Experimental and numerical investigations of free convection heat transfer in solar oven

Sobamowo M.G., Ogunmola B.Y., Ismail S.O.

Department of Mechanical Engineering, University of Lagos, Akoka, Lagos, Nigeria.

Phone: +2348055444285, +2347034717417 ; **E-mail**- mikegbeminiyiprof@yahoo.com

Abstract

The use of solar energy for baking, heating or drying represents a sustainable way of solar energy applications with negligible negative effects. Solar oven is an alternative to conventional oven that rely heavily on coal and wood or Electric oven that uses the power from the National grid of which the end users have little or no control. Since the Solar oven uses no fuel and it cost nothing to run, it use are widely promoted especially in situations where minimum fuel consumption or fire risks are considered highly important. As useful as the Solar Oven proved, it major setback in the area of applications has been its future sustainability. For the use of Solar Oven/Cookers to be sustained in the future, the design and development of solar oven must rely on sound analytical tools. Therefore, this work focused on the design and development of the solar oven. To test the performance of the Small Solar Oven a 5000cm^3 beaker of water was put into the Oven and the temperature of the water was found to reach 81^{0}C after about 3hrs under an average ambient temperature of 30^{0}C. On no load test, the oven reached a maximum temperature of 112°C in 6hrs. In order to carry out the parametric studies and improve the performance of the Solar Oven, Mathematical models were developed and solved by using Characteristics-Based Split (CBS) Finite Element Method. The Model results were compared with the Experimental results and a good agreement ware found between the two results.
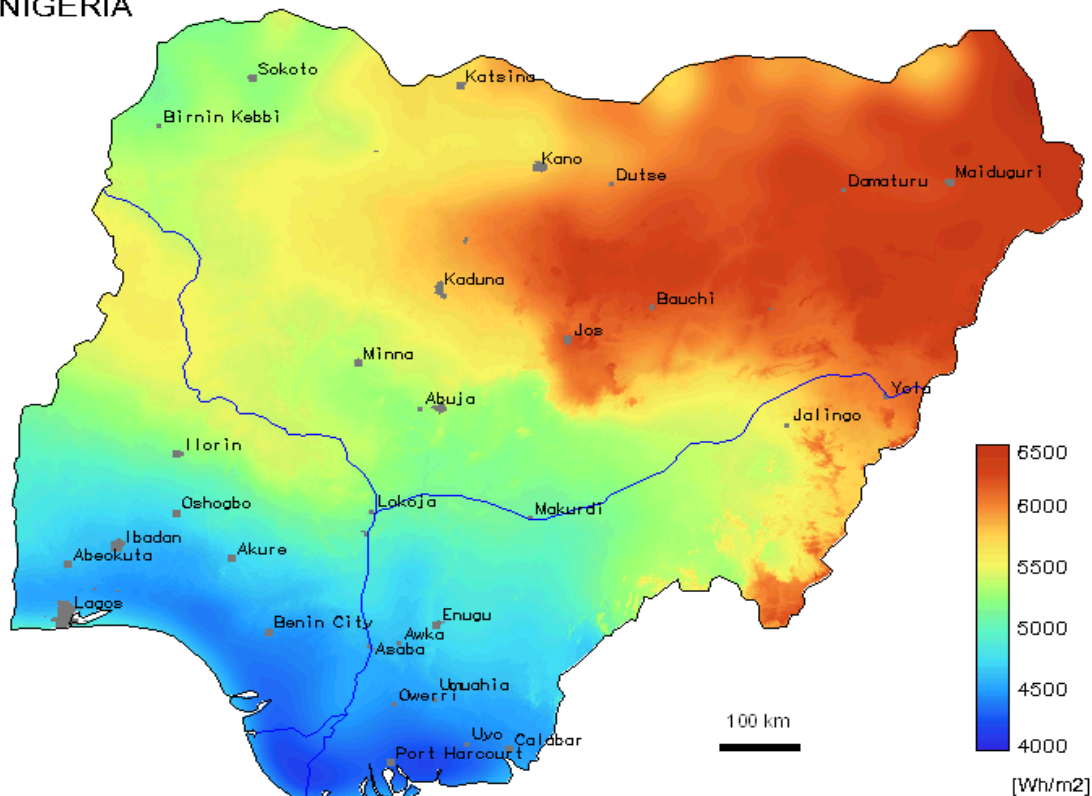
Key words: Solar Oven; Free Convection; Heat Transfer; CBS Finite Element Method

1. Introduction

The world rapidly growing population and exploding energy demands coupled with the climatic profile, socio-economic development, energy supply infrastructures and the threats on health and environment accompanying the utilization of fossil fuels strongly suggest the use of renewable energy powered systems. In fact, the strategies for attaining balance between energy demands and effects in various countries has provoked vast majority in considering the alternative sources. Furthermore, the convergence of rapidly rising costs, dwindling and threatened supply of fossil fuel are stimulating renewed efforts to find viable energy alternatives. Some of the proposed alternatives such as large scale use of bio-fuels may not be achievable or sustainable-or, eventually practical. Some other interesting possibilities such as geothermal energy may be viable but are local and limited. Solar energy that is virtually an inexhaustible natural source produces little or no greenhouse gases. It is known as a clean and environmental friendly energy source. Aside from the indisputable advantages of solar energy mentioned above, the consideration of harnessing solar energy among all other alternative sources of energy sources to meet the needs especially in developing countries is based on well established visibility studies. Firstly, most of the countries called developing, like Nigeria, are in or adjacent to the tropics and have good solar radiation available as shown in Fig.1. Secondly, energy is a critical need of these countries but they do not have widely distributed readily available supplies of conventional energy resources. Thirdly, most of the developing countries are characterized by arid climates, dispersed and inaccessible populations and a lack of investment capital and are thus faced with practically insuperable obstacles to the provision of energy by conventional means, for example, by electrification. In contrast to this, solar energy is readily available and is already distributed to the potential users. Fourthly, because of the diffuse nature of solar energy the developments all over the world, the energy source has been in smaller units which fit well into the pattern of rural economics.

