is a global health issue in animals, primarily resulting from poor hygiene. Parasites, such as mosquitoes, leeches, ticks, hookworms, and lice, are typically significantly smaller than their hosts, do not immediately kill their hosts, and often reside within their hosts for extended periods [4-6]. Over time, parasites can cause harm to the host, potentially leading to death if not removed.

The Lotka-Volterra equations, a pair of first-order nonlinear differential equations, were designed to model predator-prey relationships and are frequently used to describe the dynamic interactions between predators and prey [7-9]. This study investigates the parasitic relationship between ticks and dogs. Ticks are ectoparasites that attack the body surface and can transmit diseases to humans and animals alike. The cuticle of hard ticks can expand to accommodate the large volumes of blood ingested, which, in adult ticks, can be anywhere from 200 to 600 times their unfed body weight [10, 11].

This research is of considerable importance to scientists, particularly zoologists and veterinarians, as well as dog owners. It examines the effects of ticks on dogs and offers insights into the parasitic relationship between these two species. Given the public health concerns associated with the spread and control of tick-borne diseases, this research is of vital importance. It also investigates a dynamic system of two linear equations that explain tick dynamics to address issues related to the spread of tick-borne diseases in infected dogs.

Studies [12-14] describe the structure, feeding pattern and the biology of Ticks. While studies [15-18] describe the structure, feeding pattern and the biology of dogs.

Research such as those conducted studies [12-14] investigate the structure, feeding patterns, and biology of ticks, while others [15-18] delve into the same aspects for dogs.

Ticks, the second most prevalent blood-feeding parasites after mosquitoes [19, 20], destroy blood cells leading to anemia and are carriers of various Protozoa, Viruses, and Bacteria, which can result in tick-borne diseases (TBDs) [21]. These diseases encompass both emerging and re-emerging infectious diseases. The symptoms of infection typically manifest 2-7 days post the tick bite. However, the onset of paralysis usually requires multiple simultaneous tick bites. Symptoms in the dog may include hind leg weakness and poor coordination, difficulty swallowing, breathing, and chewing, despite the absence of fever or classic signs of illness. The dog may also appear listless and less mobile. If not promptly addressed, respiratory failure can ensue within hours due to the paralysis of chest muscles.

Experimental studies [22-24] have revealed that among the diverse species of ticks infesting dogs, the brown tick (Rhipicephalus sanguineus) is the most widespread. Other relevant works [25-29]. Studies relating to the impact of ticks on dogs can be found in the studies [30-34]. Opanuga et al. [35] and Edeki et al. [36] provided the underlying differential equations which is useful in the current study. Studies [27-39] relate to tick-borne infectious diseases affecting dogs other related studies [40-43]. Another non-linear differential approach was provided by Adesina et al. [44]. Various studies on dogs and human tick-borne infections can be found in studies [45-50]. Agboola et al. [51] presented the solution of third order ordinary differential equations using differential transform method which is relevant to the current study.

The objective of this study is to delineate the relationship between two biological species, ticks and dogs, utilizing a numerical computational scheme predicated on the Lotka-Volterra non-linear first-order ordinary differential equations. Specifically, this research aims to (i) assess the parasitic effect of ticks on dogs, (ii) evaluate the influence of pesticides on system stability, and (iii) analyze the impact of the dog's inter-competition coefficient on the system.

This study intends to augment the existing body of literature in mathematical modeling and computational mathematics, providing insights into the relationship between these two biological species. It seeks to elucidate the mutual effects of these species on each other and the overall impact of the parasite (ticks) on the host (dogs). Additionally, it aims to guide scientists in monitoring the survival of biological species.

