DESIGN OF MULTIPLE DEPENDENT STATE SAMPLING PLAN USING ZECH DISTRIBUTION WITH APPLICATION TO REAL LIFE DATA

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Abstract

In this work, a multiple dependent state sampling plan, which is an inspection procedure that determines whether an attribute is conforming or non-conforming to a specific requirement, in which the decision criterion for each lot dictates whether to accept the lot; reject the lot; or conditionally accept or reject the lot based on the disposition of future related lots, is introduced. This plan has some advantages over other acceptance sampling plans, like increased efficiency, improved ability to discriminate between acceptable and non – acceptable lots or batches, flexibility in designing the sampling process, and improved cost-effectiveness. To reject a lot, the plan made use of the properties of the sampled current and preceding lots. The study aims to reduce the average sampling number by using a non-linear optimization problem that is subjected to some constraints. With regards to a life test that is truncated in time, the product's median life was used for the proposed sampling plan assuming that the lifetime of the product follows Zech distribution. The usage of median life was necessitated because Zech distribution is an asymmetric distribution with longer tail to the right. Two points on the operating characteristic curve were used for the proposed sampling plan and the following parameters were found; number of preceding lots which is required for deciding if the current lot should be accepted or rejected, the size of the sample, rejection number, and acceptance number. For different shape parameters, we constructed tables for various combinations of consumers' and producers' risks. A real example was provided which showed that a multiple dependent state sampling plan is a good sampling plan for fitting the datasets. Comparing the proposed plan with a single sampling plan, the results reveal that the proposed plan is more effective at securing the consumer and the producer with less inspection. The approach introduced in this study provides an ample opportunity for the manufacturers to reduce the cost and time of inspection by having the sample size reduced without compromising the decision-making accuracy. By implementing the findings of this study, the consumers are confident that their hard-earned money is not used to purchase sub-standard goods.

Keywords: Acceptance Sampling, Zech Distribution, MDSSP, Single Sampling Plan, Operating Characteristic Curve, Average Sampling Number.

1. Introduction

There are growing concerns all over the world about the quality of products in the market this day. It is no longer a hearsay that there is a sharp decline in the quality of some essential commodities from what it used to be some decades ago. The reasons for this could be attributed to a surge in population leading to increase in demand which forces the producers not to take necessary precautions in ascertaining the quality of their products before being rolled to the markets. Some producers are too mindful of the profits thereby doing everything possible to cut the cost of production to the barest minimum. In all of this, the consumers are at the receiving end. In recent times, there are lots of building collapse all over the world which claimed so many lives. Many of these buildings failed integrity tests as a result of usage of poor quality materials when erecting those structures. Several patients have been sent to the early graves due to drugs/vaccines of poor qualities administered on them. It is therefore imperative that products of good qualities are produced by the producers while products of poor qualities are rejected by the consumers. To do this, the two areas of quality control used in monitoring production processes are acceptance sampling and quality control. Acceptance sampling helps to decide whether a lot will be accepted or rejected at the minimum inspection cost in terms of time and money while satisfying not only the producer's but also the consumer's risks at the same time. Producer's risk denoted as α is the risk incurred by the producer when good lots are rejected by the consumer while the consumer's risk, β is the risk incurred by the consumer for accepting a bad lot from the producer.