Yearly average of daily sums of global horizontal irradiation
(HelioClim-1/PVGIS data, period 1985-2004)

NIGERIA

PVGIS (c) European Communities 2002-2006
HelioClim-1 (c) Ecole des Mines de Paris/Armines 2001-2006

http://re.jrc.ec.europa.eu/pvgis/

Fig.1 Solar insolation in different States in Nigeria

The abundant solar radiation in the tropical and subtropical regions proved solar energy as the most attractive a1ternative energy source for baking, cooking, drying and heating. A number of government, non-government and private organizations along with a few research institutions and other individual researchers (Reddy et al, 1979; Mumba, 1995; Oosthuizen, 1995; Wisniewski and Pietruszko, 1997, Adelaja et al, 2009, Muhammad *et al*, 2011) have applied the knowledge of solar radiation on the earth surface for the fabrication, promotion and dissemination of different types of solar dryer (direct, indirect and mixed-mode types) in developing countries. In meeting the essential and basic need of human being, wood and fossil fuel as sources of energy for cooking food have played tremendous and invaluable roles. However, the direct combustion of wood and fossil fuel as the major sources of cooking has immensely contributed to global

warming and acid rains. Aside from these adverse effects, the inhalation of these gases irritates the lungs and the eyes and can cause diseases such as pneumonia. In order to obviate or eliminate this problem, a lot of researchers have developed different solar ovens/cookers over the years (Lof, 1965; Duffie *et al,* 1974; Daniel, 1990; Halacy *et al*; 1992, Wary, 1992, Warren, 1994, Barbara, 1994, Malik et al, 1996, Frederick, *et al*, 1997, Kulkarni *et al*, 1997, Adegoke, *et al*, 1998; Patel *et al*, 2000, Felix, *et al*, 2002, Amer, 2003; Ekechukwu *et al*, 2003; Folmer *et al,* 2003, Volker, 2003, 2004, Fronco et al, 2004, Negi *et al*, 2005; Hussein *et al*, 2008; Uhuegbu, 2011; Tuara and Musa, 2012). The range of temperature achieved in most of these ovens/cookers (80°C-100°C) is most suitable for cooking by boiling. Talking about the advantages, this application of solar energy has been assisting in preventing deforestation, saving of fossil fuel and reduction in

utility bills in most countries. Also, since the solar oven uses no fuel, it cost nothing to run and therefore, it use are widely promoted especially in situations where minimum fuel consumption or fire risks are considered highly important. However, as useful as the Solar Oven proved, it major setback in the area of applications has been its future sustainability. For the use of Solar Oven/Cookers to be sustained in the future, the design and development of solar oven must rely on sound analytical tools. Also, to understand the physical phenomena of solar oven and to optimize the performance of the forced convection solar dryer, theoretical investigation is essential. An adventure into the analytical methods in solving the resulting conservation equations for the theoretical investigations of the performance and optimisation of the Oven has proven abortive and over the years, recourse has been made to numerical methods such as Finite difference or Finite volume methods in analysing such class of problems involving conduction, convection and sometimes diffusion. The finite element method has been somehow stated to be limited to Computational Solid Mechanic or at its best in Thermo-Fluid problems, conductive heat transfer. Actually, for most conduction equations in heat transfer problems, the finite element method especially Galerkin Method is straight forward. However, if similar Garlerkin type approximation is applied in the solution of convection equations, the results will be marked with spurious oscillations in space if certain parameters exceed a critical value, element peclet number. This method is not unique to finite elements as all other spatial discretization techniques techniques have difficulties (Lewis et al, 2009). Direct application of the Characteristic Galerkin scheme to solve the momentum equation is difficult. Therefore, In this work, a Characteristic Based Split (CBS) scheme which satisfies the well-known Babuska-Brezzi condition (21, 22) as proposed by Lewis et al (20) is used to analysis and simulate the heat transfer in a photovoltaic-powered forced convection solar dryer. In order to optimize the performance of the solar dryer, the developed finite element models were used to investigate the effects of the blower speeds and the solar dryer chamber aspect ratios on the temperature distribution, velocity distribution and the pressure drop in the drying chamber.

2. The Working Principle of Solar Oven and Experimental Set up

When solar energy is radiated on the solar collector of the solar oven, some of energy is reflected, some is absorbed; or some of it passes right through. But only the absorbed energy is converted heat energy in the enclosure of the solar oven. When food is placed in this solar oven enclosure, moisture at the surface is evaporated and removed by the hot air and the low humidity of air in the oven. This establishes moisture vapour pressure gradient, which causes movement of moisture form the interior of the food to the surface. Depending on the nature of the food and the rate of heating when the extent of moisture loss exceeds the rate of movement from the interior, the zone of evaporation moves inside the food, the surface dried out, the temperature rise to the temperature of the hot air (119°-240°C) and a crust is formed (Halacy, 1978) but the internal temperature of the food does not exceed 100°C cause baking takes place at atmospheric pressure and moisture escape freely from the food.

Fig. 2 Physical model of the solar oven

Since the food does not reach too high temperature in the oven, it can safely be left in the cooker all day without burning. Depending on the latitude and The two test carried above was conducted on solar oven. The results obtained are shown in the Fig.13 and Fig .14. The test at load which involved placing of 500cm^3 beaker of water.

3. Model development for the solar oven

The numerical investigations of the solar oven involve setting up mathematical models. In order to achieve this, a set of conservation equations were set up for the numerical analysis

x-momentum equation.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y}\right] \qquad 2$$

y-momentum equation.

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + \beta g(T - T_o) \qquad 3$$

Energy Equation.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \qquad 4$$

The initial conditions used in this model are:

weather, food can be cooked or baked either early or later in the day. The cooker can also be used to warm food, drinks and pasteurize water or milk.

3.1 Conservation Equations for the Analysis of Solar Oven

The following set of conservation equations were used for the finite element analysis.

Continuity Equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad 1$$

$$t = 0, \quad u = 0, \quad v = 0, \quad T = T_\infty, \quad p = p_\infty \ \ for \ 0 < x < L, 0 < y < L$$

The boundary conditions used in this model are:

$$t > 0, \quad y = 0, \quad u = v = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0 \ \ for \ 0 < x < L$$

$$t > 0, \quad y = L, \quad u = v = 0, \quad T = T_w, \quad p = p_m \ \ for \ 0 < x < L$$

$$t > 0, \quad x = 0 \quad u = v = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0 \ \ for \ 0 < y < L$$

$$t > 0, \quad x = L \quad u = v = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0 \ \ for \ 0 < y < L$$

5a

To obtain the non-dimensional form of the equation, the following non-dimentional scales are used.