2. METHODOLOGY

2.1 Model formation

Considering the relationship between two biological species where one of the species N_1 (ticks) depend on the other species N_2 (dog), the modified system of Lotka-Volterra non-linear first order ordinary differential equations of the form of the Lotka-Volterra logistic model is considered as given [52]:

$$\frac{dN_1}{dt} = a_1 N_1 - a_2 N_1^2 + \alpha N_1 N_2 - \rho_1 N_1 \qquad N1(0) = N10 \geq 0 \tag{1}$$

$$\frac{dN_2}{dt} = b_1 N_2 - b_2 N_2^2 - \beta N_1 N_2 \qquad N2(0) = N20 \geq 0 \tag{2}$$

2.2 Mathematical formulation

Considering the two biological species of Lotka-Volterra logistic model with one species obtaining resource from the other, this situation leads to a relationship between the species causing both species to experience a parasitic interaction. The system above can be clearly explained using non-linear first order differential equation. The parameters in the model are contained in the governing pair of first-order nonlinear differential equations. The parameters sufficiently explain the prey-predator interactions. The parameters are defined as follows:

N_1 is the population size of the first species (ticks).
N_2 is the population size of the second species (dog).
a_1 is the intrinsic growth rate of the first species.
a_2 is the intra-competition coefficient of the first species.
b_1 is the intrinsic effect on the second species.
b_2 is the intra-competition coefficient of the second species.
α is the inter-competition co-efficient of the first species.
β is the inter competition coefficient of the second species.
ρ_1 is the pesticide to inhibit the growth of N_1.

It is imperative to note that both Eqs. (1)-(2) conform with the logistic equation whereby the tick species affects the growth of the second species growth through the parasitic relationship that exist between the two species. ρ_1 represents a control mechanism to inhibit the excessive growth of the first species.

2.3 Determination of the steady state solution

A system is said to reach a steady state or equilibrium when it exhibits no further tendency to change its property over time. That is, if the system is in a steady-state at time to then it will stay there for all times $t \geq t_0$. A detailed definition and mathematical analysis of the concept of steady-state and its stability is reported [52-54]. According to linear stability analysis, a steady-state solution is stable if all the Eigen values of the Jacobins matrix evaluated at that steady state solution have negative real parts. The study [55] is a related ordinary differential equations approach.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$$

For Eq. (1),

$$\frac{dN_1}{dt} = a_1 N_1 - a_2 N_1^2 + \alpha N_1 N_2 - \alpha_1 N_1 \tag{3}$$

Again, from Eq. (2),

$$\frac{dN_2}{dt} = b_1 N_2 - b_2 N_2^2 + \beta N_1 N_2 \tag{4}$$

Since the right-hand side of the equation is not equal to zero, Eq. (1) gives:

$$\left.\begin{array}{c} a_1 N_1 - a_2 N_1^2 + \alpha N_1 N_2 - \alpha_1 N_1 = 0 \\ N_2(a_1 - a_2 N_1 + a N_2 - a_1) = 0 \\ N_1 = 0 \ or \ N_1 = \dfrac{1}{a_2}(a_1 + a N_2 - a_1) \end{array}\right\} \tag{5}$$

Similarly, Eq. (2) gives:

$$\left.\begin{array}{c} b_1 N_2 - b_2 N_2^2 - \beta N_1 N_2 = 0 \\ N_2(b_1 - b_2 N_2 - \beta N_1) = 0 \\ N_2 = 0 \ or \ N_2 = \dfrac{1}{b_2}(b_1 - \beta N_1) \end{array}\right\} \tag{6}$$

Thus, when N_1=0 and N_2=0 is the point (0, 0) which is the trival steady state solution. This implies that both species have gone into extinction.

For N_1=0 and $N_2 \neq 0$, then $N_2 = \frac{1}{b_2}(b_1 + \rho_1) = N_2^*$, therefore $(0, N_2^*)$ is a steady state solution where the second species (Dog) has not been infested yet.

For $N_1 \neq 0 \ and \ N_2 = 0$, then $N_1 = \frac{1}{a_2}(a_1 - \alpha_1) = N_1^*$, also, the above expression gives $(N_1^*, 0)$, which is a steady-state solution where the first species (Ticks) is healthy and the second species has been infested.

