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HEAT TRANSFER IN BOUNDARY LAYER VISCOLASTIC FLUID FLOW OVER ANEXPONENTIALLY STRETCHING SHEET

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Abstract

The paper presents the study of momentum and heat transfer characteristics in a visco-elastic boundary layer fluid flow over an exponentially stretching continuous sheet with non-uniform heat source. The flow is generated solely by the application of two equal and opposite forces along the x-axis such that stretching of the boundary surface is of exponential order in x and influenced by uniform magnetic field applied vertically. The non-linear boundary layer equation for momentum is converted into ordinary differential equation by means of similarity transformation. Approximate analytical similarity solutions is obtained for the dimensionless stream function and velocity distribution function after transforming the boundary layer equation into Riccati type and solving it sequentially. Heat transfer equation is then solved using Runge-Kutta fourth order method. The accuracy of the analytical solutions is also verified by comparing the solutions obtained to those in literature when Hartmann number is zero. The effects of various physical parameters on velocity, skin friction, temperature and Nusselt number profiles are presented graphically.

Key words: Exponential Stretching sheet, Prandtl number, Non-uniform heat source/sink, Similarity Solution, visco- elastic fluid, boundary layer flow, skin-friction.

1. Introduction

Momentum and heat transfer in a visco-elastic boundary layer flow over a stretching sheet have been studied extensively in the recent past because of its ever increasing usage in polymer processing industry, in particular in manufacturing process of artificial film and artificial fibres in some applications of dilute polymer. The transport of momentum heat and mass in laminar boundary layers on the moving inextensible stretching surfaces has considerable practical relevance. For example, in electrochemistry (Chin (1975), Gorla (1978)), polymer processing (Giffth (1964), Erickson (1966)) and in fibre industries.

In view of increasing importance of non-Newtonian flows, Rajgopal et al (1984), examined a special class of Visco-elastic fluids known as second order fluids. Subhas and Siddappa(1986) studied the flow of Visco-elastic fluids of the type Walter's liquid B past a stretching sheet. Since the Walter's liquid B is a fluid that has a short memory, they arrived at the some non-linear equation as that can be derived with the boundary layer approximation.

Subhas (2002) studied visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity. All these studies deals with the studies concerning non-Newtonian flows and heat transfer in the absence of magnetic fields, but present years we find several industrial applications such as polymer technology and metallurgy (Chakrabarti and Gupta (1979)) where the magnetic field is applied in the visco-elastic fluid flow. Andersson (1992) investigated the flow problem of electrically

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Nomenclature			
f	dimensionless stream function	p	scalar pressure
k_0	elastic parameter	u_w	main stream velocity/free stream
k_1	viscoelastic parameter	u, v	the velocity components
l	reference length	x, y	the coordinate axis
Re	non-dimensional Reynolds number	C_p	specific heat
T	stress tensor	Q	Heat source/Sink parameter
¦	similarity variable	**Greek letters**	
$\sim_1, \sim_2$	the normal stress moduli		acceleration/deceleration parameter
A_1 and A_2	kinematical tensors;	«	dynamic viscosity
B_0	applied magnetic field	з	kinematic viscosity
M	MHD parameter	°	Fluid density
Ec	Eckert number	±	electrical conductivity
c_f	dimensionless skin-friction coefficient	¤	Heat source/sink parameter
Pr	Prandtl number		

conducting visco-elastic fluid past a flat and impermeable elastic sheet. Lawrence and Rao (1992) studied the non-uniqueness of the MHD flow of second order fluid past a stretching sheet.

A new dimension was added to this investigation by Elbashbeshy (2001) who examined the flow and heat transfer characteristics by considering an exponentially stretching continuous surface. Subhas *et al* (2001), in their paper report heat transfer in a MHD Visco-elastic fluid over a stretching surface. Kumari and Nath (2001), studied MHD flow of Non-Newtonian fluids over continuously moving surface with a parallel free stream. Subhas *et al* (2004), investigate Non-Newtonian Magneto hydrodynamic flow over a stretching surface with heat and mass transfer.

