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Radiation and viscous dissipation effects for the Blasius and Sakiadis flows with a convective surface boundary condition

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Abstract

This study is devoted to investigate the radiation and viscous dissipation effects on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Using a similarity variable, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations, which are solved numerically by using shooting technique along side with the sixth order of Runge-Kutta integration scheme and the variations of dimensionless surface temperature and fluid-solid interface characteristics for different values of Prandtl number Pr, radiation parameter N_R, parameter a and the Eckert number Ec, which characterizes our convection processes are graphed and tabulated. Quite different and interesting behaviours were encountered for Blasius flow compared with a Sakiadis flow. A comparison with previously published results on special cases of the problem shows excellent agreement.

Keywords: Heat transfer; Blasius/Sakiadis flows; Thermal radiation; Eckert number; Convective surface boundary condition.

1. Introduction

Investigations of boundary layer flow and heat transfer of viscous fluids over a flat sheet are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Among these studies, Sakiadis [1] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows. Tsou et al. [2] analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis [1]. Erickson et al. [3] extended the work of Sakiadis [1] to include blowing or suction at the stretched sheet surface on a continuous solid surface under constant speed and investigated its effects on the heat and mass transfer in the boundary layer. The related problems of a stretched sheet with a linear velocity and different thermal boundary conditions in Newtonian fluids have been studied, theoretically, numerically and experimentally, by many researchers, such as Crane [4], Fang [5-8], Fang and Lee [9]. The classical problem (i.e., fluid flow along a horizontal, stationary surface located in a uniform free stream) was solved for the first time in 1908 by Blasius [10]; it is still a subject of current research [11,12] and, moreover, further study regarding this subject can be seen in most recent papers [13,14]. Recently, Aziz [15], investigated a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Very more recently, Makinde & Olanrewaju [16] studied the effects of thermal buoyancy on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition.

Olanrewaju & Makinde [17] presented the combined effects of internal heat generation and buoyancy force on boundary layer over a vertical plate with a convective surface boundary condition.

On the other hand, convective heat transfer with radiation studies are very important in process involving high temperatures such as gas turbines, nuclear power plants, thermal energy storage, etc. In light of these various applications, Hossain & Takhar [18] studied the effect of thermal radiation using Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Furthermore, Hossain et al. [19,20] have studied the thermal radiation of a gray fluid which is emitting and absorbing radiation in a non-scattering medium. Moreover, Bataller [21] presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow). This study is an extension of those analyses. It is aimed at analysing the effect of viscous Eckert number Ec, radiation parameter N_R on both Blasius and Sakiadis thermal boundary layers over a horizontal plate with a convective boundary condition. This boundary condition scarcely appears in the pertinent literature. Sajid and Hayat [22] examined the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. The most recent attempt for the Blasius and Sakiadis flows but without viscous dissipation term has been developed by Bataller [21] whose results we used for comparison including Aziz [15] and Makinde & Olanrewaju [16] which discussed Blasius flow. Interaction of thermal radiation and Eckert number with wall convection is included. Our results have been displayed for range of given parameters. Makinde and Maserumule [23] examined the inherent irreversibility and thermal stability for steady flow of variable viscosity liquid film in a cylindrical pipe with convective cooling at the surface. Makinde [24] investigated the similarity of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. Similarly, Makinde and Aziz [25] studied the MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. Makinde [26] examined the MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition.

The aim of the present paper is to report the effects of thermal radiation and Eckert number as well as Prandtl number Pr and convective parameter a on both Blasius and Sakiadis thermal boundary layers under a convective boundary condition.

2. Problems formulation

Taking into account the viscous dissipation and the thermal radiation terms in the energy equation, the governing equations of motion and heat transfer for the classical Blasius flat-plate flow problem can be summarized by the following boundary value problem [15-16,21]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2. \qquad (3)$$

The boundary conditions for the velocity field are:

$$\begin{aligned} &u=v=0 \;\; at \;\; y=0; \;\; u=U_\infty \quad at \;\; x=0, \\ &u\to U_\infty \;\; as \;\; y\to\infty, \end{aligned} \qquad (4)$$

for the Blasius flat-plate flow problem, and

$$\begin{aligned} &u=U_w; \quad v=0 \quad at \quad y=0, \\ &u\to 0 \quad as \quad y\to\infty, \end{aligned} \qquad (5)$$

for the classical Sakiadis flat-plate flow problem, respectively.