For $N_1 \neq 0$ and $N_2 \neq 0$, then $N_1 = \frac{1}{a_2}(a_1 + \alpha N_2 - \alpha_1)$,

$$N_1 = \frac{1}{a_2}\left[a_1 + \alpha(\frac{1}{b_2}(b_1 - \beta N_1)) - \alpha_1\right]$$
$$= \frac{1}{a_2}\left[a_1 - \frac{\alpha b_1}{b_2} - \frac{\alpha b_1 N_1}{b_1} - \alpha_1\right]$$
$$a_2 N_1 = a_1 - \frac{\alpha b_1}{b_2} - \frac{\alpha b_1 N_1}{b_2} - \alpha_1$$
$$a_2 N_1 + \frac{\alpha \beta N_1}{b_2} = \frac{b_2 a_1 - \alpha b_1 - \alpha_1 b_2}{b_2}$$
$$\frac{a_2 b_2 N_2 + \beta N_1}{b_2} = \frac{b_2 a_1 - \alpha b_1 - \alpha_1 b_2}{b_2}$$
$$N_1(a_2 b_2 - \alpha\beta) = a_1 b_2 - \alpha b_1 - \alpha_1 b_2$$

$$N_1 = \frac{1}{a_2 b_2 + \alpha\beta}(a_1 b_2 - \alpha b_1 - \alpha_1 b_2) = N_1^{**} \quad (7)$$

Similarly,

$$N_2 = \frac{1}{b_2}(b_1 - \beta N_1)$$
$$\frac{1}{b_2}[b_1 + \beta(\frac{1}{a_2}(a_1 + \alpha N_2 - \alpha_1))]$$
$$\frac{1}{b2}[b_1 - \frac{\beta a_1}{a_2} - \frac{\alpha\beta N_2}{a_2} + \frac{\beta\alpha_1}{a_2}]$$
$$b_2 N_2 + \frac{\alpha\beta N_2}{a_2} = b_1 - \frac{\beta a_1}{a_2} + \frac{\beta\alpha_1}{a_2}$$
$$a_2 b_2 N_2 + \alpha\beta N_2 = a_2 b_1 - \beta a_1 + \beta\alpha_1$$
$$N_2(a_2 b_2 + \alpha\beta) = a_2 b_1 - \beta a_1 + \beta\alpha_1$$

$$N_2 = \frac{1}{a_2 b_2 + \alpha\beta}(a_2 b_1 - \beta a_1 + \alpha\beta) = N_2^{**} \quad (8)$$

At this point **(N_1^{**}, N_2^{**}),** there is a co-existence of both species.

2.3.1 Characterization of the steady state solution of the interacting function

In characterization of the steady state solution, steady state equation is generalized using state variables in order to obtain Jacobian Matrix elements as given by:

$$a_1 N_1 - a_2 N_1^2 + \alpha N_1 N_2 - \alpha_1 N_1 = 0$$

Let,

$$f(N_1, N_2) = a_1 N_1 - a_2 N_1^2 - \alpha N_1 N_2 - \alpha_1 N_1$$

And,

$$f(N_1, N_2) = b_1 N_2 - b_2 N_2^2 - \beta N_1 N_2 \quad (9)$$

N_1 and N_2 is this instance are state variables. Differentiating the above equations with respect to state variables to obtain Jacobian elements gives:

$$J_{11} = \frac{\partial y}{\partial N_1} = a_1 - 2a_2 N_1 + \alpha N_2 - \alpha_1$$
$$J_{12} = \frac{\partial y}{\partial N_2} = \alpha N_1$$
$$J_{21} = \frac{\partial y}{\partial N_1} = -\beta N_2$$
$$J_{22} = \frac{\partial y}{\partial N_2} = b_1 - 2b_2 N_2 + \beta N_2 - \beta N_1$$

At the trivial steady state solution (0, 0),

$$J_{11} = a_1 - 2a_2(0) + \alpha(0) - \alpha_1 = a_1 - \alpha_1$$
$$J_{12} = \alpha(o) = 0$$
$$J_{21} = -\beta(0) = 0$$
$$J_{22} = b_1 - 2b_2(0) + \beta(0) - \beta(0) = b_1$$

The Jacobian matrix becomes,

$$J_1 = \begin{bmatrix} a_1 - \alpha_1 & 0 \\ 0 & b_1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The characteristic equation is,

$$Det\ (J_1 - \lambda I) = 0$$
$$\begin{vmatrix} a_1 - \alpha_1 - \lambda & 0 \\ 0 & b_1 - \lambda \end{vmatrix} = 0$$
$$a_1 - \alpha_1 - \lambda = 0 \text{ and } b_1 - \lambda = 0$$
$$\lambda_1 = a_1 - \alpha \text{ and } \lambda_2 = b_1$$

Therefore, $\lambda_1 = a_1 - \alpha_1$ and $\lambda_2 = b_1$ are the eigenvalues. The trivial steady state solution is unstable since both eigenvalues are positive.