Recently, Siddheshwar and Mahabaleshwar (2005), examined the effects of radiation and heat source on MHD flow of a visco-elastic liquid and heat transfer over a stretching sheet. In his work Sujit (2006) obtained first and second order similarity solution of boundary layer visco-elastic fluid flow over an exponentially stretching sheet with uniform heat source. In 2012, Midya present the study of diffusion of reactive species undergoing first-order chemical reaction in a boundary layer flow of an incompressible homogeneous second order fluid over a linearly shrinking sheet in the presence of a transverse magnetic field. The study reveals that the velocity is getting closer towards wall for increasing magnetic parameter whereas it is going away from the wall for increasing visco-elastic parameter. It is also found that the diffusion of reactive species is considerably reduced with increasing values of Schmidt number, magnetic and reaction rate parameter whereas it is increased for enhanced values of visco-elastic parameter. Negative concentration is observed in some cases which may not have real world applications.

In reality, most of the fluids considered in industrial applications are more non-Newtonian in nature, especially of visco-elastic type than viscous type. Motivated by all these studies we intend to study the visco-elastic fluid flow and heat transfer over stretching sheet in the presence of uniform magnetic field in the boundary layer region. Since for MHD flows, the effect of internal heat generation is important and hence, it is taken into consideration, and combined effect of visco-elasticity, magnetic field and Reynolds number on the skin friction coefficient are considered.

2. Formulation of the problem

2.1. Preliminaries

The constitutive equation of an incompressible second order fluid is given by

$$T = -pI + «A_1 + \sim_1 A_2 + \sim_2 A_1^2 \qquad (2.1)$$

Kinematical tensors A_1 and A_2 are defined by

$$A_1 = (grad\ q) + (grad\ q)^T \cdot A_1$$
$$A_2 = \frac{dA_1}{dt} + A_1 \cdot (grad\ q) + (grad\ q)^T \cdot A_1 \qquad (2.2)$$

Equation (2.1) was derived by Coleman and Noll (1960) using the postulates of gradually fading memory. Using some experimental data verification Fosdick and Rajagopal (1979) gave the range of values of « , $\sim_1$ and $\sim_2$ as

$$« \ge 0,\ \ \sim_1 \le 0,\ \ \sim_1 + \sim_2 \ne 0. \tag{2.3}$$

Making use of Eq.(2.1) Beard and Walters (1964) derived the steady state two-dimensional boundary layer equation for a visco-elastic fluid flow in the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = ³\,\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} \tag{2.4}$$

This equation has been derived with the assumption that the normal stress is of the same order of magnitude as that of the shear stress, in addition to the usual boundary layer approximations.

2.2. Flow governing equations

In formulating the problem we consider the following assumption.

(i) The boundary sheet is assumed to be moving axially with a velocity of exponential order in the axial direction and generating the boundary layer type of flow.

(ii) A steady two-dimensional laminar flow of an incompressible, electrically conducting visco-elastic liquid (Walters' liquid B model) due to an exponentially stretching sheet is considered.

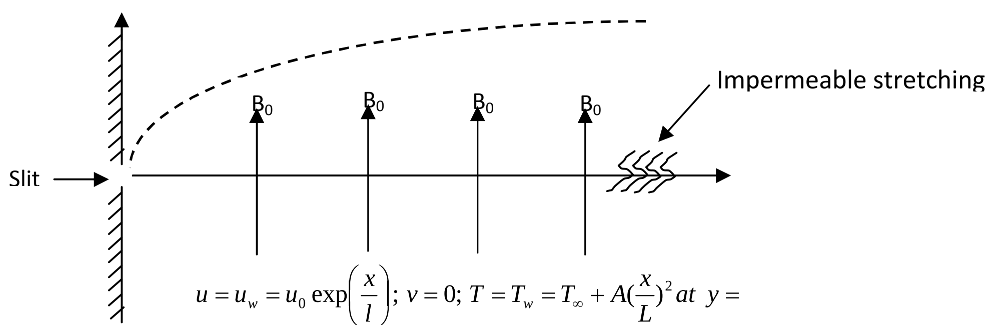

Figure 1: Boundary layer over an impermeable exponentially stretching sheet.