Limiting quality control (LQL) is the quality level attached to consumer's risk while acceptable quality control (AQL) is the quality level attached to producer's risk. The field of acceptance sampling has gained a lot of patronage in the literature. There are several types of acceptance sampling, of which, the simplest and easiest for practical implementation is the single sampling acceptance plan. Others are double acceptance plan, group acceptance plan, sequential acceptance plan, multiple dependent state sampling plan, modified multiple dependent plan, adjusted multiple dependent plan, rank sampling plan, to mention but a few. Kantam et al. [1] proposed an economic reliability test plan using Log –logistic distribution, Tzong – Ru and Shou – Jye Wu [2] developed acceptance sampling based on truncated life tests for generalized Rayleigh distribution, Syed et al. [3] proposed Mean ranked acceptance sampling plan under exponential distribution, Wenhao and Shangli [4] developed acceptance sampling plans based on truncated life tests for Gompertz distribution, Rao et al. [5] developed acceptance sampling plan for Marshall – Olkin extended Lomax distribution, Al-Omari et al. [6] developed a single sampling plan for the three – parameter Lindley distribution. Al – Omari [7] considered acceptance sampling plans based on truncated life tests for Sushila distribution, Mahdy and Basma [8] proposed double acceptance sampling plan using new distributions, Ramasamy and Sutharani [9] designed double acceptance sampling plans based on truncated life tests in Rayleigh distribution using minimum angle method. For an exponentiated Frechet distribution with known shape parameters, a double acceptance sampling plan was devised by Babu et al. [10]. Wortham and Baker [11] proposed a multiple deferred state sampling inspection. It is a modification to the chain sampling plan and also intends to supplement existing plans like dependent stage sampling, chain sampling, exponential smoothed sampling and fixed deferred state sampling. It is perhaps, very useful where the lots are serially submitted for inspection. The main aim of acceptance sampling is to reduce the sample size and this can be achieved by implementing multiple dependent state sampling plan. The reason for this is because the results of samples drawn by the experimenter from both the current and successive lots is used to make decision concerning the disposition of the current lot. Several works have been done on multiple deferred state sampling plans (MDSSP), among whom are Davood et al. [12] who designed the Bayesian multiple dependent (deferred) sampling plan based on the process capability index. Yan et al. [13] developed a multiple dependent state sampling plan based on the coefficient of variation. The design of multiple deferred state sampling plans for exponentiated half-logistic distribution was proposed by Rao et al. [14], a multiple dependent state sampling plan in a Fuzzy environment was developed by Afshari et al. [15]. Balamulari and Jun. [16] proposed a multiple deferred state sampling plans for lot acceptance based on measurement data. Aslam et al. [17] developed plans for sampling several dependent state variables that take process loss into account, Balamurali et al. [18] designed multiple deferred state sampling plan for generalized inverted exponential distribution. Recently, some scholars extended the multiple dependent state sampling plan, among whom are: Nadi et al. [19] developed a group multiple dependent state sampling plan using truncated life tests for the Weibull distribution, Aslam et al. [20] proposed generalized multiple dependent state sampling state sampling plans for various life distributions, Cheng et al. [21] proposed adjustable variables multiple dependent state sampling plan based on a process capability index, Aslam et al. [22] developed modified multiple dependent state sampling plan. In this work, we developed a multiple dependent state sampling plan using the median life of the product when the test is truncated at a pre – determined time t, assuming that the lifetime of the product follows Zech distribution. Similar approach can be seen in Rao et al. [23]. To showcase the performance of the proposed plan on Zech distribution, a comparison was made between multiple dependent state sampling plans and single sampling plan to reveal which is better. The MDSSP's operating procedure for Zech distribution is detailed in section 4.

2. Zech Distribution

Zech distribution proposed by Adeyeye, et al. [24] is a heavy – tailed, three – parameter distribution derived by finding the inverse of Gompertz Inverse-Exponential distribution. The cumulative distribution function (cdf) of Zech distribution is given as follows:

$$G(t) = e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}} \; ; \; t > 0, \gamma > 0, \delta > 0, \theta > 0 \tag{1}$$

Its probability density function (pdf) is given as follows:

$$g(t) = \gamma\theta e^{-\theta t}\left[1-e^{-\theta t}\right]^{-\delta-1} e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}}; \quad t > 0, \gamma > 0, \delta > 0, \theta > 0 \tag{2}$$

The shape parameters are γ and δ while the scale parameter is θ. Assuming γ and δ are known, the cdf depends on θx and the qth quantile of the products.

The qth quantile of a product's lifetime which follows Zech distribution is given by (3)

$$t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma}\ln q\right)^{-\frac{1}{\delta}}\right]\right\} = \frac{\xi_q}{\theta} \quad where \;\; \xi_q = -\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma}\ln q\right)^{-\frac{1}{\delta}}\right]\right\} \tag{3}$$

The median is derived when $q = 0.5$ and is given by (4)

$$x_{0.5} = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma}\ln(0.5)\right)^{-\frac{1}{\delta}}\right]\right\} \tag{4}$$

Before the experiment time t_0, the probability of failure of products under the Zech distribution is given by

$$p = e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t_0}\right]^{-\delta}\right\}} \tag{5}$$

The experiment is stopped at the time t_q^0 indicated by $t_0 = a t_q^0$, where t_0 is the termination time. The scale parameter θ can be expressed as

$$\theta = \frac{\xi_q}{t_q} \tag{6}$$

By substituting θ in equation (5), the probability of failure of the item is obtained as follows

$$p = exp\left(\frac{\gamma}{\delta}\left\{1-\left[1-exp\left(\frac{-a\xi_q}{(t_q/t_q^0)}\right)\right]^{-\delta}\right\}\right) \tag{7}$$

Then in expanded form, equation (7) becomes

$$p = exp\left(\frac{\gamma}{\delta}\left\{1-\left[1-exp\left(\left(aln\left\{\left[1-\left(1-\frac{\delta}{\gamma}lnq\right)^{-\frac{1}{\delta}}\right]\right\}\right)^{-\delta}\left(\frac{t_q}{t_q^0}\right)^{\delta}\right)\right]\right\}\right) \tag{8}$$

The quantile ratio is $\left(\frac{t_q}{t_q^0}\right)$, when it is greater than one, the failure probability in (8) is regarded as acceptable quality level (AQL) (p_1). It is regarded as limiting quality level (LQL) (p_2) when it is equal to 1.