At the trivial steady state $\left(0, \frac{b_1}{b_2}\right)$,

$$J_{11} = a_1 - 2a_2 N_1 + \rho N_2 - \rho_1$$
$$a_1 + \rho N_2 - \rho_1$$
$$a_1 + \rho\left(\frac{b_1}{b_2}\right) - \rho_1$$
$$a_1 + \frac{\rho b_1}{b_2} - \rho_1$$
$$J_{12} = \rho N_1 = 0$$
$$J_{21} = -\beta N_2 = -\beta\left(\frac{b_1}{b_2}\right) = \left(\frac{-\beta b_1}{b_2}\right)$$
$$J_{22} = b_1 - 2b_2 N_2 + \beta N_2 - \beta N_1 = b_1 - 2b_2\left(\frac{b_1}{b_2}\right)$$
$$= b_1 - 2b_1 = -b_1$$

The Jacobian matrix is,

$$J_2 = \begin{bmatrix} a_1 - \frac{\alpha b_1}{b_2} & 0 \\ \frac{\beta b_1}{b_2} & -b_1 \end{bmatrix}$$

The characteristic equation is,

$$Det(J_2 - \lambda I) = \begin{vmatrix} a_1 - \frac{\alpha b_1}{b_2} - \rho_1 - \lambda & 0 \\ \frac{\beta b_1}{b_2} & -b_1 - \lambda \end{vmatrix} = 0$$
$$a_1 - \frac{\alpha b_1}{b_2} - \rho_1 - \lambda = 0$$

$$\lambda_1 = a_1 - \frac{\rho b_1}{b_2} - \rho_1 \quad (10)$$

And,

$$-b_1 - \lambda = 0$$
$$\lambda_2 = -b_1 \quad (11)$$

The steady state at $(0, \frac{b_1}{b_2})$ is unstable since the eigenvalues are positive and negative.

At the trivial steady state $\left(\frac{a_1-\rho_1}{a_2}, 0\right)$,

$$J_{11} = a_1 - 2a_2\left[\frac{a_1-\rho_1}{a_2}\right] + \rho(0) - \rho_1$$
$$= a_1 - 2a_1 + 2\rho_1 - \rho_1$$
$$= -a_1 + \rho_1$$
$$J_{12} = \rho\left[\frac{a_1-\rho_1}{a_2}\right]$$
$$J_{21} = -\beta(0) = 0$$
$$J_{22} = b_1 - 2b_2(0) + \beta\left[\frac{a_1-\rho_1}{a_2}\right]$$
$$b_1 - \beta\left[\frac{a_1-\rho_1}{a_2}\right]$$

The Jacobian matrix is:

$$J_3 = \begin{bmatrix} \rho_1 - a_1 & \rho\left(\frac{a_1-\alpha_1}{a_2}\right) \\ 0 & b_1 - \beta\left(\frac{a_1-\rho}{a_2}\right) \end{bmatrix}$$

The characteristic equation is:

$$Det(J_3 - \lambda I) = \begin{vmatrix} \alpha_1 - a_1 - \lambda & \rho\left(\frac{a_1-\alpha_1}{a_2}\right) \\ 0 & b_1 - \beta\left(\frac{a_1-\alpha_1}{a_2}\right) - \lambda \end{vmatrix} = 0$$
$$= \rho_1 - a_1 - \lambda = 0, \lambda_1 = \rho_1 - a_1$$

And,

$$\lambda_2 = b_1 - \beta\left[\frac{a_1-\alpha_1}{a_2}\right] - \lambda \tag{12}$$

Considering the eigenvalues which are positive, this means that the steady state solution is unstable.