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{t u_\infty}{L}, \quad U = \frac{u}{U_\infty}, \quad V = \frac{v}{U_\infty}, \quad P = \frac{p}{\rho U^2_\infty}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

5b

Where L is the characteristics dimention, T_w is the wall temperature (reference temperature) and U_∞ is the free stream velocity.

On substituting the above scales into the equations(1-4), we have:

Continuity equation.
$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad\qquad 6$$

X-momentum equation.
$$\frac{\partial V}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial X^2}\right) \qquad\qquad 7$$

Y-momentum equation.
$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial X^2}\right) + \frac{Gr}{Re^2}\theta \qquad\qquad 8$$

Energy Equation.
$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re P_r}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial X^2}\right) \qquad\qquad 9$$

or

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pe}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial X^2}\right)$$

10

The initial and Boundary conditions in dimensionless form are:

$$\tau > 0, \quad Y = 0, \quad U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \frac{\partial P}{\partial Y} = 0 \quad for \ 0 < X < 1$$

$$\tau > 0, \quad Y = 1, \quad U = V = 0, \quad T = 1, \quad P = p_m/\rho U^2 \quad for \ 0 < X < 1$$

$$\tau > 0, \quad X = 0 \quad U = V = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad \frac{\partial P}{\partial X} = 0 \ for \ 0 < y < 1$$

$$\tau > 0, \ X = 1 \quad U = V = 0, \frac{\partial \theta}{\partial X} = 0, \frac{\partial P}{\partial X} = 0 \ for \ 0 < y < 1$$

11

3.2 Characteristics-Based Split (CBS) Finite Element Analysis of the Conservation Equations

For the continuity equation

$$\frac{U_{i+1,j}^n - U_{i,j}^n}{\Delta X} + \frac{V_{i+1,j}^n - V_{i,j}^n}{\Delta Y} = 0 \quad or \quad \frac{\partial U_{ij}^n}{\partial X} + \frac{\partial V_{ij}^n}{\partial X}$$

12

For the x momentum equation, the semi-discrete form is:

$$\frac{U_{ij}^{n+1} - U_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 U_{ij}}{\partial X^2} + \frac{\partial^2 V_{ij}}{\partial Y^2}\right)^n - \frac{\partial P_{ij}}{\partial X} + U_{ij}\frac{\Delta \tau}{2}\frac{\partial}{\partial X}\left[U_{ij}^n\frac{\partial U_{ij}^n}{\partial X} + \right.$$
$$\left. V_{ij}^n\frac{\partial U_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X}\right] + V_{ij}\frac{\Delta \tau}{2}\frac{\partial}{\partial Y}\left[U_{ij}^n\frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n\frac{\partial U_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X}\right]$$

13a

For y momentum equation, the semi-discrete form is:

$$\frac{V_{ij}^{n+1} - V_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial V_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial V_{ij}^n}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V_{ij}}{\partial X^2} + \frac{\partial^2 V_{ij}}{\partial Y^2}\right)^n - \frac{\partial P_{ij}}{\partial X} + U_{ij}^n\frac{\Delta \tau}{2}\frac{\partial}{\partial X}\left[U_{ij}^n\frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n\frac{\partial}{\partial}\right.$$
$$\left.\frac{\partial P_{ij}^n}{\partial X}\right] + V_{ij}^n\frac{\Delta \tau}{2}\frac{\partial}{\partial Y}\left[U_{ij}^n\frac{\partial V_{ij}^n}{\partial X} + V_{ij}^n\frac{\partial V_{ij}^n}{\partial Y} + \frac{\partial P_{ij}^n}{\partial X}\right] + \frac{Gr}{Re^2}\theta_{ij}^n$$

13b

On removing the pressure from equations 13a and 13b, we arrived equation 14-15

$$\frac{U_{ij} - U_{ij}^n}{\Delta \tau} = U_{ij}^n \frac{\partial U_{ij}^n}{\partial X} - V_{ij}^n \frac{\partial U_{ij}^n}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 U_{ij}}{\partial X^2} + \frac{\partial^2 U_{ij}}{\partial Y^2}\right)^n + U_{ij}^n\frac{\Delta \tau}{2}\frac{\partial}{\partial X}\left(U_{ij}^n\frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n\frac{\partial U_{ij}^n}{\partial Y}\right)$$

$$+ V_{ij}^n\frac{\Delta \tau}{2}\frac{\partial}{\partial Y}\left(U_{ij}^n\frac{\partial U_{ij}^n}{\partial X} + V_{ij}^n\frac{\partial U_{ij}^n}{\partial Y}\right)$$

14

$$\frac{V_{ij}-v_{ij}^n}{\Delta\tau} = U_{ij}^n\frac{\partial v_{ij}^n}{\partial X} - V_{ij}^n\frac{\partial v_{ij}^n}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V_{ij}}{\partial X^2}+\frac{\partial^2 V_{ij}}{\partial Y^2}\right)^n + U_{ij}^n\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(U_{ij}^n\frac{\partial U_{ij}^n}{\partial X}+V_{ij}^n\frac{\partial U_{ij}^n}{\partial Y}\right) +$$
$$V_{ij}^n\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(U_{ij}^n\frac{\partial v_{ij}^n}{\partial X}+V_{ij}^n\frac{\partial v_{ij}^n}{\partial Y}\right) + \frac{Gr}{Re^2}\theta_{ij}^n$$

15

On substracting equation 14 and 15 from equations 13a and 13b respectively, we obtained the real velocity field equations(velocity or momentum correction equations) as:

$$\frac{U_{ij}^{n+1}-\tilde{U}_{ij}}{\Delta\tau} = -\frac{\partial P^n}{\partial X} + U\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(\frac{\partial P}{\partial X}\right)^n + V\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(\frac{\partial P}{\partial X}\right)^n$$

16

$$\frac{v_{ij}^{n+1}-\tilde{V}_{ij}}{\Delta\tau} = -\frac{\partial P^n}{\partial Y} + U\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(\frac{\partial P}{\partial X}\right)^n + V\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(\frac{\partial P}{\partial Y}\right)^n$$

17

On differentiating equation 17 with respect to X and also with respect to Y and adding the resulting equations together neglecting third-order terms gives.

$$\frac{\partial U_{ij}^{n+1}}{\partial X} + \frac{\partial v_{ij}^{n+1}}{\partial Y} - \frac{\partial \tilde{U}_{ij}}{\partial X} - \frac{\partial \tilde{V}_{ij}}{\partial Y} = -\Delta\tau\left(\frac{\partial^2 P_{ij}}{\partial X^2}+\frac{\partial^2 P_{ij}}{\partial Y^2}\right)^n$$