3. Multiple Dependent State Sampling Plan for Zech Distribution

This section is sub–divided into two:

(i) The procedures of operation of MDSSP on Zech distribution.
(ii) Determination of Average Sampling Number.
3.1 Procedures of Operation: this is carried out as itemized in the steps below:
Step 1: A random sample, of size n units will be selected from the current lot. Having fixed a specified time t_0, all units are subjected to a life test at the same time.
Step 2: If d is at most c_1, the present lot is accepted and rejected if d is greater than c_2. The tests ends here but if none of these conditions is met, go to step 3.
Step 3: If d surpasses c_1 and is not greater than c_2 the decision to accept the current lot is made , i.e. $c_1 < d \leq c_2$ provided the succeeding m (preceding m) lots will be accepted with condition $d \leq c_1$.
The four parameters characterizing the proposed MDSSP are: n, m, c_1 and c_2 which represent the sample size, number of preceding lots the experimenter needs to make a good decision, maximum number of failed items for unrestricted (unconditional) acceptance and maximum number of failed items for restricted (conditional) acceptance respectively.
It is important to note that single sampling plan (SSP) can be generalized to form an attribute MDSSP; this is so because it reduces to SSP when either $m \to \infty$ or $c_1 = c_2 = c$
According to Rao [3], the operating characteristic (OC) function of the proposed plan is denoted by

$$P_a(p) = p(d \leq c_1) + p(c_1 < d \leq c_2)\big(p(d \leq c_1)\big)^m \tag{9}$$

To accept a lot, equation (10) provides the probability of failure p if a binomial distribution is considered.

$$P_a(p) = \sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p^d (1-p)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} \right)^m \tag{10}$$

3.2 Determination of ASN: The main aim of any sampling plan is to reduce the average sampling number (ASN). This will of course, reduce the inspection time and cost. It will also be of immense benefit to the producers in case of destructive sampling. In this study, we seek to reduce the ASN of the suggested MDSSP for the Zech distribution under truncated life testing. This is achieved by using a non-linear optimization problem in which the objective function is the minimization of ASN at p subject to some constraints. The optimization problem is as follows:
Minimize Average Sample Number, ASN $(P_1) = n$
Subject to

$$P_a(P_1) \geq 1 - \alpha, \tag{11}$$

$$P_a(P_2) \leq \beta \tag{12}$$

$n > 1, m > 1, c_2 > c_1 \geq 0,$
Failure at the producer's risk has a chance of p_1, while failure at the consumer's risk has a probability of p_2. The ratio $\frac{t_q}{t_q^0}$ is known as the quality level or true life quantile ratio, and it aids the producer in ensuring that the quality of his products is good and acceptable. Equations (13) and (14) are respectively, the probabilities of the acceptance of the lot under the modified sampling scheme at acceptable quality level (AQL) and limiting quality level (LQL).

$$P_a(p_1) = \sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} \right)^m \tag{13}$$

$$P_a(p_2) = \sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p_2^d (1-p_2)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d} \right)^m \tag{14}$$

The median ratio, $\frac{t_q}{t_q^0}$ whose values are 2, 4, 6, 8 and 10 is considered at the risk of the producer. This will make it more likely that people will accept the high quality product while the mean ratio $\frac{t_q}{t_q^0} = 1$ is considered at the consumer' risk to ensure that the product of poor quality is rejected.

4. Simulation Studies

The optimal parameters of the proposed plan for Zech distribution with $\gamma = 0.5, \delta = 0.5$; $\gamma = 1.0, \delta = 1.0$ and $\gamma = 1.5, \delta = 1.5$ are presented in the following tables (Table 1-3). Values assumed for consumer's risks $\beta = 0.25, 0.10, 0.05, 0.01$ while the producer's risk was kept at $\alpha = 0.05$ at 50th percentile. The values considered for termination ratio are *a*= 0.5, 0.7 and 1.0

Results of simulation studies observed from Tables 1-3 when parametric combinations are fixed are as follows:

i. Inverse relationship is observed between sample size and consumers risk. In all the Tables 1-3 sample size increases when consumers risk decreases.
ii. As the termination ratio a rises from 0.5 to 1.0, the sample size drops.
iii. Probability of lot acceptance increases along with quantile ratio. As $\frac{t_q}{t_q^0}$ approaches 10, probability of lot acceptance also increases and approximates to almost 1.