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$\begin{aligned} &-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right], \\ &T(x,\infty)=T_\infty. \end{aligned} \qquad (6)$$

Here u and v are the velocity components along the flow direction (x-direction) and normal to flow direction (y-direction), υ is the kinematic viscosity, k is the thermal conductivity, c_p is the specific heat of the fluid at constant pressure, ρ is the density, g is the acceleration due to gravity, μ is the dynamic viscosity, q_r is the radiative heat flux in the y-direction, T is the temperature of the fluid inside the thermal boundary layer, U_∞ is a constant free stream velocity and U_w is the plate velocity. It is assumed that the physical properties of the fluid are constant, and the Boussinesq and boundary layer approximation may be adopted for steady laminar flow. The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium.

The radiative heat flux q_r is described by Roseland approximation such that

$$q_r = -\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y}, \qquad (7)$$

where σ^* *and* K' are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Bataller [21], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \approx 4T_\infty^3 T-3T_\infty^4. \qquad (8)$$

Using (7) and (8) in (3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3K'}\frac{\partial T^4}{\partial y}. \tag{9}$$

In view of eqs. (9) and (8), eq. (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p K'}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\alpha}{k}\left(\frac{\partial u}{\partial y}\right)^2, \tag{10}$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity.

From the equation above, it is clearly seen that the influence of radiation is to enhance the thermal diffusivity. If we take $N_R = \frac{kK'}{4\sigma^* T_\infty^3}$ as the radiation parameter, (10) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2} + \frac{\alpha\mu}{k}\left(\frac{\partial u}{\partial y}\right)^2, \tag{11}$$

where $k_0 = \frac{3N_R}{3N_R + 4}$. It is worth citing here that the classical solution for energy equation, eq. (11), without thermal radiation and viscous dissipation influences can be obtained from the above equation which reduces to $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$, as $N_R \to \infty (i.e., k_0 \to 1)$.

We introduce a similarity variable η and a dimensionless stream function f(η) as

$$\eta = y\sqrt{\frac{U}{\upsilon x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}, \frac{u}{U} = f', v = \frac{1}{2}\sqrt{\frac{U\upsilon}{x}}(\eta f' - f), \tag{12}$$

where prime denotes differentiation with respect to η and Re_x is the local Reynolds number ($=\frac{Ux}{\upsilon}$), we obtain by deriving eq. (12)

$$\frac{\partial u}{\partial x} = -\frac{U}{2}\frac{\eta}{x}f''; \frac{\partial v}{\partial y} = \frac{U}{2}\frac{\eta}{x}f'' \tag{13}$$

And the equation of continuity is satisfied identically.

$$\frac{\partial u}{\partial y} = Uf''\sqrt{\frac{U}{\upsilon x}}; \quad \frac{\partial^2 u}{\partial y^2} = \frac{U^2}{\upsilon x}f'''. \tag{14}$$

Nothing that in eqs. (12)-(14) $U = U_\infty$ represents Blasius flow, whereas $U = U_w$ indicates Sakiadis flow, respectively. We also assume the bottom surface of the plate is heated by convection from a hot fluid at uniform temperature T_f which provides a heat transfer coefficient h_f.

Defining the non-dimensional temperature $\theta(\eta)$ and the Prandtl number Pr as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \mathrm{Pr} = \frac{\upsilon}{\alpha}, Ec = \frac{U^2}{k(T_w - T_\infty)} \tag{15}$$

We substitute eqs. (12)-(14) into eqs. (2) and (11) we have:

$$f''' + \frac{1}{2}ff'' = 0, \tag{16}$$

$$\theta'' + \frac{\mathrm{Pr}\,k_0}{2}f\theta' + Ec\,\mathrm{Pr}(f'')^2 = 0. \tag{17}$$

Where Ec is the Eckert number. When k_0 = 1 and *Ec = 0*, the thermal radiation and the viscous dissipation effects are not considered.
The transformed boundary conditions are:

$$f=0,\ \ f'=0, \theta'=-a[1-\theta(0)]\ \ at \eta=0,$$
$$f'\to 1 as\ \theta\to 0\ as\ \eta\to\infty \tag{18}$$

for the Blasius flow, and

$$f=0,\ \ f'=1, \theta'=-a[1-\theta(0)]\ \ at \eta=0,$$
$$f'\to 0 as\ \theta\to 0\ as\ \eta\to\infty \tag{19}$$

for the Sakiadis case, respectively. Where

$$a=\frac{h_f}{k}\sqrt{\upsilon x/U_\infty}. \tag{20}$$

For the momentum and energy equations to have a similarity solution, the parameters a must be constants and not functions of x as in eq. (20). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$.
We therefore assume

$$h_f=cx^{-1/2}. \tag{21}$$

Where c is constant. Putting eq. (21) into eq. (20), we have

$$a=\frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}}. \tag{23}$$

Here, a is defined by eq. (23), the solutions of eqs. (16)-(19) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever a is defined as in eq. (20).