18

Which gives the pressure caculation

$$\left(\frac{\partial^2 P_{ij}}{\partial X^2}+\frac{\partial^2 P_{ij}}{\partial Y^2}\right)^n = \frac{1}{\Delta\tau}\left(\frac{\partial \tilde{U}_{ij}}{\partial X}^n+\frac{\partial \tilde{V}_{ij}}{\partial Y}^n\right)$$

19

Since (continuity equation);

$$\frac{\partial U_{ij}^{n+1}}{\partial X} + \frac{\partial v_{ij}^{n+1}}{\partial Y} = 0$$

20

Temperature Calculation
On applying the characteristic Galerkin procedure to the temperature equation (11), it gives:

$$\frac{\theta_{ij}^{n+1}-\theta_{ij}^{n}}{\Delta t} = U_{ij}^{n}\frac{\partial\theta^{n}}{\partial X} - V_{ij}^{n}\frac{\partial\theta_{ij}^{n}}{\partial Y} + \frac{1}{RePr}\left(\frac{\partial^{2}\theta_{ij}}{\partial X^{2}} + \frac{\partial^{2}\theta_{ij}}{\partial Y^{2}}\right)^{n} + U\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right) +$$
$$V\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right)^{n}$$

21

In summary, we have the intermediate X-momentum equation is:

$$\frac{\tilde{U}_{ij}-U_{ij}^{n}}{\Delta t} = U\frac{\partial U_{ij}^{n}}{\partial X} - V\frac{\partial U_{ij}^{n}}{\partial X} + \frac{1}{Re}\left(\frac{\partial^{2}U_{ij}}{\partial X^{2}} + \frac{\partial^{2}U_{ij}}{\partial Y^{2}}\right)^{n} + U_{ij}^{n}\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(U_{ij}^{n}\frac{\partial V_{ij}}{\partial X} + V\frac{\partial V_{ij}}{\partial Y}\right)^{n} +$$
$$V_{ij}^{n}\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(U_{ij}^{n}\frac{\partial U_{ij}}{\partial X} + V\frac{\partial U_{ij}}{\partial Y}\right)$$

22

And the intermediate Y-momentum equation is:

$$\frac{\tilde{V}_{ij}-V_{ij}^{n}}{\Delta t} = U_{ij}\frac{\partial V_{ij}^{n}}{\partial X} - V\frac{\partial V_{ij}^{n}}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^{2}V_{ij}}{\partial X^{2}} + \frac{\partial^{2}V_{ij}}{\partial Y^{2}}\right)^{n} + U_{ij}^{n}\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(U_{ij}^{n}\frac{\partial V_{ij}}{\partial X} + V\frac{\partial V_{ij}}{\partial Y}\right)^{n} +$$
$$V_{ij}^{n}\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(U\frac{\partial U_{ij}}{\partial X} + V\frac{\partial U_{ij}}{\partial Y}\right)$$

23

Pressure Calculation

$$\left(\frac{\partial^{2}P_{ij}}{\partial X^{2}} + \frac{\partial^{2}P_{ij}}{\partial Y^{2}}\right)^{n} = \frac{1}{\Delta\tau}\left(\frac{\partial\tilde{U}_{1}}{\partial X} + \frac{\partial\tilde{V}}{\partial Y}\right)$$

24

Temperature equation

$$\frac{\theta_{ij}^{n+1}-\theta_{ij}^{n}}{\Delta t} = U_{ij}^{n}\frac{\partial\theta^{n}}{\partial X} - V_{ij}^{n}\frac{\partial\theta_{ij}^{n}}{\partial Y} + \frac{1}{RePr}\left(\frac{\partial^{2}\theta_{ij}}{\partial X^{2}} + \frac{\partial^{2}\theta_{ij}}{\partial Y^{2}}\right)^{n} + U\frac{\Delta\tau}{2}\frac{\partial}{\partial X}\left(U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right) +$$
$$V\frac{\Delta\tau}{2}\frac{\partial}{\partial Y}\left(U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y}\right)^{n}$$

25

Spatial Discretization
Applying the standard Galerkin approximation for solving equations 20-25, assuming linear interpolation functions for all variables, the spatial variation for a linear triangular element may be written as :

$$U = N_{i}U_{i} + N_{j}U_{j} + N_{k}U_{k} = [N][U]$$

26

$$V = N_{i}V_{i} + N_{j}V_{j} + N_{k}V_{k} = [N][V]$$

27

$$P = N_{i}P_{i} + N_{j}P_{j} + N_{k}P_{k} = [N][P]$$

28

$$\theta = N_i\theta_i + N_j\theta_j + N_k\theta_k = [N][\theta]$$

After performing spatial discretization, the Element Mass Matrix from the Characteristics-Based Split scheme is

$$[M_e] = \frac{A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

30

And the Element Convection Matrix

$$[C_e] = $$

$$\frac{1}{24}\begin{bmatrix} (wsw + u_i)b_i & (wsw + u_i)b_j & (wsw + u_i)b_k \\ (wsw + u_j)b_i & (wsw + u_j)b_j & (wsw + u_j)b_k \\ (wsw + u_k)b_i & (wsw + u_k)b_j & (wsw + u_k)b_k \end{bmatrix} +$$

$$\frac{1}{24}\begin{bmatrix} (wrw + u_i)c_i & (wrw + u_i)c_j & (wrw + u_i)c_k \\ (wrw + u_j)c_i & (wrw + u_j)c_j & (wrw + u_j)c_k \\ (wrw + u_k)c_i & (wrw + u_k)c_j & (wrw + u_k)c_k \end{bmatrix}$$

31

Where
$$wsw = u_i + u_j + u_k$$

$$wrw = v_i + v_j + v_k$$

$$[K_e] = \frac{Re}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{Re}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

32

For the momentum diffussion, we have;
$$[K_{te}] = \frac{Pr}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{Pr}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