Table 1: *The developed MDSSP's ideal (optimal) parameters for the Zech distribution with* $\gamma = 0.5, \delta = 0.5$

β	$\frac{t_q}{t_q^0}$	a=0.5					a=0.7					a=1.0				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	18	3	7	2	0.9551	16	4	14	2	0.9508	16	6	8	2	0.9563
	4	5	0	4	2	0.9531	7	1	2	1	0.9870	5	1	2	1	0.9758
	6	5	0	4	2	0.9921	3	0	2	3	0.9798	3	0	2	1	0.9708
	8	5	0	4	2	0.9984	3	0	2	3	0.9945	3	0	2	1	0.9902
	10	5	0	4	2	0.9996	3	0	2	3	0.9983	3	0	2	1	0.9963
0.10	2	33	5	8	1	0.9547	29	7	17	2	0.9534	26	9	19	2	0.9513
	4	13	1	2	1	0.9868	9	1	8	2	0.9751	7	1	3	1	0.9574
	6	8	0	1	1	0.9859	5	0	4	2	0.9642	7	1	3	1	0.9957
	8	8	0	1	1	0.9970	5	0	4	2	0.9900	4	0	1	1	0.9773
	10	8	0	1	1	0.9993	5	0	4	2	0.9969	4	0	1	1	0.9913
0.05	2	41	6	10	1	0.9528	38	9	19	2	0.9542	34	11	18	1	0.9521
	4	16	1	3	1	0.9880	11	1	10	2	0.9522	11	2	5	1	0.9820
	6	10	0	1	1	0.9785	7	0	2	1	0.9638	8	1	3	1	0.9929
	8	10	0	1	1	0.9954	7	0	2	1	0.9900	5	0	2	1	0.9742
	10	10	0	1	1	0.9989	7	0	2	1	0.9969	5	0	2	1	0.9901
0.01	2	64	9	16	1	0.9510	55	12	18	1	0.9505	50	16	22	1	0.9525
	4	21	1	11	2	0.9528	19	2	4	1	0.9782	17	3	8	1	0.9824
	6	15	0	3	1	0.9678	15	1	4	1	0.9950	11	1	5	1	0.9805
	8	15	0	3	1	0.9930	10	0	2	1	0.9804	7	0	2	1	0.9524
	10	15	0	3	1	0.9983	10	0	2	1	0.9938	7	0	2	1	0.9813

Table 2: *The developed MDSSP's ideal (optimal) parameters for the Zech distribution with* $\gamma = 1.0, \delta = 1.0$

β	$\frac{t_q}{t_q^0}$	a=0.5					a=0.7					a=1.0				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	8	0	1	2	0.9581	8	1	2	2	0.9736	7	2	3	2	0.9661
	4	8	0	1	2	1.0000	4	0	1	2	0.9997	3	0	2	1	0.9971
	6	8	0	1	2	1.0000	4	0	1	2	1.0000	3	0	2	1	1.0000
	8	8	0	1	2	1.0000	4	0	1	2	1.0000	3	0	2	1	1.0000
	10	8	0	1	2	1.0000	4	0	1	2	1.0000	3	0	2	1	1.0000
0.10	2	22	1	5	2	0.9928	11	1	4	2	0.9529	12	3	5	1	0.9755
	4	13	0	10	2	1.0000	7	0	1	1	0.9995	4	0	1	1	0.9931

	6	13	0	10	2	1.0000	7	0	1	1	1.0000	4	0	1	1	0.9999
	8	13	0	10	2	1.0000	7	0	1	1	1.0000	4	0	1	1	1.0000
	10	13	0	10	2	1.0000	7	0	1	1	1.0000	4	0	1	1	1.0000
0.05	2	27	1	2	1	0.9804	18	2	3	1	0.9640	13	3	12	2	0.9502
	4	17	0	1	1	1.0000	8	0	7	2	0.9991	5	0	2	1	0.9921
	6	17	0	1	1	1.0000	8	0	7	2	1.0000	5	0	2	1	0.9999
	8	17	0	1	1	1.0000	8	0	7	2	1.0000	5	0	2	1	1.0000
	10	17	0	1	1	1.0000	8	0	7	2	1.0000	5	0	2	1	1.0000
0.01	2	37	1	3	1	0.9742	24	2	5	1	0.9546	22	5	8		0.9702
	4	25	0	10	2	0.9999	13	0	3	1	0.9988	7	0	2	1	0.9850
	6	25	0	10	2	1.0000	13	0	3	1	1.0000	7	0	2	1	0.9998
	8	25	0	10	2	1.0000	13	0	3	1	1.0000	7	0	2	1	1.0000
	10	25	0	10	2	1.0000	13	0	3	1	1.0000	7	0	2	1	1.0000

Table 3: *The developed MDSSP's ideal parameters for the Zech distribution with* $\gamma = 1.5, \delta = 1.5$.