3. Numerical procedure

The coupled nonlinear eqs. (16) and (17) with the boundary conditions in eqs. (18) and (19) are solved numerically using the sixth-order Runge-Kutta method with a shooting integration scheme and implemented on Maple [27]. The step size 0.001 is used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence.

4. Results and discussion

Numerical computations have been carried out for different embedded parameters coming into the flow model controlling the fluid dynamics in the flow regime. The Prandtl number used are 0.72, 1, 3, 5, 7.1, 10 and 100; the convective parameters a used are 0.1, 0.5, 1.0, 5.0, 10, and 20; the radiation parameters N_R used are 0.7, 5.0, 10, and 100; and Eckert number(Ec) used are Ec > 0. Comparisons of the present results with previously works are performed and excellent agreements have been obtained. We obtained the results as shown in Tables 1 - 4 and figures 1-8 below.

Table 1 shows the comparison of Aziz [15] and Makinde and Olanrewaju [16] (in the absent of radiation and viscous dissipation parameters) work with the present work for Prandtl numbers (Pr = 0.72, and 10) and it is noteworthy to mention that there is a perfect agreement in the absence of radiation parameter and the viscous dissipation term. Table 2 shows the comparison of Bataller [21] work for Blasius and Sakiadis flows for Prandtl numbers (Pr = 0.72, 1.0, 5.0, 10 and 100) and radiation parameter (N_R = 0.7, 5.0, 10 and 100) and it is noteworthy to mention that there is a perfect agreement in the absence of viscous dissipation parameter. Accurately, the results at a = 0.5, Pr = 5 and N_R = 0.7 for the missed plate temperature θ(0) values were numerically obtained as θ(0) = 0.55489763 for Blasius flow, and θ(0) = 0.44474556 for Sakiadis flow, respectively (see table 2). In table 3, we show the influence of the embedded flow parameters on the temperature at the wall plate for the Blasius and Sakiadis flow. It is clearly seen that when Biot number a increases the wall temperature for Blasius and Sakiadis flow increases while increase in Prandtl number Pr, radiation parameter N_R, and Eckert number Ec decreases the wall temperature for both Blasius and Sakiadis flow. Table 3 shows the influence of the flow parameters on the Nusselt number and the Skin friction for Blasius flow. Increase in the convective parameter a, Prandtl number Pr, thermal radiation parameter N_R, and the Eckert number Ec bring an increase in the Nusselt number. Skin friction increases with an increase in the convective parameter and the Eckert number while increase in the Prandtl number and the radiation parameter decreases the Skin friction at the wall plate. In table 4, we show the effect of flow embedded parameters on the Nusselt number and the Skin friction for Sakiadis flow. Increase in all the flow parameters brings an increase in the Nusselt number and also in the Skin friction except the Eckert number.

Table 1: Values of $\theta(0)_{Blasius}$ for different values of a without thermal radiation and viscous dissipation term. Parenthesis indicates results from Ref. [15,16].

a	Pr = 0.72	Pr = 10	Pr = 0.1
0.05	0.14466116 (0.1447)	0.06425568 (0.0643)	0.25357322(0.2536)
0.20	0.40352252 (0.4035)	0.21548442 (0.2155)	0.57606722(0.5761)
0.60	0.66991555 (0.6699)	0.45175915 (0.4518)	0.80301752(0.8030)
1.00	0.77182214 (0.7718)	0.57865638 (0.5787)	0.87170149(0.8717)
10.0	0.97128537 (0.9713)	0.93212791 (0.9321)	0.98549531(0.9855)
20.0	0.98543355 (0.9854)	0.96487184 (0.9649)	0.99269468(0.9927)

Table 2: Values of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ for different values of a, Pr, and N_R in the absent of viscous dissipation parameter. Parenthesis indicates results from Ref. [21].