2.4 Method of solution

The numerical simulation was conducted using MATLAB software and the programming language provided in the package (oDE45) with reference to the numerical system of Eqs. (1)-(2). Following the procedure outlined [52], the following parameters were obtained a_1=5, a_2=0.22, α=0.007, b_1=3, b_2=0.26, β=0.008 while the values of ρ_1=3.5, β_1=1.4 are randomly selected. N_1 and N_2 are obtained based on Eqs. (7)-(8), λ_1 and λ_2 are obtained based on Eq. (10) and Eq. (12) respectively. We present the simulation scheme based on the Eqs. (1)-(12) in Table 1, as follows:

Table 1. Simulation scheme

Case	Effect	On
1	$-\Delta\rho_1$	N_1, N_2
2	$+\Delta\rho_1 n$	N_1, N_2
3	$-\Delta\rho_1$	λ_1 and λ_2
4	$+\Delta\rho_1$	λ_1 and λ_2
5	$-\Delta N_1$	λ_1 and λ_2
6	$-\Delta N_2$	λ_1 and λ_2
7	$(-\Delta N_1\ and\ -\Delta N_2)$	λ_1 and λ_2
8	β	N_1, N_2
	β	λ_1 and λ_2

where,

$-\Delta\rho_1$ is the decrease in pesticides
$+\Delta\rho_1$ is the increase in pesticides
$-\Delta N_1$ is decrease in ticks population
$-\Delta N_2$ is decrease in ticks population
β is inter-competition of the 2nd species

Given the parameters, the study seeks to obtain the results outlined in Table 1 as follows:

(i) the impact of decrease in pesticide $-\Delta\rho_1$ on ticks size of ticks, N_1 and dog size N_2 is sought.
(ii) the impact of increase in pesticide, $+\Delta\rho_1$, on ticks size of ticks, N_1 and dog size N_2 is sought
(iii) the impact of $(-\Delta\rho_1)$ on tick on the stability λ_1 and λ_2 of the system.
(iv) the impact of $(+\Delta\rho_1)$ on tick on the stability λ_1 and λ_2 of the system is sought.
(v) the effect of decreasing the tick's population $(-\Delta N_1)$ on the stability λ_1 and λ_2 of the system.
(vi) the effect of decreasing the dog's population$(-\Delta N_2)$ on the stability λ_1 and λ_2 of the system is sought.
(vii) the effect of simultaneously decreasing the population of both species $(-\Delta N_1\ and\ -\Delta N_2)$ on the stability λ_1 and λ_2 of the system is sought.
(viii) the effect of the inter-competition of the 2nd species (β), on the population of competing species, N_1 and N_2.
(ix) the effect of the inter-competition of the 2nd species (β), on the stability λ_1 and λ_2 of the system is sought.

3. RESULTS

Table 2 shows that as the volume of pesticide increase, the number of ticks increase, and the number of dogs increases. By implication, the mortality rate of dogs decreases.

Table 3 shows that the increase in volume of pesticides, results in a significant decrease in the size of the ticks, and a resultant gradual increase in the size of the dogs.

Table 4 shows that decreasing the impact of pesticide on ticks' results in a stable dynamical system, by implication, there wouldn't be increase without bound in the number either dog or tick in a given dynamical ecological system.

Table 2. Impact of decreasing the effects of pesticides, ρ_1, on the populations of competing species, N_1 and N_2

$+\Delta\rho_1$	N_1	N_2
3.5	7.1783	11.3167
3.3250	7.9730	11.2921
3.1500	8.7677	11.2675
2.9750	9.5624	11.2430
2.8000	10.3572	11.2184
2.6250	11.1521	11.1930
2.4500	11.9470	11.1688
2.2750	12.7430	11.1451
2.1000	13.5447	11.1282
1.9250	14.3316	11.0980
1.7500	15.1297	11.0741
1.5750	15.9203	11.0487
1.4000	16.7150	11.0242
1.2250	17.5103	10.9998
1.0500	18.3059	10.9754
0.8750	19.0996	10.9509
0.7000	19.8946	10.9265
0.5250	20.6899	10.9020
0.3500	21.4857	10.8776
0.1750	22.2805	10.8532