β	$\frac{t_q}{t_q^0}$	a=0.5					a=0.7					a=1.0				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	13	0	10	3	0.9988	5	0	4	2	0.2381	5	1	2	1	0.9829
	4	13	0	10	3	1.0000	5	0	4	2	0.2381	3	0	2	1	1.0000
	6	13	0	10	3	1.0000	5	0	4	2	0.2381	3	0	2	1	1.0000
	8	13	0	10	3	1.0000	5	0	4	2	0.2381	3	0	2	1	1.0000
	10	13	0	10	3	1.0000	5	0	4	2	0.2381	3	0	2	1	1.0000
0.10	2	21	0	1	2	0.9974	8	0	1	1	0.0978	7	1	3	1	0.9705
	4	21	0	1	2	1.0000	8	0	1	1	0.0978	4	0	1	1	0.9999
	6	21	0	1	2	1.0000	8	0	1	1	0.0978	4	0	1	1	1.0000
	8	21	0	1	2	1.0000	8	0	1	1	0.0978	4	0	1	1	1.0000
	10	21	0	1	2	1.0000	8	0	1	1	0.0978	4	0	1	1	1.0000
0.05	2	27	0	1	2	0.9957	10	0	1	1	0.0485	8	1	3	1	0.9537
	4	27	0	1	2	1.0000	10	0	1	1	0.0485	5	0	2	1	0.9999
	6	27	0	1	2	1.0000	10	0	1	1	0.0485	5	0	2	1	1.0000
	8	27	0	1	2	1.0000	10	0	1	1	0.0485	5	0	2	1	1.0000
	10	27	0	1	2	1.0000	10	0	1	1	0.0485	5	0	2	1	1.0000
0.01	2	42	0	1	1	0.9938	22	1	3	1	0.0095	14	2	6	1	0.9682
	4	42	0	1	1	1.0000	15	0	1	1	0.0090	7	0	2	1	0.9999
	6	42	0	1	1	1.0000	15	0	1	1	0.0090	7	0	2	1	1.0000
	8	42	0	1	1	1.0000	15	0	1	1	0.0090	7	0	2	1	1.0000
	10	42	0	1	1	1.0000	15	0	1	1	0.0090	7	0	2	1	1.0000

5. Application of the Proposed Sampling Plan to Cancer Data

In this section, the application of multiple dependent state sampling plan to real life data on survival time for 44 patients diagnosed by Head and Neck cancer disease is considered. The data consists of 44 observations. It has been used by Efron [25] for logistic regression and recently analyzed by Sule et al. [26] in Topp Leone Kumaraswamy–G family of distributions. The data are as follows:
12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194,195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776

The application is done by first checking if the data sets fit the distribution well. Zech distribution, being a heavily right – tailed distribution, the histogram of the data drawn depicts that the data is also positively skewed. The descriptive statistics of the data like minimum,1st Quartile, median, mean, 3rd quartile, maximum, coefficient of skewness, coefficient of kurtosis and standard deviation are shown in the Table 4. Also, Estimates of the parameters, standard errors, negative log-likelihood value, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) together with the goodness of fit tests, viz., Kolmogorov Smirnov Statistic (K –S), and P-value are given in Table 5.

Table 4 : *Summary of head and neck cancer data.*

Min	1st Quartile	Median	Mean	3rd Quartile	Max.	Standard Deviation	Skewness	Kurtosis
12.20	67.21	128.50	223.48	219.00	1776.00	305.4282	3.38382	16.5596

Table 4 above shows that the data is asymmetric with longer tail to the right.

Table 5 : *Goodness of fit tests of Zech distribution using head and neck cancer data.*

Distribution	Estimates	Std Error	-LL	AIC	BIC	KS	P - value
Zech	$\hat{\gamma} =$ 0.2731 $\hat{\delta} =$ 0.7736 $\hat{\theta} =$ 0.0023	0.08364 0.1735 0.0001944	-277.5201	561.0402	566.3928	0.074069	0.9546

Table 5 shows the estimates of parameters, standard errors and goodness of fit tests of Zech distribution for the head and neck cancer data. The parameters are estimated via the method of maximum likelihood. Empirical and theoretical pdf's and cdf's, Quantile – Quantile plot, and Probability-Probability plot are shown in Figure 1 to showcase the goodness of fit of Zech distribution. Judging from the plots in Figure 1, it is not out of place to say that Zech distribution yields a good fit for the head and neck cancer data.

