SOLVABILITY OF RESONANT FRACTIONAL ORDER BOUNDARY VALUE PROBLEMS WITH TWO-DIMENSIONAL KERNEL ON THE HALF-LINE

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A THESIS SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF DOCTOR OF PHILOSOPHY (Ph.D) IN INDUSTRIAL MATHEMATICS IN THE DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE AND TECHNOLOGY, COVENANT UNIVERSITY, OTA, NIGERIA

FEBRUARY, 2024

ACCEPTANCE

This is to attest that this thesis is accepted in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy in Industrial Mathematics in the Department of Mathematics, College of Science and Technology, Covenant University, Ota, Ogun State, Nigeria.

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(Dean, School of Postgraduate Studies) **Signature and Date**

DECLARATION

I, **OJO, EZEKIEL KADEJO (17PAD01720)** declare that this research work was carried out by me under the supervision of Prof. Samuel A. Iyase and Prof. Timothy A. Anake of the Department of Mathematics, College of Science and Technology, Covenant University, Ota, Ogun State, Nigeria. I attest that this thesis has not been presented either partially or wholly for the award of any degree elsewhere. All scholarly materials used in this research work are duly cited and acknowledged.

OJO, EZEKIEL KADEJO

Signature and Date

CERTIFICATION

We certify that this thesis titled **"SOLVABILITY OF RESONANT FACTIONAL ORDER BOUNDARY VALUE PROBLEMS WITH TWO-DIMENSIONALKERNEL ON THE HALF-LINE**" is an original work carried out by **OJO, EZEKIEL KADEJO (17PAD01720)**, in the Department of Mathematics, College of Science and Technology, Covenant University, Ota, Ogun State, Nigeria, under the supervision of Prof. Samuel A. Iyase and Prof. Timothy A. Anake. We have examined and found the work acceptable as part of the requirements for the award of Doctor of Philosophy (Ph.D.) degree in Industrial Mathematics.

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Prof. Akan B. Williams
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DEDICATION

To God who enabled me to start and complete the study. And to others who stood by me to give needed encouragement and support.

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LIST OF SYMBOLS

$\in$	Member of a set
Σ	Summation
Ω	Open-bounded subset
$\overline{\ }$	Closure of Ω
$\partial\ \Omega$	Boundary of Ω
$\exists$	There exists
ϵ	Epsilon
$L^1[0,\infty)$	L^1 function space
$\|\cdot\|$	Norm
$\subset$	Subset
$\cap$	Intersection
$\cup$	Union

LIST OF ABBREVIATIONS

a.e.	Almost everywhere
IVP	Initial Value Problem
IVPs	Initial Value Problems
BVP	Boundary Value Problem
BVPs	Boundary Value Problems
dom	Domain
ker	Kernel
Im	Image
deg	Degree
coker	Cokernel
lim	Limit
dim	Dimension
dim ker	Dimension of kernel
codim	Co dimension

ABSTRACT

This research derives conditions for existence of solutions for resonant fractional order boundary value problems with multi-point and integral boundary conditions when the dimension of the kernel of the differential operator equals two on the half-line. Two classes of fractional order boundary value problems were investigated. The first class included two problems with linear differential operator of Riemann-Liouville type. Existence results were established by using Mawhin's coincidence degree theory. The fractional order differential equations under consideration were transformed to abstract equation $Lx(t) = Nx(t)$. The corresponding homogeneous equations were solved to establish conditions critical for resonance. For the first class of problems, it was shown that L is a Fredholm map of index zero and N is $L-$compact. The existence lemmas and theorem were stated and proved to establish that solutions exist for the two problems. The second class contained two p-Laplacian fractional order boundary value problems with nonlinear differential operator. Riemann-Liouville and Caputo type of fractional derivatives were involved. The extension of coincidence degree theory by Ge and Ren was applied to establish existence of solutions for the two problems. Conditions for resonance were derived by solving the corresponding homogeneous fractional p-Laplacian BVPs. The BVPs were transformed to abstract equations $Mx(t) = N_\lambda x(t)$, $\lambda \in [0,1]$. It was shown that M is a quasi-linear operator and N_λ is $M-$compact. The results obtained generalize and complement existing results in the literature, which are applicable in the sciences, engineering, finance and business. Examples were provided to substantiate the results obtained.

Keywords: Fractional order, half-line, integral boundary conditions, Mawhin's coincidence degree, multi-point, p-Laplacian, resonance, two-dimensional kernel.