The sheet lies in the plane y = 0 with the flow being confined to y > 0. The coordinate x is being taken along the stretching sheet and y is normal to the surfaced, two equal and opposite forces are applied along the x-axis, so that the sheet is stretched, keeping the origin fixed. This flow obeys the rheological equation of state derived by Beard and Walters (1964).For ease of analysis, we assume that dissipation due to elastic stresses is negligible. It is also assumed that boundary layer approximations can be used to simplify the basic equations governing the transport of momentum and heat. Further, this flow exposed under the influence of uniform transverse magnetic field. The set of basic equations, together with the continuity equation, then become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.5}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=³\,\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\}-\frac{±B_0^2}{°}u \tag{2.6}$$

$$°\,c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+«\left(\frac{\partial u}{\partial y}\right)^2+Q\left(T-T_\infty\right)+±B_0^2u^2 \tag{2.7}$$

where all parameters have their usual meanings. The appropriate boundary conditions for this purpose are:

$$\begin{aligned}&at\ y=0; \qquad u=u_w=u_0\exp\left(\frac{x}{l}\right);\ v=0;\ T=T_w=T_\infty+A(\frac{x}{L})^2\\ &at\ y=\infty; \qquad u\to 0;\ T\to T_\infty\end{aligned} \tag{2.8}$$

where A is a positive parameter.

Assuming that material properties appearing in the above equation are constant, the momentum equation can be solved regardless of the energy equation.

3. Solution of the boundary layer equations

At this stage we try to find a suitable similarity variable, ¦ , such that the problem can be reduced to a set of ODEs instead of PDEs. And it turns out that the following similarity variable can meet our purpose:

$$¦=y\sqrt{\frac{u_w}{2³\,l}}\,e^{\frac{x}{2l}} \tag{3.1}$$

Using this similarity variable, the stream function $·\,(x,y)$ and the temperature field $T(y)$ can be made dimensionless as f and ¯ respectively:

$$·\,(x,y)=\sqrt{2³\,lu_w}\,f(¦)e^{\frac{x}{2l}};\ ¯(¦)=\frac{T-T_\infty}{T_w-T_\infty} \tag{3.2}$$

Assuming the flow to be two-dimensional, the velocity components can be related to the stream unction as:

$$u=\frac{\partial ·}{\partial y}=u_w f'e^{\frac{x}{l}}\ ,\quad v=-\frac{\partial ·}{\partial x}=-\sqrt{\frac{³u_w}{2l}}e^{\frac{x}{2l}}\left(f+¦f'\right) \tag{3.3}$$

Substituting these dimensionless parameters into the governing equations (2.1) to (2.4) one would obtain:

$$2f'^2-f\,f''=f'''-k_1\left\{3f'f'''-\frac{1}{2}ff^{iv}-\frac{3}{2}f''^2\right\}-Mf' \tag{3.4}$$

$$¯''-\Pr\left(f'-f\right)¯'+\Pr ¤¯=\Pr Ec\left(f''^2+Mf'^2\right) \tag{3.5}$$

$$\Pr=\frac{«c_p}{k},\ ¤=\frac{Q}{a«c_p},\ M=\frac{±B_0^2u_w}{°c_p},\ Ec=\frac{³^2l^2}{u_w^{\ 2}c_p},\ k_1=\frac{u_wk_0}{³}$$

In terms of dimensionless parameters, the boundary conditions required to solve Equations(3.4) and (3.5) are:

$$\left.\begin{aligned}&f(0)=0,\quad f'(0)=1,\quad f'(\infty)=0,\quad f''(\infty)=0\\ &¯(0)=1\ ,\quad ¯(\infty)=0\end{aligned}\right\} \tag{3.6}$$

Integrating equation (3.4), we obtain

$$f''+ff'=-s-Mf+\int_0^{¦}3f'^2+k_1\left\{3f'f'''-\frac{1}{2}ff'^{v}-\frac{3}{2}f''^2\right\}d¦ \tag{3.7}$$

where $s=f''(0)$.