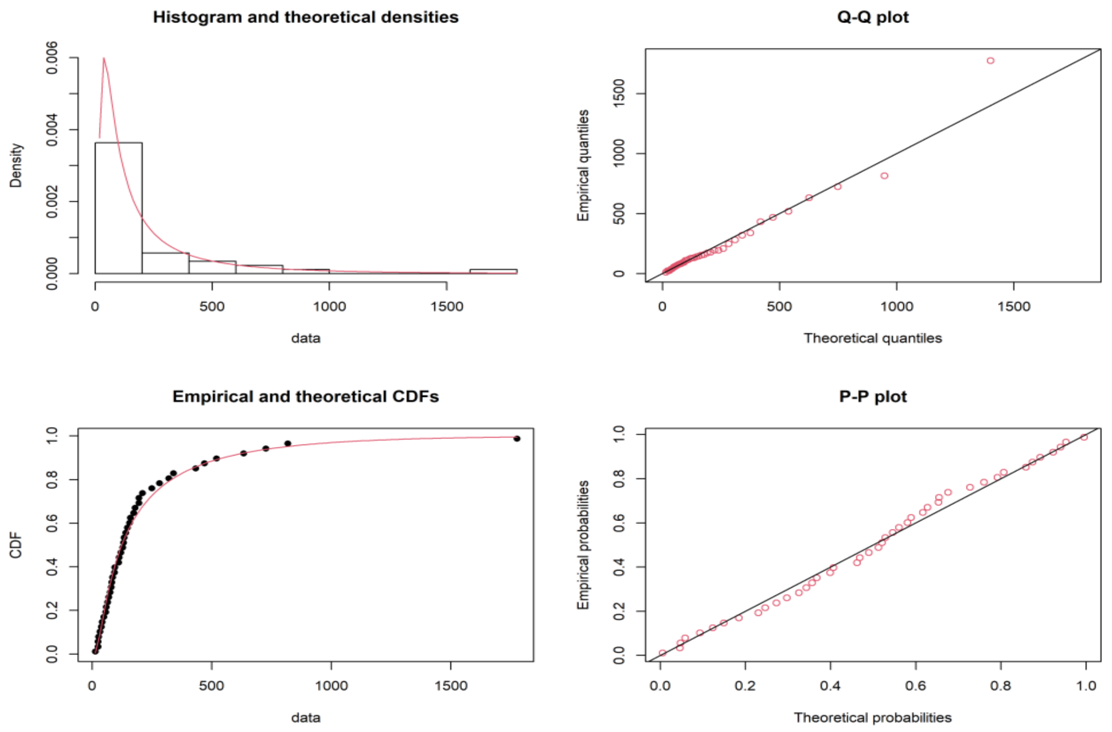

Figure 1: *Empirical and theoretical PDF's, empirical and theoretical CDF'S, Q- Q plots and P-P plots for Zech distribution using Head and Neck cancer data.*

For illustration, the medical practitioner would prefer to employ the developed sampling plan to implement the percentile of the median life of the product where the product's lifetime follows Zech distribution with the shape parameters are $\hat{\gamma} =\ 0.2731$ and $\hat{\delta} = 0.7736$. The medical practitioners suggest that given median survival time is 45 days whereas the medical practitioners expected that the median survival time 90 days. The risk of the consumer is 0.05 provided the actual median survival time 45 days while that of the producer is 0.10 given that the true median survival time 90 days. The ideal (optimal) parameters chosen from Table 6 under these restrictions (constraints) are n=13, c_1=3, c_2=6, and m=2 with values of $\hat{\gamma} =\ 0.2731$ and $\hat{\delta} = 0.7736$, t_{q_0} =45, α =0.05, β=0.25, t_q / t_{q_0} =2 at a=0.7. The MDS sampling plans are illustrated as follows:

A sample of 13 patients' survival time of Head and Neck cancer disease will be chosen at random from the group of Head and Neck cancer disease patients and their survival time is 45 days. If the survival time before 45 days is 3 patients then a group of the population will be allowed (accepted) and the group of the population will be denied (rejected) if it is greater than 6 patients in a group. In the event that a group of patients' survival time for Head and Neck cancer disease is between 3 and 6, the choice of the group of the population will be delayed until the two preceding groups of the population have been tested. For the purpose of this real world illustration, 7 people in the community with Head and Neck cancer disease survived before the survival time of 45 days. Hence, disregard the survival time of the Head and Neck cancer disease patients in a group of the population. So, doctors could advise the government or general public that the median survival time of people with Head and Neck cancer disease in a particular population group is at an undesirable level.