33

And for the Heat diffusion and Stabilization Matrix

$$[K_{se}] = \frac{U_{av}}{12}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{U_{av}}{12}\begin{bmatrix} b_i c_i & b_i c_j & b_i c_k \\ b_j c_i & b_j c_j & b_j c_k \\ b_k c_i & b_k c_j & b_k c_k \end{bmatrix} + \frac{V_{av}}{12A}\begin{bmatrix} c_i b_i & c_i b_j & c_i b_j \\ c_j b_i & c_j b_j & c_j b_j \\ c_k b_i & c_k b_j & c_k b \end{bmatrix}$$

$$\frac{V_{av}}{12A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

34

Where U_{av} and V_{av} are the average values of U and V over the element. The discretization of the CBS steps requires three more matrices and four forcing vectors to complete the process. The matrix from the discretized second-order terms for pressure calculations is:

$$K = \frac{1}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{1}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix}$$

35

The first gradient matrix in the X-direction is:

$$[G_u] = \frac{1}{6}\begin{bmatrix} b_i & b_j & b_k \\ b_i & b_j & b_k \\ b_i & b_j & b_k \end{bmatrix}$$

36

And the second-gradient matrix in the Y-direction is

$$[G_v] = \frac{1}{6}\begin{bmatrix} c_i & c_j & c_k \\ c_i & c_j & c_k \\ c_i & c_j & c_k \end{bmatrix}$$

37

The forcing vectors of the X-component of the momentum equation is:

$$[f_1] = \frac{\Gamma}{4A} Re \begin{bmatrix} b_i u_i & b_j u_j & b_k u_k \\ b_i u_i & b_j u_j & b_k u_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} Re \begin{bmatrix} c_i u_i & c_j u_j & c_k u_k \\ c_i u_i & c_j u_j & cb_k u_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v$$

38

Where $i\,j$ is the boundary edge of an element.

The forcing vector of the Y- component momentum equation is:

$$[f_2] = \frac{\Gamma}{4A} Re \begin{bmatrix} b_i v_i & b_j v_j & b_k v_k \\ b_i v_i & b_j v_j & b_k v_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} Re \begin{bmatrix} c_i v_i & c_j v_j & c_k v_k \\ c_i v_i & c_j v_j & c_k v_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \qquad 39$$

The forcing vector from the discretion of the second-order pressure terms in pressure calculations is:

$$[f_3] = \frac{\Gamma}{4A} \begin{bmatrix} b_i P_i & b_j P_j & b_k P_k \\ b_i P_i & b_j P_j & b_k P_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} \begin{bmatrix} c_i P_i & c_j P_j & c_k P_k \\ c_i P_i & c_j P_j & c_k P_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \qquad 40$$

Finally, the forcing term due to the discretization of the second-order terms in the equation is:

$$[f_4] = \frac{\Gamma}{4A} Pr \begin{bmatrix} b_i \theta_i & b_j \theta_j & b_k \theta_k \\ b_i \theta_i & b_j \theta_j & b_k \theta_k \\ 0 & 0 & 0 \end{bmatrix}^n n_u + \frac{\Gamma}{4A} P_r \begin{bmatrix} c_i \theta_i & c_j \theta_j & c_k \theta_k \\ c_i \theta_i & c_j \theta_j & c_k \theta_k \\ 0 & 0 & 0 \end{bmatrix}^n n_v \qquad 41$$

The four steps of the characteristic based split scheme may now be written in matrix form.

Step 1: Intermediate Velocity Calculation X-Component

$$\left[M \frac{\Delta [\tilde{v}]}{\Delta t} \right] = -[C][U]^n - [K_m][U]^n - [K_b][U]^n + [f_1] \qquad 42$$

Intermediate Velocity Calculation Y-Component

$$\left[M\frac{\Delta[\tilde{U}]}{\Delta t}\right] = -[C][V]^n - [K_m][V]^n - [K_s][V]^n + [f_2]$$ 43

Step 2: Pressure Calculation

$$[K][P]^n = -\frac{1}{\Delta t}[G_u][\tilde{U}] + [G_u][\tilde{V}] + [f_3]$$ 44

Step 3: Velocity Correction

$$[M][U]^{n+1} = [M][\tilde{U}] - \Delta\tau[G_v][P]^n$$ 45a

$$[M][V]^{[n+1} = [M][\tilde{V}] - \Delta\tau[G_v][P]^n$$ 45b

Step 4: Temperature Computation

$$\left[M\frac{\Delta[\theta]}{\Delta t}\right] = -[C][\theta]^n - [K_f][\theta]^n - [K_s][\theta]^n + [f_4]$$ 46