For ¯ $\to\infty$, we get

$$s = -\int_0^\infty 3f'^2 + k_1 \left\{ 3ff''' - \frac{1}{2} ff^{iv} - \frac{3}{2} f''^2 \right\} d\eta \tag{3.8}$$

We integrate equation (3.7) once again and apply boundary conditions (3.6). This yield

$$f' + \frac{1}{2} f^2 = 1 - s\eta - \int_0^\eta \left[Mf + \int_0^\xi \left(3f'^2 + k_1 \left\{ 3ff''' - \frac{1}{2} ff^{iv} - \frac{3}{2} f''^2 \right\} \right) d\tau \right] d\xi \tag{3.9}$$

Now, the solution procedure of the equation (3.9) may be reduced to the sequential solutions of the Riccati-type equations

$$f'^{(n)} + \frac{1}{2} f^{(n)^2} = RHS\left(f'^{(n-1)}, f''^{(n-1)}, f'''^{(n-1)}, f^{iv(n-1)} \right) \tag{3.10}$$

This iteration algorithm has to be solved by substituting suitable zero-order approximation $f'^{(0)}(\eta)$ for $f'(\eta)$ on the *R.H.S* of equation (3.9).

We assume zero-order approximation of $f'^{(0)}(\eta)$ as

$$f'^{(0)}(\eta) = \exp(-s_0\eta) \tag{3.11}$$

which satisfies the boundary conditions at infinity. Integrating the Eq.(3.10) and making use of boundary condition at $\eta = 0$ of (3.6) we get

$$f^{(0)}(\eta) = \frac{1 - \exp(-s_0\eta)}{s_0} \tag{3.12}$$

Substituting all the derivatives of zero-order approximation $f'^{(0)}(\eta)$ into *R.H.S* of equation (3.9) and assuming that first-order iteration $f'^{(1)}(\eta)$ on the L.H.S of equation (3.9) satisfying all the boundary conditions at $\eta = 0$ (3.6) we obtain the value of s as

$$s = \sqrt{\frac{3+M}{4(1-k_1)}} \tag{3.13}$$

Here the equation for first-order iteration f takes the form

$$f'^{(1)}(\eta) + \frac{1}{2} f^{(1)^2}(\eta) = 1 + \frac{(3 + k_1 s_0^2)}{4s_0^2}\left(e^{-2s_0\eta} - 1\right) + \frac{k_1}{2}\left(e^{-s_0\eta} - 1\right) + Me^{-s_0\eta} \tag{3.14}$$

Equation (3.14) is the non-linear Riccati equation and this can be solved for $f^{(1)}(\eta)$ in terms of confluent hyper-geometric Whittaker function (Abramowitz and Stegun 1964):
Letting

$$b_1 = \frac{3}{4}s^2 + \frac{1}{4}k_1,\ b_2 = 1 - b_1 - \frac{1}{2}k_1,\ b_3 = \frac{1}{2}k_1 + M,$$

$$a_1 = -\frac{b_2}{4s}\frac{\sqrt{2}}{\sqrt{b_1}},\ a_2 = \frac{\sqrt{2b_3}}{2s}, a_3 = \frac{\sqrt{2b_1}}{s},\ a_5 = 2b_1^{3/2}s - cb_1b_2,\ \ a_6 = -\frac{2C_1b_1^{3/2}s}{S_1} + \frac{2\sqrt{2}C_1b_1b_2}{S_1}$$

we have

$$f^{(1)}(¦)=\left[\begin{array}{l}\dfrac{1}{2}\dfrac{\left(\dfrac{2C_1b_1^2\sqrt{2}Whitta\ker\left(a_1,a_2,a_3e^{-s¦}\right)}{S_1}-2\sqrt{2}Whitta\ker\left(a_1,a_2,a_3e^{-s¦}\right)b_1^2\right)e^{-s¦}}{b_1^{\left(3/2\right)}\left(\dfrac{C_1Whitta\ker W\left(a_1,a_2,a_3e^{-s¦}\right)}{S_1}+M_1\right)}\\ -\dfrac{2C_1sWhitta\ker W\left(1-a_1,a_2,a_3e^{-s¦}\right)}{S_1C_2Whitta\ker W\left(a_1,a_2,a_3e^{-s¦}\right)}\\ +\dfrac{S_2Whitta\ker M\left(1-a_1,a_2,a_3e^{-s¦}\right)}{b_1^{\left(3/2\right)}\left(-\dfrac{C_1Whitta\ker W\left(a_1,a_2,a_3e^{-s¦}\right)}{S_1}\right)+Whitta\ker M\left(a_1,a_2,a_3e^{-s¦}\right)}\\ +\dfrac{1}{2}\dfrac{\left(a_5Whitta\ker M\left(a_1,a_2,a_3e^{-s¦}\right)+a_6Whitta\ker W\left(a_1,a_2,a_3e^{-s¦}\right)\right)}{b_1^{\left(3/2\right)}\left(-\dfrac{C_1Whitta\ker W\left(a_1,a_2,a_3e^{-s¦}\right)}{S_1}\right)+Whitta\ker M\left(a_1,a_2,a_3e^{-s¦}\right)}\end{array}\right] \quad (3.15)$$