Table 6: *Optimal parameters of the proposed MDS plan for Zech Distribution with* $\hat{\gamma}$= 0.2731 *and* $\hat{\delta}$=0.7736

β	$\frac{t_q}{t_q^0}$	a=0.5					a=0.7					a=1.0				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	N	c_1	c_2	m	$P_a(p_1)$
0.25	2	15	2	12	2	0.9683	13	3	6	2	0.9635	14	5	13	2	0.9670
	4	5	0	1	2	0.9915	3	0	2	4	0.9629	5	1	2	1	0.9912
	6	5	0	1	2	0.9997	3	0	2	4	0.9969	3	0	2	1	0.9916
	8	5	0	1	2	1.0000	3	0	2	4	0.9997	3	0	2	1	0.9986
	10	5	0	1	2	1.0000	3	0	2	4	1.0000	3	0	2	1	0.9998
0.10	2	24	3	8	2	0.9620	23	5	15	2	0.9612	21	7	11	2	0.9519
	4	8	0	7	2	0.9827	6	0	2	1	0.9603	7	1	3	1	0.9856
	6	8	0	7	2	0.9994	5	0	1	2	0.9948	4	0	1	1	0.9807
	8	8	0	7	2	1.0000	5	0	1	2	0.9995	4	0	1	1	0.9967
	10	8	0	7	2	1.0000	5	0	1	2	1.0000	4	0	1	1	0.9994
0.05	2	33	4	14	2	0.9607	29	6	16	2	0.9505	29	9	15	1	0.9571
	4	10	0	3	2	0.9739	11	1	2	1	0.9909	8	1	3	1	0.9768
	6	10	0	3	2	0.9990	7	0	1	1	0.9939	5	0	2	1	0.9780
	8	10	0	3	2	1.0000	7	0	1	1	0.9994	5	0	2	1	0.9963
	10	10	0	3	2	1.0000	7	0	1	1	0.9999	5	0	2	1	0.9993
0.01	2	47	5	9	1	0.9522	45	9	19	2	0.9501	43	13	20	1	0.9535
	4	16	0	2	1	0.9663	15	1	3	1	0.9885	14	2	6	1	0.9877
	6	16	0	2	1	0.9987	10	0	1	1	0.9879	7	0	2	1	0.9592
	8	16	0	2	1	0.9999	10	0	1	1	0.9989	7	0	2	1	0.9929
	10	16	0	2	1	1.0000	10	0	1	1	0.9999	7	0	2	1	0.9987

Excerpts from Table 6 are as follows:

i. As consumer's risk decreases, sample size increases
ii. As the termination ratio rises from 0.5 to 1.0, the sample size falls.
iii. As the quantile ratio rises, the likelihood that the lot will be accepted increases. As the quantile ratio approaches 10, probability of lot acceptance also increases and almost approximates to 1

6. Comparative Study

To determine the efficiency of MDSSP over Single sampling plan, a comparison study was made between the two plans when the underlying distribution of data follows Zech distribution. From Table 7, the quantile ratios 2, 4, 6, 8 and 10 were considered for each of the consumer's risks $\beta = 0.25, 0.10, 0.05, 0.01$ while the producer's risk was kept at $\alpha = 0.05$. The sample size n was compared to the probability of acceptance $P_a(p_1)$. The results reveal that the sample size of the single sampling plan is higher than that of the developed MDSSP. For plan ratio 2 under the consumer's risk, $\beta = 0.01$, the parameters for the MDSSP are: $n = 42, c_1 = 0, c_2 = 1$ and $m = 1$. For the single sampling plan, the parameters of the plan are $n = 61$ and $c = 1$ with corresponding probabilities of 0.9938 and 0.9956 for MDSSP and SSP respectively when the termination ratio is 0.5

Also, for plan ratio 2 under the consumer's risk,, $\beta = 0.01$, the plan parameters for MDSSP are: $n = 14, c_1 = 2, c_2 = 6$ and $m = 1$. For the single sampling plan, the plan parameters are $n = 19$ and $c = 4$ with corresponding probabilities of 0.9682 and 0.9569 for MDSSP and SSP respectively when the termination ratio is 1.0

It shows that the proposed MDSSP is more efficient than the existing SSP for Zech distribution. The operating characteristic curve in Figure 2 also emphasizes the MDSSP is more efficient than the existing SSP.