a	Pr	N_R	$\theta(0)_{Blasius}$	$\theta(0)_{Sakiadis}$
0.1	5	0.7	0.19957406 (0.1996265)	0.13807609 (0.1380922)
0.5	5	0.7	0.55489763 (0.5548979)	0.44474556 (0.4447517)
1.0	5	0.7	0.71374169 (0.7137422)	0.61567320 (0.6156583)
10	5	0.7	0.96143981 (0.9614407)	0.94124394 (0.9412387)
20	5	0.7	0.98034087 (0.9803475)	0.96973278 (0.9697438)
1	0.72	0.7	0.83312107 (0.8334487)	0.84297896 (0.8623452)
1	1.0	0.7	0.81555469 (0.8156143)	0.81785952 (0.8281158)

1	5	0.7	0.71374169 (0.7137422)	0.61567320 (0.6156583)
1	10	0.7	0.66301284 (0.6630187)	0.51639994 (0.5163969)
1	100	0.7	0.47592614 (0.4759402)	0.23747971 (0.2374795)
5	5	0.7	0.92574298 (0.9257453)	0.88900927 (0.8890038)
5	5	5	0.90376783 (0.9037694)	0.83172654 (0.8317292)
5	5	10	0.90044458 (0.9004477)	0.82284675 (0.8228368)
5	5	100	0.89700322 (0.8970060)	0.81361511 (0.8136082)

Table 3: Values of $f''(0)_{Blasius}$, $\theta'(0)_{Blasius}$ *and* $\theta(0)_{Blasius}$ for several values of the parameters entering the problem.

a	Pr	N_R	Ec	$\theta(0)_{Blasius}$	$-\theta'(0)_{Blasius}$	$f''(0)_{Blasius}$
0.1	5	0.7	2	0.19753138	0.08024686	0.35549045
0.5	5	0.7	2	0.54658747	0.22670626	0.39549180
1.0	5	0.7	2	0.70501920	0.29498079	0.41312720
10	5	0.7	2	0.95932704	0.40672956	0.44083687
20	5	0.7	2	0.97922106	0.41557870	0.44297550
1	0.72	0.7	2	0.82436476	0.17563523	0.47982849
1	1.0	0.7	2	0.80642320	0.19357679	0.46710681
1	5	0.7	2	0.70501920	0.29498079	0.41312720
1	10	0.7	2	0.65528104	0.34471895	0.39495048
1	100	0.7	2	0.47246774	0.52753225	0.35527077
5	5	0.7	2	0.92196930	0.39015346	0.43680984
5	5	5	2	0.89982474	0.50087626	0.41431353
5	5	10	2	0.89649412	0.51752937	0.41156519
5	5	100	2	0.89304899	0.53475500	0.40886337
5	5	0.7	5	0.91891118	0.40544409	0.53184089
5	5	0.7	10	0.91631148	0.41844259	0.62020358
5	5	0.7	20	0.91403503	0.42982480	0.70358410

Table 4: Values of $f''(0)_{Sakiadis}$, $\theta'(0)_{Sakiadis}$ *and* $\theta(0)_{Sakiadis}$ for several values of the parameters entering the problem.

a	Pr	N_R	Ec	$\theta(0)_{Sakiadis}$	$-\theta'(0)_{Sakiadis}$	$-f''(0)_{Sakiad}$
0.1	5	0.7	2	0.13775550	0.08622444	0.43350645
0.5	5	0.7	2	0.44265532	0.27867233	0.41073121
1.0	5	0.7	2	0.61292892	0.38707107	0.39814732
10	5	0.7	2	0.94027601	0.59723989	0.37420826
20	5	0.7	2	0.96920391	0.61592164	0.37210806
1	0.72	0.7	2	0.83116411	0.16883588	0.30704987
1	1.0	0.7	2	0.80572810	0.19427189	0.32277934
1	5	0.7	2	0.61292892	0.38707107	0.39814732
1	10	0.7	2	0.51532923	0.48467076	0.41618829
1	100	0.7	2	0.23744803	0.76255196	0.43957601
5	5	0.7	2	0.88737565	0.56312172	0.37805517
5	5	5	2	0.83090242	0.84548788	0.40210478
5	5	10	2	0.82209565	0.88952171	0.40463142
5	5	100	2	0.81293145	0.93534270	0.40703672
5	5	0.7	5	0.88591024	0.57044879	0.31474240
5	5	0.7	10	0.88457452	0.57712737	0.25349452
5	5	0.7	20	0.88334270	0.58328646	0.19398242