where

$$M_1=Whitta\ker M\left(a_1,a_2,a_3\right),\ M_2=Whitta\ker M\left(1-a_1,a_2,a_3\right),$$
$$W_1=Whitta\ker W\left(a_1,a_2,a_3\right),\ W_2=Whitta\ker W\left(1-a_1,a_2,a_3\right),$$

and

$$C_1=\sqrt{2}M_1b_1\left(s\sqrt{2b_1}-b_2-2b_1\right)+\sqrt{2}M_2b_1\left(s\sqrt{2b_1}+b_2-2\sqrt{b_1b_3}\right)$$
$$C_2=-\sqrt{2}M_1b_1\left(b_2+2b_1\right)+\sqrt{2}M_2b_1\left(b_2-2\sqrt{b_1b_3}\right)$$

with

$$S_1=\sqrt{2}W_1b_1\left(s\sqrt{2b_1}-b_2\right)+2W_2b_1\left(2s\sqrt{b_1}+b_1\sqrt{2}\right)$$
$$S_2=\sqrt{2}b_1b_2-2\sqrt{2b_1b_3}-2sb_1^{3/2}$$

The dimensionless skin-friction coefficient c_f is expressed as

$$c_f=\frac{3\dfrac{\partial^2u}{\partial y^2}-k_0\left\{u\dfrac{\partial^2u}{\partial x\partial y}-2\dfrac{\partial u}{\partial y}\dfrac{\partial v}{\partial y}\right\}-\dfrac{\pm B_0^2}{°}u}{u_w^2\exp\left(\dfrac{2x}{l}\right)}\text{ at }y=0$$

$$c_f=\frac{-s_0\left(1-\dfrac{7}{2}k_1\right)-M}{\mathrm{Re}}$$

Here, $\mathrm{Re}=\dfrac{u_wl}{3}$ is the non-dimensional Reynolds number.

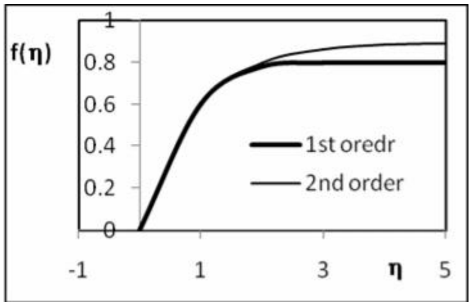

Figure 2: Vertical velocity profile obtained from zero and first order solution when $M = 5$ and $k_1 = 0.2$

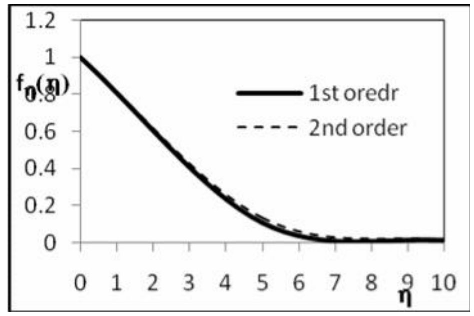

Figure 3: Horizontal velocity profile obtained from zero and first order solution when $M = 5$ and $k_1 = 0.2$

4. Heat Transfer

The governing boundary layer heat transport equation in the presence of space- and temperature- dependent internal heat generation/ absorption for two-dimensional flow is

$$\theta'' - \Pr\left(f' - f\right)\theta' + \Pr\,\text{¤}\,\theta = \Pr Ec\left(f''^2 + Mf'^2\right) \tag{4.1}$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{4.2}$$

The energy equation (4.1) together with the boundary condition (4.2) is a linear second order ordinary differential equation with variable coefficient, *f*(*η*), which is known from the solution of the flow equation (3.12) and (3.15) and the Prandtl number Pr is assumed constant. From Figures (2) and (3) above, it is clear that both zero and first order solutions are very close. Hence use have been made of zero order solution in solving equation (4.1) numerically under the boundary condition (4.2) using central differences for the derivatives and Thomas' algorithm for the solution of the set of discritized equations. The resulting system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the *η*-direction can be used instead with *η* chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each of uniform step size 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of everyone of *f*, f', f'', or f''' for the last 2 approximations differed from unity by less than 10^{-5} at all values of *η* in $0 < \eta < \eta_\infty$.