Table 3. Impact of increasing the effects of pesticides, ρ_1, on the populations of competing species, N_1 and N_2

$+\Delta\rho_1$	N_1	N_2
3.5	7.1783	11.3167
3.6756	6.3836	11.3413
3.8500	5.5890	11.3658
4.0250	4.7943	11.3903
4.3750	3.2049	11.4392
4.5500	2.4101	11.4636
4.7250	1.6144	11.4881
4.9000	0.8218	11.5124
5.0750	0.1925	11.5316
5.2500	0.0140	11.5371
5.4250	0.0006	11.5376
5.6000	1.9570×10^{-5}	11.5376
5.7750	6.3650×10^{-7}	11.5369
5.9500	2.0995×10^{-8}	11.5382
6.1250	1.0935×10^{-9}	11.5379
6.3000	1.4951×10^{-10}	11.5371
6.4750	1.2559×10^{-11}	11.5355
6.6500	1.4784×10^{-11}	11.5362
6.8250	1.1880×10^{-12}	11.5348
7.0000	6.4500×10^{-13}	11.5378

Table 4. Impact of decreasing the effects of pesticides, ρ_1, on the stability of the system (ToS)

$-\Delta\rho_1$	λ_1	λ_2	ToS
3.5	-1.7415	-2.9383	Stable
3.3250	-1.9171	-2.9307	Stable
3.1500	-2.0933	-2.9226	Stable
2.9750	-2.2704	-2.9136	Stable
2.8000	-2.4496	-2.9025	Stable
2.6250	-2.6358	-2.8841	Stable
2.4500	-2.8441	-2.8441	Stable
2.2750	-2.9287	-2.9287	Stable
2.1000	-3.0954	-2.9371	Stable
1.9250	-3.2864	-2.9078	Stable
1.7500	-3.4686	-2.8955	Stable
1.5750	-3.6445	-2.8855	Stable
1.4000	-3.8209	-2.8771	Stable
1.2250	-3.9971	-2.8695	Stable
1.0500	-4.1729	-2.8622	Stable
0.8750	-4.3477	-2.8551	Stable
0.7000	-4.5229	-2.8481	Stable
0.5250	-4.6981	-2.8413	Stable
0.3500	-4.8734	-2.8346	Stable
0.1750	-5.0483	-2.8280	Stable

Table 5 is a replica of Table 4, which shows that increasing the impact of pesticide on ticks' results in a stable dynamical system, by implication, there wouldn't be increase without bound in the number either dog or tick in a given dynamical ecological system. This shows that variations in the effects of pesticide while other model parameters are fixed results in a stable system.

Table 6 shows that as N_1 decreases, the dynamical system is stable to a point, util it gets to a point when it becomes progressively unstable as N_1 further approach zero.

Table 7 shows that as N_1 decreases, the dynamical system is stable to a point, until it gets to a point when it becomes progressively unstable as N_1 further approach zero.

In Table 8, a simultaneous decrease in the size of interacting species N_1 and N_2, the dynamical system is stable to a point, until it gets to a point when it becomes progressively unstable as N_1 further approach zero.

Table 5. Impact of increasing the effects of pesticides, ρ_1, on the stability of the system

$+\Delta\rho_1$	λ_1	λ_2	ToS
3.5	-1.7415	-2.9383	Stable
3.6750	-1.5661	-2.9456	Stable
3.8500	-1.3910	-2.9526	Stable
4.0250	-1.2160	-2.9595	Stable
4.3750	-0.8662	-2.9730	Stable
4.5500	-0.6914	-2.9797	Stable
4.7250	-0.5162	-2.9863	Stable
4.9000	-0.3424	-2.9928	Stable
5.0750	-0.2404	-2.9979	Stable
5.2500	-0.3369	-2.9994	Stable
5.4250	-0.5060	-2.9995	Stable
5.6000	-0.6808	-2.9995	Stable
5.7750	-0.8558	-2.9992	Stable
5.9500	-1.0308	-2.9998	Stable
6.1250	-1.2058	-3.0000	Stable
6.3000	-1.3808	-2.9993	Stable
6.4750	-1.5557	-2.9985	Stable
6.6500	-1.7308	-2.9988	Stable
6.8250	-1.9057	-2.9981	Stable
7.0000	-2.0808	-2.9996	Stable