Temperature profiles

The influences of various embedded parameters on the fluid temperature are illustrated in Figs. 1 to 8. Fig. 1 depicts the effect of Eckert number on the temperature profile for Blasius flow and it is seen that increase in the Eckert number increases the thermal boundary layer thickness across the plate. We can see also that the same effect was seen for Sakiadis flow (see fig. 5). Fig. 2 depicts the curve of temperature against spanwise coordinate η for various values of convective parameter a. It is clearly seen that increases in the convective parameter decreases the temperature profile and thereby reduce the thermal boundary layer thickness. Similar effect was seen also in fig. 6 for Sakiadis flow. It is interesting to note that at a≥ 10 the temperature remain the same meaning that it has reach a steady state. Fig. 3 also represents the curve of temperature against Spanwise coordinate η for various values of Prandtl number. Increase in Prandtl number leads to an increase in the temperature profile until η = 2.3 and θ = 0.8 before obeying literature. It is also interesting to note that the same effect was experienced in fig. 7. This could be caused by the flow governing parameters. At high Prandtl fluid has low velocity, which in turn also implies that at lower fluid velocity the specie diffusion is comparatively lower and hence higher specie concentration is observed at high Prandtl number. Fig. 4 depicts the effect of radiation parameter on the temperature profile for Sakiadis flow and it is seen that increase in the radiation parameter decreases the thermal boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in fig. 8. We can see also that the same effect was seen for Sakiadis flow.

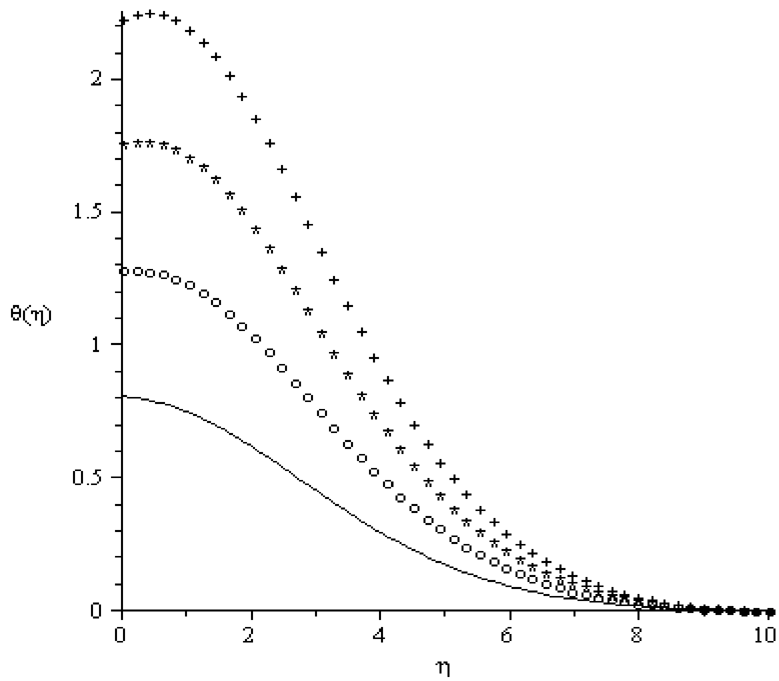

Figure 1: Temperature profiles of ——— Ec = 1, ooooo Ec = 2, ****** Ec = 3, ++++++ Ec = 4 for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 (Blasius **flow)**

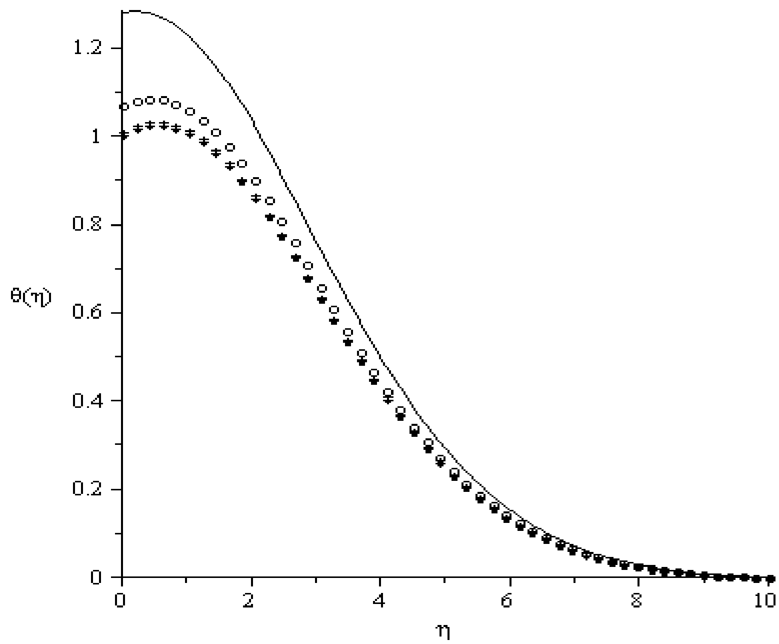

Figure 2: Temperature profiles of ——— a = 0.1, ooooo a = 1, ****** a = 10, ++++++ a = 20 for embedded parameter Pr = 0.72, a = 0.1, N_R = 0.7 (**Blasius flow)**