5. Results and Discussion:

In the paper we investigate the boundary layer flow and heat transfer in a visco-elastic liquid over an exponentially stretching sheet in presence of non-uniform heat source. Similarity solution is used to obtain the velocity distribution, which is governed by non-linear differential equation.

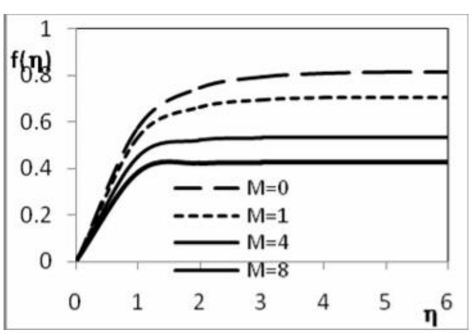

Figure 4: Variation of vertical velocity for various values of M

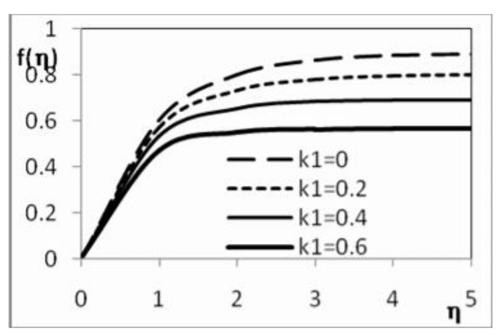

Figure 5: Variation of vertical velocity for various values of k_1

Figures 4 and 6 is a graphical representation which depicts the effect of Magnetic field Parameter M on the velocity profiles $f(\eta)$ and $f_\eta(\eta)$ respectively. It is found that the effect of Magnetic field Parameter M is to reduce the velocity, significantly in the visco-elastic flow in comparison with the viscous flow, this is due to the fact that increase of M signifies the increase of Lorentz force, which opposes the flow in the reverse direction. The graphs for the non-dimensional velocity profiles $f(\eta)$ and $f_\eta(\eta)$ for different values of the visco-elastic parameter k_1 are shown in Figures 5 and 7 respectively. The analysis of the figure demonstrates that the effect of the visco- elastic parameter k_1 is to decrease velocity throughout the boundary layer flow field, which is quite obvious.