Table 6. The effect of decreasing the population of ticks on the stability of the system

$-\Delta N_1$	λ_1	λ_2	ToS
7.1783	-1.7415	-2.9383	Stable
6.8194	-1.5829	-2.9361	Stable
6.4605	-1.4245	-2.9337	Stable
6.1016	-1.2662	-2.9312	Stable
5.7426	-1.1080	-2.9286	Stable
5.3837	-0.9498	-2.9260	Stable
5.0245	-0.7916	-2.9234	Stable
4.6659	-0.6335	-2.9207	Stable
4.3070	-0.4754	-2.9180	Stable
3.9481	-0.3173	-2.9153	Stable
3.5892	-0.1593	-2.9126	Stable
3.2302	-0.0012	-2.9298	Stable
2.8713	0.1568	-2.9071	Unstable
2.5124	0.3148	-2.9043	Unstable
2.1535	0.4728	-2.9015	Unstable
1.7946	0.6308	-2.8987	Unstable
1.4357	0.7888	-2.8959	Unstable
1.0767	0.9468	-2.8931	Unstable
0.7178	1.1048	-2.8903	Unstable
0.3589	1.2628	-2.8875	Unstable

Table 7. The effect of decreasing the population of Dog, N_2, on the stability of the system

$-\Delta N_2$	λ_1	λ_2	ToS
11.3167	-1.7415	-2.9383	Stable
10.7509	-1.7385	-2.6431	Stable
10.1850	-1.7364	-2.3470	Stable
9.6192	-1.7378	-2.0474	Stable
9.0534	-1.7435	-1.7435	Stable
8.4875	-1.7032	-1.4856	Stable
7.9217	-1.7079	-1.1827	Stable
7.3559	-1.7064	-0.8861	Stable
6.7900	-1.7035	-0.5907	Stable
6.2242	-1.7002	-0.2958	Stable
5.6584	-1.6967	-0.0011	Stable
5.0925	-1.6931	0.2934	Unstable
4.5267	-1.6893	0.5879	Unstable
3.9608	-1.6856	0.8823	Unstable
3.3950	-1.6817	1.1767	Unstable
2.8292	-1.6779	1.4710	Unstable
2.2633	-1.6740	1.7654	Unstable

$-\Delta N_2$	λ_1	λ_2	ToS
1.6975	-1.6702	2.0597	Unstable
1.1317	-1.6663	2.3540	Unstable
0.5658	-1.6624	2.6483	Unstable

Table 8. The effect of simultaneously decreasing the population of both species, N_1 and N_2, on the stability of the system

$-\Delta N_1$	λ_1	λ_2	ToS	ToS
7.1783	11.3167	-1.7415	-2.9383	Stable
6.8194	10.750	-1.5796	-2.6412	Stable
6.4605	10.185	-1.4179	-2.3439	Stable
6.1016	9.6192	-1.2562	-2.0467	Stable
5.7426	9.0534	-1.0946	-1.7493	Stable
5.3837	8.4875	-0.9331	-1.4517	Stable
5.0248	7.9217	-0.7721	-1.1537	Stable
4.6659	7.3559	-0.6122	-0.8547	Stable
4.3070	6.7900	-0.4578	-0.5500	Stable
3.9481	6.2242	-0.2744	-0.2843	Stable
3.5892	5.6584	-0.1107	0.0208	Unstable
3.2302	5.0925	0.0463	0.3228	Unstable
2.8713	4.5267	0.2067	0.6214	Unstable
2.5124	3.9608	0.3678	0.9193	Unstable
2.1535	3.3950	0.5293	1.2168	Unstable
1.7946	2.8292	0.6909	1.5141	Unstable
1.4357	2.2633	0.8527	1.8114	Unstable
1.0767	1.6975	1.0144	2.1086	Unstable
0.7178	1.1317	1.1763	2.4058	Unstable
0.3589	0.5658	1.3381	2.7029	Unstable

Table 9 shows that in other a decrease in β, increases the population of both species, with the increment more significant in N_2.