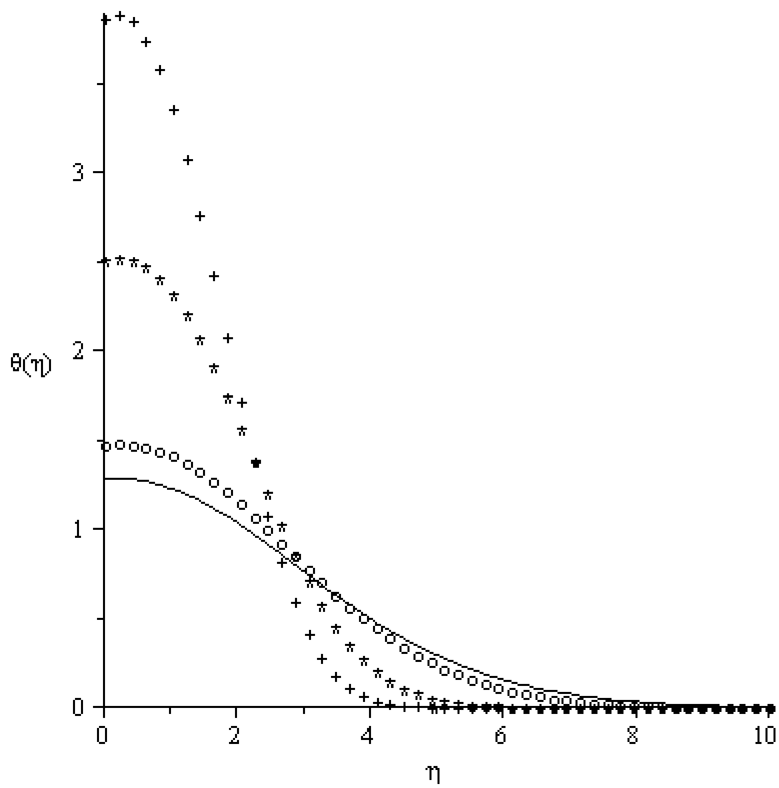

Figure 3: Temperature profiles of ——— Pr = 0.72, ooooo Pr = 1, ****** Pr = 3, ++++++ Pr = 7.1 for embedded parameter Pr = 0.72, a = 0.1, N_R = 0.7 (**Blasius flow)**

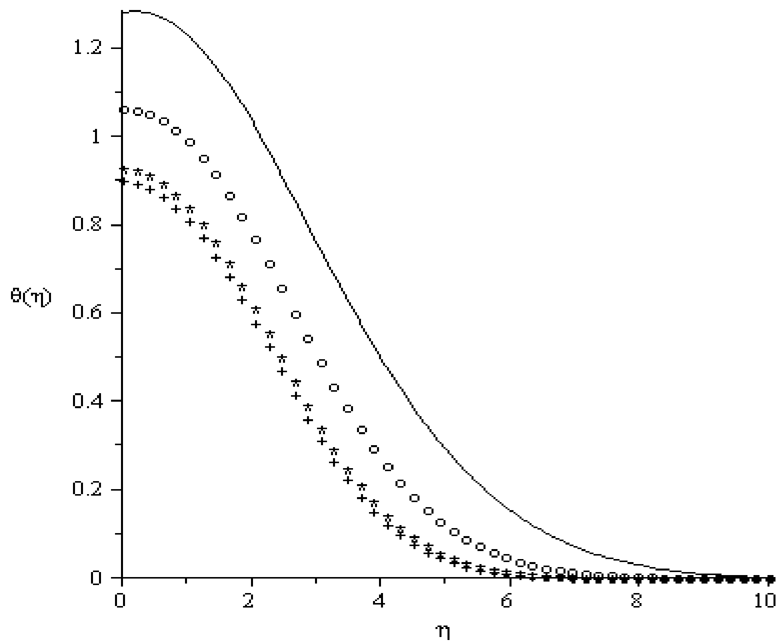

Figure 4: Temperature profiles of ——— NR = 0.7, ooooo NR = 2, ****** NR = 10, ++++++ NR = 30 for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 (**Blasius flow)**