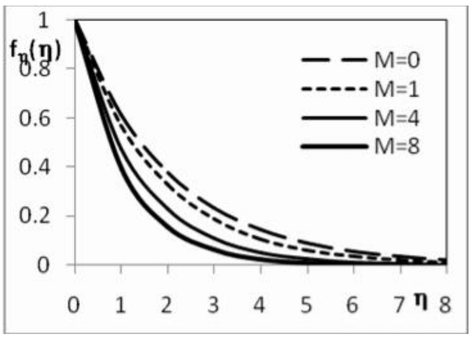

Figure 6: Effect of the parameter M on the horizontal velocity $f'(\eta)$

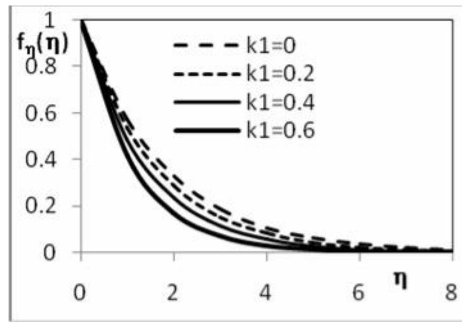

Figure 7: Effect of the parameter k_1 on the horizontal velocity $f'(\eta)$

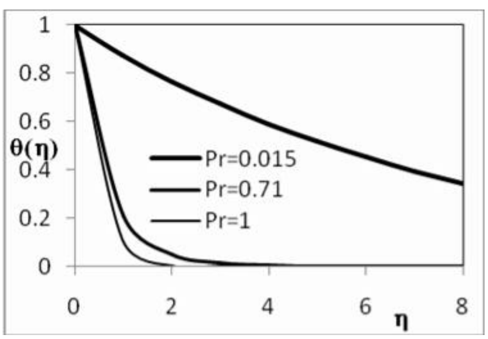

Figure 8: Temperature distribution for various values of Pr

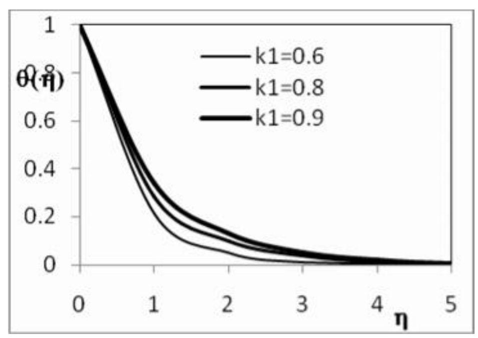

Figure 9: Temperature distribution for various values of k_1

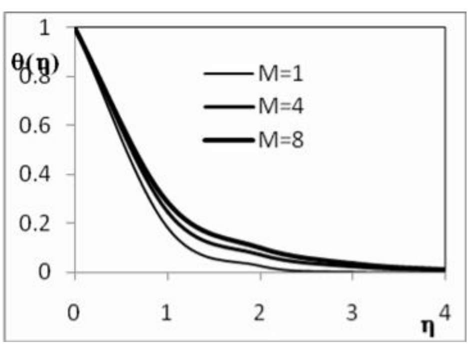

Figure 10: Temperature distribution for various values of M

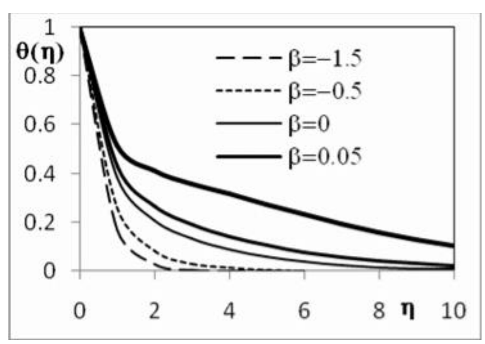

Figure 11: Temperature distribution for various values of

Figures (8) and (9) is plotted for the temperature distribution for various Prandtl number and visco-elastic parameter k_1 respectively, it is an interesting note that there is a significant enhancement of temperature in both cases. On comparison of the curves it could be seen that there would be an increase in temperature in the flow region for lower values of Prandtl number, which result in increase of thermal boundary layer thickness as Prandtl number increases. Also, increase in visco-elastic parameter k_1 brings about an increase in temperature in the flow region

In Figure 10, we displayed the effect of M temperature distribution. On comparison of the curves, it is seen that temperature increases in the flow region due to the application of magnetic field. Here increase of magnetic force causes significant increase of thermal boundary layer thickness in the fluid flow. Figure 11 show the effect of heat source/sink on the thermal boundary layer of the flow field. We observe that for heat generation temperature distribution decreases throughout the boundary layer of the flow field as heat absorption and increases with heat generation.