4. Results and Discussion
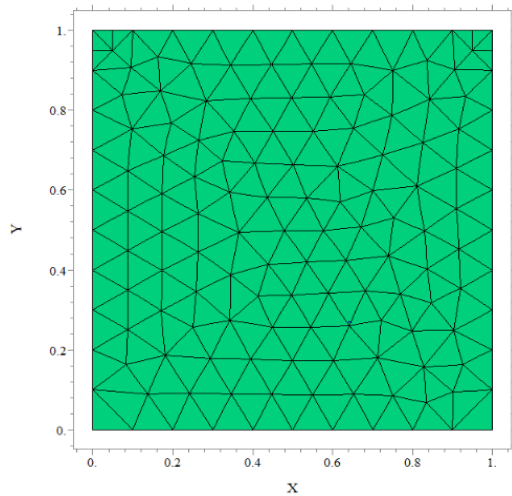
Fig.3 The Finite Element discretization of the solar oven enclosure
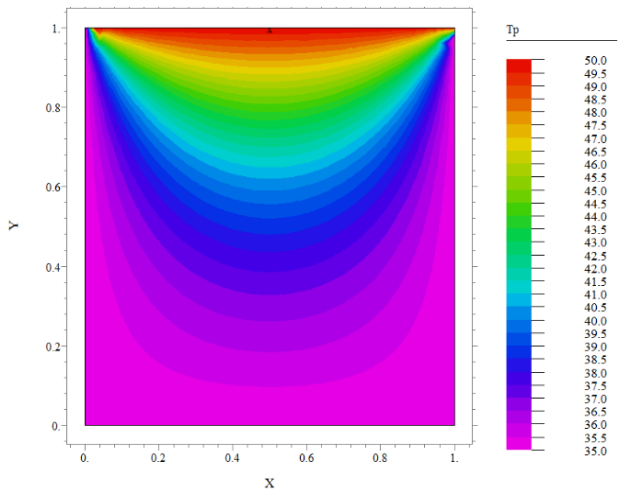
Fig.4 Temperature distribution in the oven after 30mins
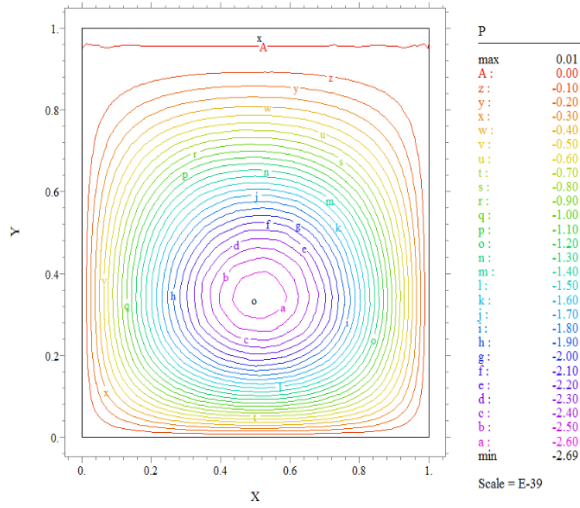
Fig.5 Pressure distribution in the oven after 30mins 30mins
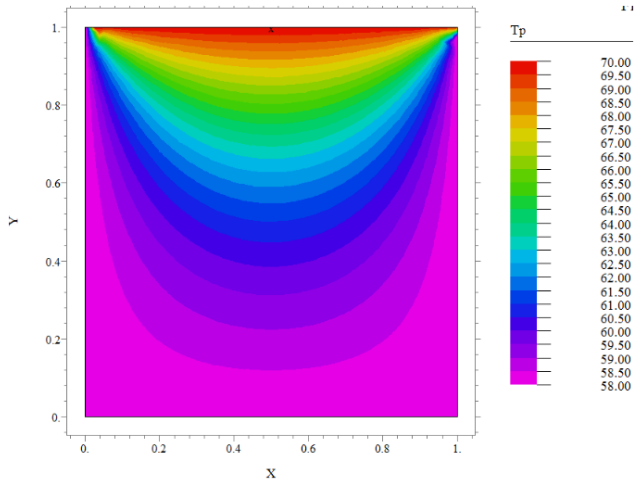
Fig.6 Temperature distribution in the oven after 2hrs
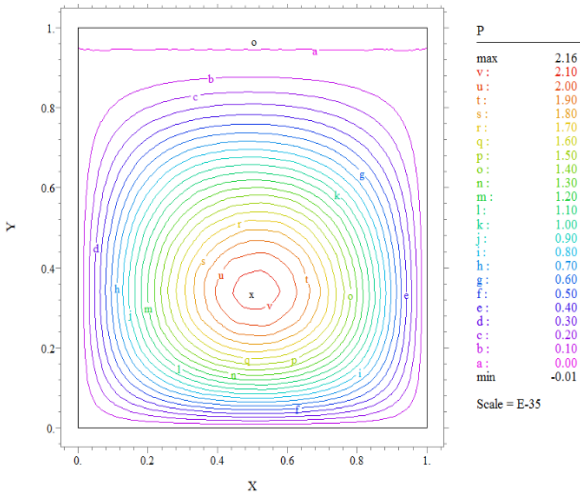
Fig.7 Pressure distribution in the oven after 2hrs 30mins 30mins
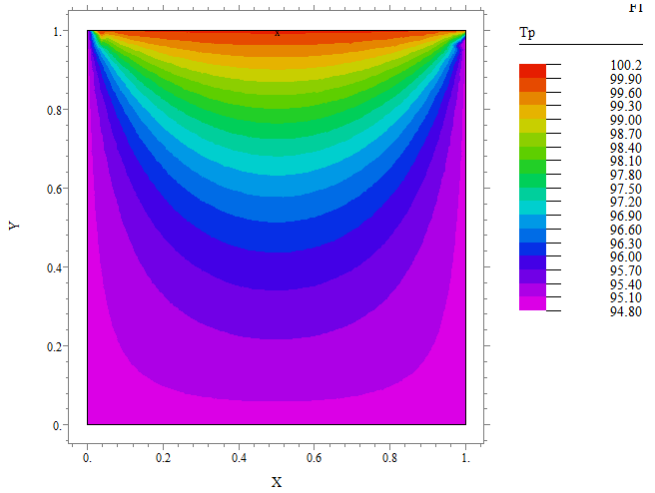

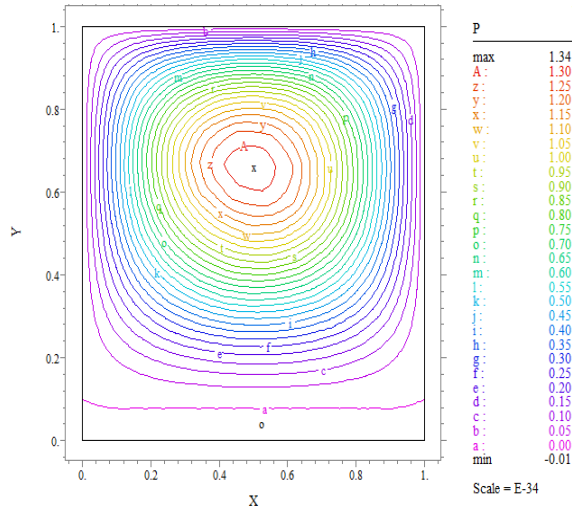Fig.8 Temperature distribution in the oven after 4hrs

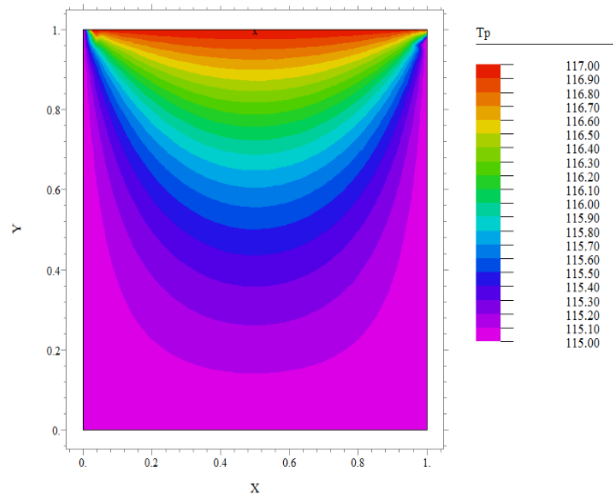Fig.9 Pressure distribution in the oven after 4hrs 30mins