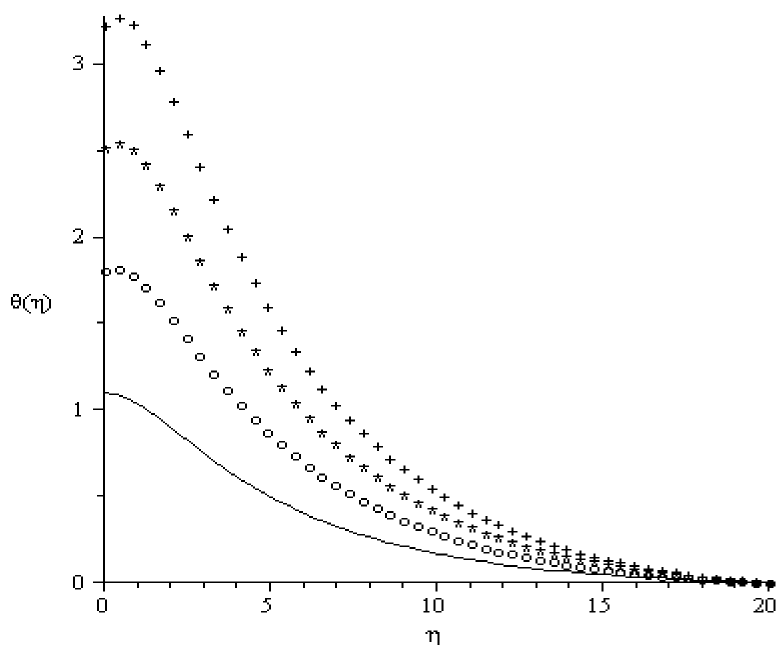

Figure 5: Temperature profiles of ——— Ec = 1, ooooo Ec = 2, ****** Ec = 3, ++++++ Ec = 4 for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 (**Sakiadis flow)**

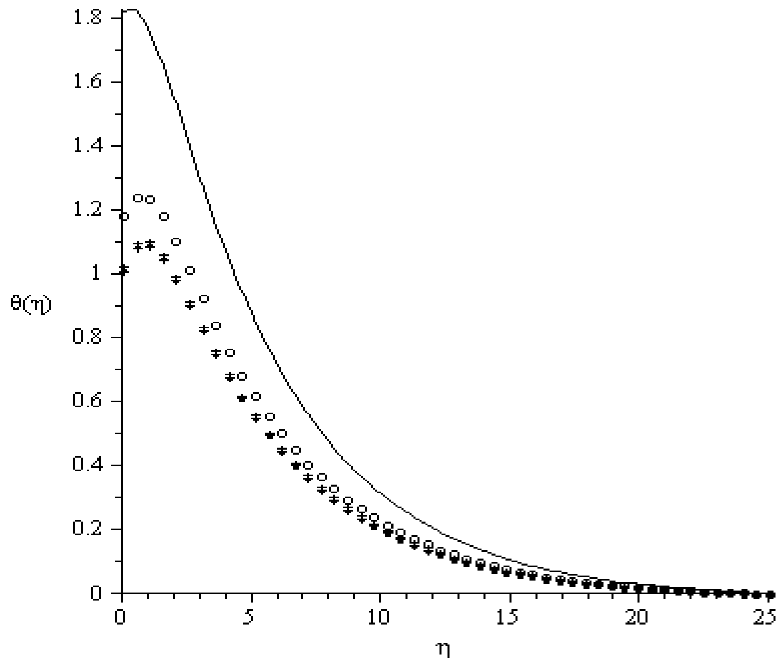

Figure 6: Temperature profiles of ——— a = 0.1, ooooo a = 1, ****** a = 10, ++++++ a = 20 for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 (**Sakiadis flow)**