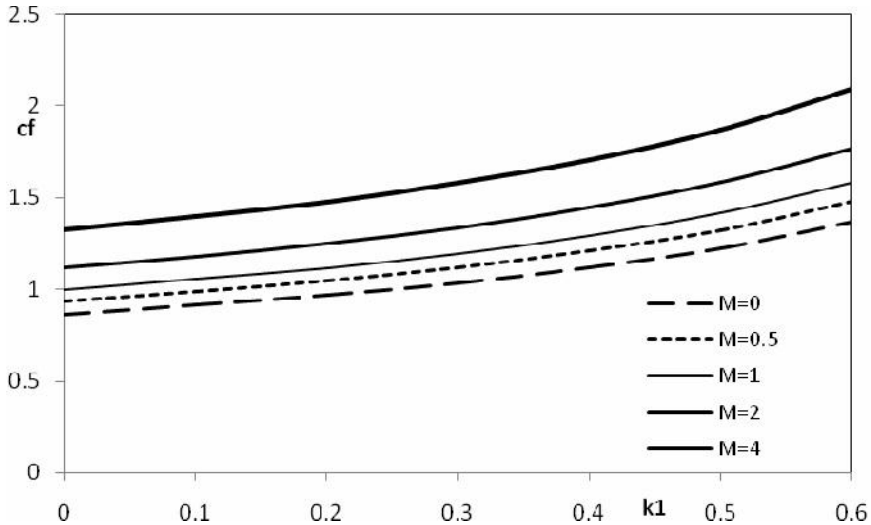

Figure 12: Skin – friction distribution for various values of Hartmann number

Table 1: Effect of Eckert number on horizontal velocity $f'(\eta)$

y	Ec=0.1	Ec=0.3	Ec=0.5
0	1	1	1
1	0.209893	0.187764	0.165634
2	0.047997	0.039665	0.031333
3	0.012325	0.009435	0.006545
4	0.003523	0.002528	0.001533
5	0.00109	0.000747	0.000405
6	0.000354	0.000236	0.000118
7	0.000118	7.77E-05	3.71E-05
8	4E-05	2.61E-05	1.22E-05
9	1.37E-05	8.88E-06	4.08E-06
10	4.69E-06	3.04E-06	1.39E-06

Table 1 show the effect of Eckert number on the temperature distribution. It could be seen that increase in Eckert number brings about decrease the temperature distribution throughout the boundary layer.

The graphs of the non-dimensional skin-friction parameter c_f against visco-elastic parameter for different values of the Hartmann number is shown in Figure 12. The figure shows that the skin friction parameter increases on the wall with the application of magnetic field. This is because of the magnetic force acts as a retarding force and causes the increase of shear stress. The combined effect of visco-elasticity and impermeability of the wall is to increase the skin friction at the wall largely. Here additional introduction of shear stress at the wall by magnetic field, non-Newtonian nature of visco-elastic flow and impermeability of the wall, thereby decreases the boundary layer thickness leads to increase the skin friction of the flow.

Table 2: Effect of flow parameters on wall sear stress

Pr	k_1	M	Nu		Ec	$(\quad < 0)$	Nu
0.015	0.2	2.0	0.1460		0.3	-0.3	1.7449
0.71	0.6	2.0	1.5796		0.5	-0.3	2.3233
1.0	0.2	2.0	1.9788		0.1	-5.0	2.4074
0.71	0.8	2.0	1.4568		0.1	-3.0	2.0523
0.71	0.9	2.0	1.3676		0.1	-1.0	1.5796
0.71	0.2	1.0	1.6076		0.1	0.0	1.1625
0.71	0.2	4.0	1.5443		0.1	0.1	1.2418
0.71	0.2	8.0	1.5122		0.1	0.2	2.8570
0.71	0.2	15	1.5007		0.1	0.3	0.9373

The heat transfer phenomena is usually analyzed from the numerical results of a physical parameter, namely wall temperature gradient and the same are documented in table 2. Analyzing the table reveals that the effect of increasing the value of Prandtl number, Eckert number and heat sink $(\quad < 0)$ is to increase the wall temperature gradient. While visco-elastic parameter, Hartmann number and heat source $(\quad < 0)$ reduces the Nusselt number. The results are in tune with what happens in regions away from the sheet.

Conclusions

A mathematical problem has been formulated for the heat and mass transfer in a visco-elastic fluid flow over an exponentially stretching impermeable sheet. In the solution procedure the non-linear differential equation is converted into an ordinary differential equation by applying similarity transformations. Sequential similarity solutions of the transformed momentum equation are obtained analytically by solving the non-linear Riccati type equation. Expressions are also obtained for the dimensionless skin-friction coefficient (c_f) and wall sear stress (Nu).

The important findings of the graphical analysis of the results of the present problem are as follows.

1. Increase in both heat generation and Hartmann number increase the temperature throughout the boundary layer.
2. The effect of increasing the values of the visco-elastic parameter k_1 is to increase the temperature distribution throughout the boundary layer and decrease the Nusselt number.
3. The effect of increasing the values of the visco-elastic parameter k_1 is to decrease the velocity through out the boundary layer.

4. The effect of increasing the values of the visco-elastic parameter k_1 is to decrease the skin-friction parameter (c_f) and the effect of increasing values of the hartmann number is to increase the skin – friction coefficient (c_f) and decrease the Nusselt number.

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