Table 7: *Comparison of optimal parameters of the proposed MDSSP and SSP for Zech distribution with* $\gamma = 1.5, \delta = 1.5$.

a = 0.5										a = 1.0							
MDSSP							SSP			MDSSP					SSP		
β	$\frac{t_q}{t_q^0}$	n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$	n	c_1	c_2	M	$P_a(p_1)$	n	c	$P_a(p_1)$
0.25	2	13	0	10	3	0.9988	13	0	0.9795	5	1	2	1	0.9829	7	2	0.9703
	4	13	0	10	3	1.0000	13	0	1.0000	3	0	2	1	1.0000	2	0	0.9968
	6	13	0	10	3	1.0000	13	0	1.0000	3	0	2	1	1.0000	2	0	1.0000
	8	13	0	10	3	1.0000	13	0	1.0000	3	0	2	1	1.0000	2	0	1.0000
	10	13	0	10	3	1.0000	13	0	1.0000	3	0	2	1	1.0000	2	0	1.0000
0.10	2	21	0	1	2	0.9974	21	0	0.9671	7	1	3	1	0.9705	12	3	0.9694
	4	21	0	1	2	1.0000	21	0	1.0000	4	0	1	1	0.9999	4	0	0.9936
	6	21	0	1	2	1.0000	21	0	1.0000	4	0	1	1	1.0000	4	0	1.0000
	8	21	0	1	2	1.0000	21	0	1.0000	4	0	1	1	1.0000	4	0	1.0000
	10	21	0	1	2	1.0000	21	0	1.0000	4	0	1	1	1.0000	4	0	1.0000
0.05	2	27	0	1	2	0.9957	27	0	0.9578	8	1	3	1	0.9537	13	3	0.9594
	4	27	0	1	2	1.0000	27	0	1.0000	5	0	2	1	0.9999	5	0	0.9921
	6	27	0	1	2	1.0000	27	0	1.0000	5	0	2	1	1.0000	5	0	1.0000
	8	27	0	1	2	1.0000	27	0	1.0000	5	0	2	1	1.0000	5	0	1.0000
	10	27	0	1	2	1.0000	27	0	1.0000	5	0	2	1	1.0000	5	0	1.0000
0.01	2	42	0	1	1	0.9938	61	1	0.9956	14	2	6	1	0.9682	19	4	0.9569
	4	42	0	1	1	1.0000	42	0	1.0000	7	0	2	1	0.9999	7	0	0.9889
	6	42	0	1	1	1.0000	42	0	1.0000	7	0	2	1	1.0000	7	0	0.9999
	8	42	0	1	1	1.0000	42	0	1.0000	7	0	2	1	1.0000	7	0	1.0000
	10	42	0	1	1	1.0000	42	0	1.0000	7	0	2	1	1.0000	7	0	1.0000

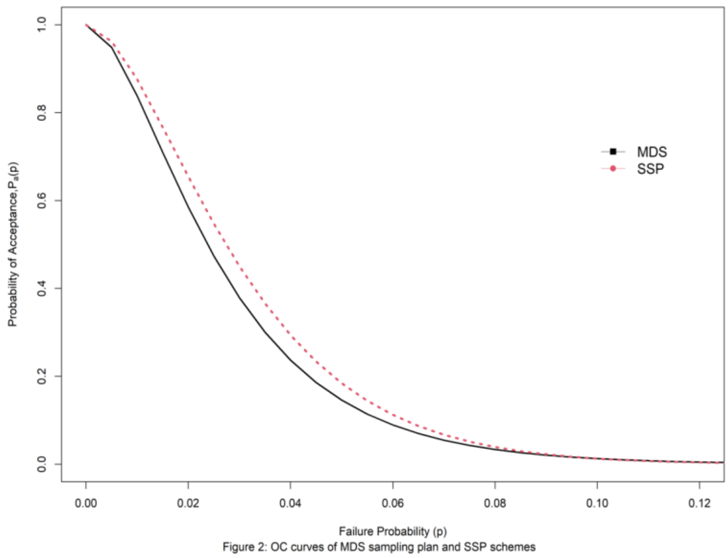

Figure 2: *The operating characteristic curve for multiple dependent state sampling plan and single sampling plan. MDSSP is more accurate than SSP, with steeper slope.*

7. Conclusions

The assumption used in this work to build the multiple deferred state sampling plan is that the product's lifespan will follow Zech distribution when the lifetime tests are truncated. By simultaneously addressing both the producer's and the consumer's risks, the optimal parameters for the proposed MDSSP were established. Application of the proposed sampling plan to real life example shows that it is suitable for the data used. The operating characteristic curve for the proposed sampling plan and single sampling plan (SSP) reveals that MDSSP is more accurate than SSP with steeper slope.

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