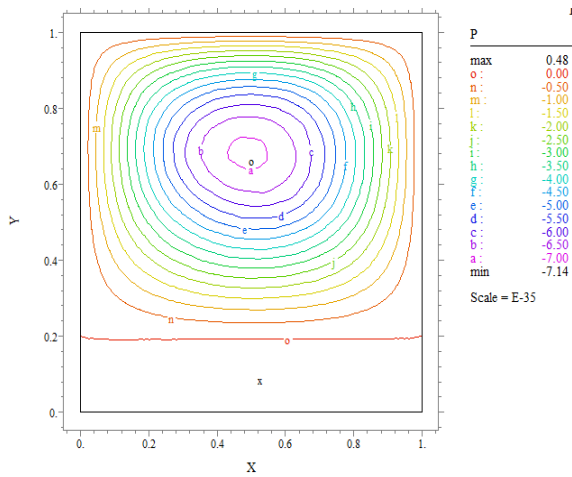Fig.10 Temperature distribution in the oven after 6hrs

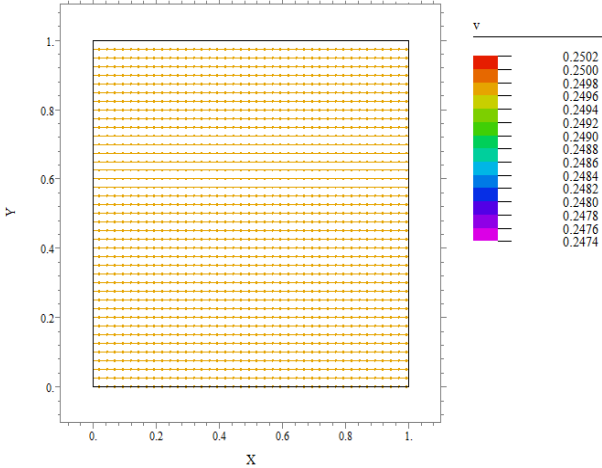Fig.11 Pressure distribution in the oven after 6hrs

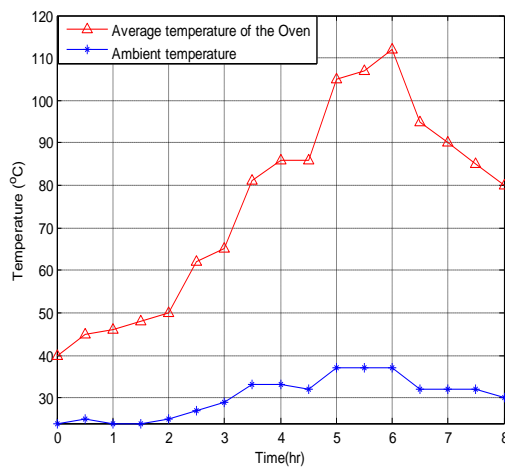Fig.12 Velocity distribution in the oven after 6hrs

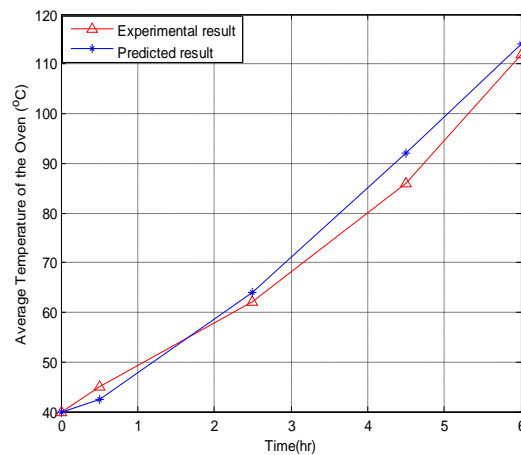Fig.13 Experimental results of the solar oven

Fig.14 Comparison of Experimental results with model results

Fig. 3-12 show the temperature pressure, and velocity distributions as predicted from the numerical analysis of the solar oven. The figures display the variation and the distributions in colours and contours. The simulated results shown above are the results of what happened in the Solar oven chamber during a period of 6hours of maximum insolation.

Fig.13 shows the experimental results of the average temperature of the solar oven and the ambient. The test was set out around 8am of 8th October, 2005. From the results, it was discovered that the time taken for the oven to attain a maximum temperature of 112°C was 6hours without the average insolation on no load. Fig. 14 shows the comparison of the predicted results with the experimental results. As it could be seen from the figure, a good agreement was achieved between the predicted results of the numerical analysis and the experimental results.

5. CONCLUSION

In this work, mathematical models were developed and solved by using Characteristics-Based Split (CBS) Finite Element Method to numerically investigate the temperature, velocity and pressure distributions in solar oven. The numerical results were compared with the Experimental results and a good agreement ware found between the two results. The work will greatly assist in developing and optimising the design solar oven especially for the developing countries.

Nomenclature

Gr   Grashof number
Pr Prandtl number
Re   Reynold Number
Pe   Peclet number
t time, s
T   temperature, ^{o}C
T_w Wall Temperature, ^{o}C
T_∞ Atmospheric temperature, ^{o}C
U   dimensionless velocity along x-axis
u_∞ Free stream velocity, m/s
V   dimensionless velocity along x-axis
x   length of the drying chamber, m
X   dimensionless length of the drying chamber
y   breadth of the drying chamber, m
Y   dimensionless breadth of the drying   chamber

Symbol

α thermal diffusivity, m^2/s
θ dimensionless temperature
τ dimensionless time
ρ density, kg/m^3
μ Coefficient of dynamic viscosity, kg/ms
β   Volumetric expansion coefficient

References

Adegoke, C. O. and Fasheun, T. A (1998): Performance evaluation of a solar cooker under a tropical humid climate, Nigeria Journal of Renewable Energy vol. 6, nos. 1&2 Pp 71-74

Amer, E. H (2003): Theoretical and experimental assessment of a double exposure solar cooker, Energy Conversion and Management vol. 44 Pp 2651-2663

ASAE Standards. Testing and Reporting Solar Cooker Performance. *ASAE S580* Jan 2003

Babuska."The finite element method with Lagrangian multipliers," Numer. Math., v. 20, 1973, pp. 179-192.