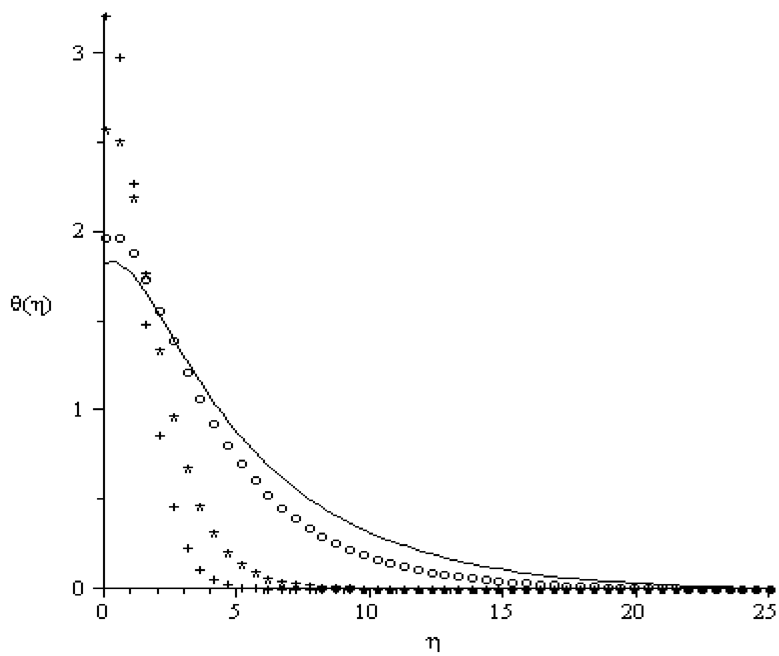

Figure 7: Temperature profiles of ——— Pr = 0.72, ooooo Pr = 1, ****** Pr = 3, ++++++ Pr = 7.1 for embedded parameter *Pr* = 0.72, *a* = 0.1, N_R = 0.7 (**Sakiadis flow)**

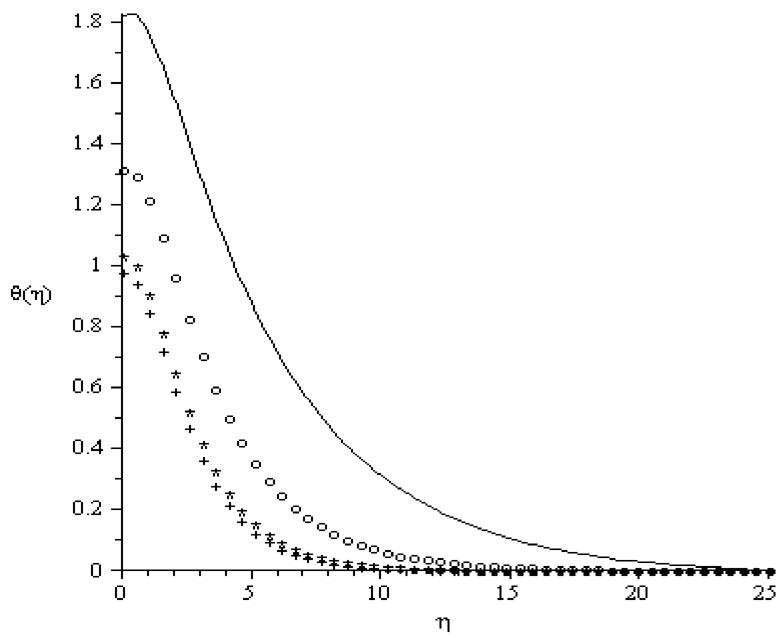

Figure 8: Temperature profiles of ——— NR = 0.7, ooooo NR = 2, ****** NR = 10, ++++++ NR = 30 for embedded parameter $Pr = 0.72$, $a = 0.1$, $N_R = 0.7$ (**Sakiadis flow)**

5. Conclusions

In this article an IVP procedure is employed to give numerical solutions of the Blasius and Sakiadis momentum, thermal boundary layer over a horizontal flat plate and heat transfer in the presence of thermal radiation and the viscous dissipation parameters under a convective surface boundary condition. The lower boundary of the plate is at a constant temperature T_f whereas the upper boundary of the surface is maintained at a constant temperature T_w. It is also noted that the temperature of the free stream is assumed as T_∞ and also we have $T_f > T_w > T_\infty$. Where T_w is the temperature at the wall surface. The transformed partial differential equations together with the boundary conditions are solved numerically by a shooting integration technique alongside with 6th order Runge-Kutta method for better accuracy. Comparisons have been analyzed and the numerical results are listed and graphed. The combined effects of increasing the Eckert number, the Prandtl number and the radiation parameter tend to reduce the thermal boundary layer thickness along the plate which as a result yields a reduction in the fluid temperature. On the contrary, the values of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ increase with increasing a and decreases with increasing Ec. In general, the Blasius flow gives a thicker thermal boundary layer compared with the Sakiadis flow, but this trend can be reversed at low values of embedded parameters controlling the flow model. Finally, in the limiting cases, $N_R \to \infty (i.e., k_0 \to 1)$ the thermal radiation influence can be neglected.

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