Table 10 shows that a decrease in β, results in a stable system as both eigenvalues, λ_1 and λ_2 , are negative. According to the linear stability analysis a dynamical system is stable if all the Eigen values of the Jacobian matrix are negative. But if one of the Eigen values is positive the system is unstable.

Table 9. Evaluating the effect of the inter-competition of the 2nd species (β), on the population of competing species, N_1 and N_2

β	N_1	N_2
0.0080	23.0841	10.8290
0.0076	23.0854	10.8645
0.0072	23.0866	10.8999
0.0068	23.0878	10.9354
0.0064	23.0890	10.9708
0.0060	23.0816	11.0060
0.0056	23.0829	11.0415
0.0052	23.0841	11.0770
0.0048	23.0845	11.1125
0.0044	23.0867	11.1479
0.0040	23.0879	11.1834
0.0036	23.0892	11.2189
0.0032	23.0905	11.2544
0.0028	23.0917	11.2899
0.0024	23.0930	11.3254
0.0020	23.0943	11.3609
0.0016	23.0956	11.3964
0.0012	23.0969	11.4319
0.0008	23.0982	11.4674
0.0004	23.0995	11.5029

Table 10. Evaluating the effect of the inter-competition of the 2nd species (β), on the stability of the system

β	λ_1	λ_2	ToS
0.0080	-5.2270	-2.8216	Stable
0.0076	-5.2281	-2.8305	Stable
0.0072	-5.2291	-2.8395	Stable
0.0068	-5.2301	-2.8484	Stable
0.0064	-5.2312	-2.8574	Stable
0.0060	-5.2284	-2.8661	Stable
0.0056	-5.2295	-2.8751	Stable
0.0052	-5.2306	-2.8840	Stable
0.0048	-5.2317	-2.8930	Stable
0.0044	-5.2328	-2.9019	Stable
0.0040	-5.2339	-2.9108	Stable
0.0036	-5.2350	-2.9198	Stable
0.0032	-5.2361	-2.9287	Stable
0.0028	-5.2372	-2.9376	Stable
0.0024	-5.2383	-2.9466	Stable
0.0020	-5.2394	-2.9555	Stable
0.0016	-5.2405	-2.9644	Stable
0.0012	-5.2417	-2.9733	Stable
0.0008	-5.2428	-2.9935	Stable
0.0004	-5.2440	-2.9911	Stable

4. CONCLUSIONS

This study underscores the importance of prompt tick identification and treatment in dogs, bringing to light the severe consequences of unchecked tick infestations. Utilizing a system of nonlinear first-order differential equations, we explored the intricate dynamics between these two biological species.

For future research, we recommend an extension of this work using a system of second-order differential equations. This could potentially provide deeper insights into the more complex interactions and dynamics that characterize this parasitic relationship. Beyond this, there may be a wealth of other parameters, such as environmental factors, the host's health status, or the specific species of ticks involved, that could influence the population dynamics of the interacting species. These parameters could be the focus of future investigations.

Moreover, exploring the effects of the competition coefficient on the populations of biological species might provide valuable information. For example, how does the presence of other parasites or potential hosts in the environment influence the tick-dog interaction? Could a higher competition coefficient lead to a decrease in tick populations, thereby reducing the risk for dogs?

Finally, future studies may consider conducting clinical trials to validate and extend the findings of this study. Real-world testing could provide a more comprehensive understanding of the practical implications of our theoretical models, helping to bridge the gap between mathematical modeling and veterinary practice.

In conclusion, this study contributes to the existing body of knowledge by shedding light on the adverse effects of tick infestations in dogs and offering a mathematical model to understand the dynamics of such parasitic relationships. We believe the pathways we have highlighted for future research will pave the way for more comprehensive investigations, ultimately benefiting both veterinary science and the welfare of animals.

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