Barabara, P. K (1994): Solar panel cookers, a new low technology design line proceeding of the second world conference on solar cookers held between July 12 -15 Heredia Costa Rica

Bello, A.M.A., Makinde, V., Sulu, H.T., (2010), performance, tests and thermal efficiency of a constructed solar Box Cooker at aGuinea Savannah Station (Nigeria). Journal of American Science**Vol**.6 pp32

Beth, Halacy and Dan, Halacy (1992): Cooking with the sun, how to build and use solar cookers Pp. 3-6

Brezzi F. "On the existence, uniqueness and approximation of saddle-point problems arising from Lagrangian multipliers." RAIRO Ser. Rouge, v. 8. 1974, pp. 129-151

Daniel, M. K (1990): Comparative study of box type solar in Nicaragua solar and wind technology vol. 7, no. 4 Pp. 463-471

Devadas, R. P. (1992): Solar cooking experiments with different foods, Proceedings of the first world conference on solar cooking held between June

19=20 at University of the Pacific Stickton, California U.S.A Pp. 77-84

Devadas, R. P (1997): An option for cooking solar energy, proceedings of the third international conference on solar cooker use and technology, Jan. 6-10, Coimbatore 641043 Tamil Nadu India Pp. 15-17

Diso D.G, Chiromawa, N.L, Hotoro, M.A.Y. and Ali M.H., (2008), A comparative study of Solar cooker with array of reflectors and single reflector.Techno Science Africana Journal

Duffie, J. A and William, A. B (1974): Solar energy thermal processes, John Wiley Interscience, New York Pp. 20

Ekechukwu, O. V and Ugwuoke, N. T (2003): Design and measured performance of a plane reflector augmented box type solar energy cooker, Renewable Energy vol. 28 Pp. 1935-1952

Felix, A. P, Karl-Heinz, R and Martin, S (2002): Solar thermal system, successful planning and construction, Solar praxis Berling, Pp. 15-17, 101-105

Folmer, J. C and Franzen, w (2003): Study of polymer glasses by modulated differential scanning calorimetry, Stefan Journal of Chemical Education 80(7) Pp. 813-818

Franco, J., Cadena, C. and Saravia, L.: "Multiple use communal solar cookers". Solar Energy 77, pp. 217–223, 2004

Frederick, J. B and Eugene, H (1997): College Physics, Schaum's outline McGraw-Hill Pp. 346

Gary, H. P (1994): Solar cooking technology and promotional programme in India, proceedings of the second world conference on solar cookers use and technology held at Universidad National, Heredia Costa Rica

Hussein, H. M. S, El-Ghetang, H. H and Nada, S.a (2008): Experimental investigation of novel indirect solar cooker with indoor PCM thermal storage and cooking unit, Energy Conversion and Management vol. 49, Pp. 2237-2246

Kimambo, C. Z. M, (2007) Development and Performance testing of Solar cooker. Journal of Energy in South Africa. Vol. 18, No 3

Kulkarni, S. S, Joshi, J. B., Pandit, A. B and Patel, S. b (1997): Development of high efficiency concentrating solar cookers, Proceedings of the third International conference on solar cookers use and technology held between Jan 6-10 in Coimbatore, Tamil Nadu, India Pp. 244-249

Kumar, S. (2005), Estimation of Design parameters for thermal of box-type Solar cooker. Renewable Energy Journal **Vol.**30, 1117 - 1126

Lewis R.W., Nithiarasu P. and SeetharamuK.N. Fundamental of the Finite Element Method for Heat and Fluid Flow John Wiley and Sons, Ltd, 2004.
Lof, G. O. G (1963): Recent investigations in the use of solar energy for cooking, SolarEnergy vol. 7 Pp. 125

Malik, A. Q and Hamid Bin Hussen (1996): Development of a solar cooker in Brunces Darussalam, Renewable Energy vol. 7 no. 4 Pp. 419-425

Mark, A. (2009) Principles of solar cooker design. www.sciencedirect.com (15/04/2009.
Mazen, M. A., Mohammed, A. H., Salah, A. and Badran, O. (2009) Evaluation Thermal

Melanic, S (2006): Field test of the hot pot, a new solar cooker in West Africa, solar household energy Incorporated.Solar cooking for Human Development and Environment Relief

Nahar,. N. M. (1988), Performance and testing of a low cost solar water heater cum solar

Nandwani, S. S (1992): Twelve years experience with solar cookers, necessary tools for a cleaner environment, Proceedings of the first world conference on solar cooking held between June 19-20 in University of the Pacific Stockton California U.S.A, Pp. 4-11

Negi, B. S and Purohit, I (2005): Experimental investigation of a box type solar cooker employing a non-tracking concentrator, Energy Conversion and Management, vol. 46, Pp. 577-605

Olusegun, H. D (2006): Design construction and testing of a solar box cooker using locally sourced material, Journal of Research Information in civil energineering, vol. 3 no. 1 Pp. 102-125

Passamai, V.: "Laboratory and field experiments with a solar oven" (Experiencias de laboratorio y de

campo con un horno solar). Avances en Energies Renovables y Medio Ambiente, Vol. 2, No. 1, pp. 02.29-02.03,1998.

Patel, N. V. and Philip, S. K (2000): Performance evaluation of three solar concentrating cookers, Renewable Energy vol. 20, Pp. 347-354

Performance of solar cookers under Jordan climate. salahabdalah@asu.edu.jo
Rai, G.D. (2008), Solar Energy Utilization, 5th edition. Khano Publishers New Delhi, India (pp 522 - 528)

Ramon, C (2004): Solar cooker review, vol. 10, no. 2, http//:solarcooking.org

Renewable Energy Journal **Vol.**30

Shaw, Shaw (2005): Development of a comparative frame work for evaluating the performance of solar cooking devices, Dept of Physics, Appied Physics and Astronomy, Renseselaes Polytechnic Institute 1108th St Troy NY 12130

Sukhatme, S. P., (2005) Solar Energy, Principles of thermal collection and storage, 2nd Edition, 10, Tata McGraw-Hill Publishing Company Limited, New Delhi, India (pp102).

Volker, Q (2003): The sun as an energy source, Renewable Energy World, vol. 6, no. 5, Pp. 90-92

Volker, Q (2004): Solar thermal water heating, Renewable Enegry World, vol. 7 no. 2 Pp95-96

Warren, R. C (1994): Solar cooking the history problems parameters and goals, Proceedings of the second world conference on solar cookers use and technology held between July 13-15 in Heredia Costa Rica

Wary, Xiping (1992): Solar cooking in China, Proceedings of the first World conference on solar cooking held between June 19-20 at University of the Pacific Stockton California U.S.A