Effects of internal heat generation, thermal radiation, and buoyancy force on boundary layer over a vertical plate with a convective boundary condition

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Abstract

In this paper we analyze the effects of internal heat generation, thermal radiation, and buoyancy force on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition. In the analysis, we assumed that left surface of the plate is in contact with a hot fluid while a stream of cold fluid flows steadily over the right surface with a heat source that decays exponentially. Similarity variable method is applied to the governing non-linear partial differential equations. The transformed into a set of coupled non-linear ordinary differential equations are solved numerically by applying shooting iteration technique together with fourth order Runge-Kutta integration scheme. The effects of Prandtl number, local Biot number, the internal heat generation parameter, thermal radiation, and the local Grashof number on the velocity and temperature profiles are illustrated and interpreted in physical terms. A comparison with previously published results in special case of the problem shows an excellent agreement.

Keywords: thermal radiation; buoyancy force; internal heat generation; vertical plate; Biot number; boundary layer.

1. Introduction

Boundary-layer flows over a moving or stretching plate are of great importance in view of their relevance to a wide variety of technical applications, particularly in the manufacture of fibers in glass and polymer industries. The first and foremost work regarding the boundary-layer behavior in moving surfaces in quiescent fluid was considered by Sakiadis [1]. Subsequently, many researchers [2-9] worked on the problem of moving or stretching plates under different situations. In the boundary-layer theory, similarity solutions are found to be useful in the interpretation of certain fluid motions at large Reynolds numbers. Similarity solutions often exist for the flow over semi-infinite plates and stagnation point flow for two dimensional, axi-symmetric and three dimensional situations. In some special cases when there is no similarity solution, one has to solve a system of non-linear partial differential equations (PDEs). For the boundary-layer flows, the velocity profiles are similar. But this kind of similarity is lost for non-similarity flows [10-14]. Obviously, the non-similarity boundary-layer flows are more general in nature and more important not only in theory but also in applications. The heat transfer analysis of boundary layer flow with radiation is further important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Extensive literature that deals with flows in the presence of radiation effects is now available. Raptis et al. [15] studied the effect of thermal radiation on the MHD flow of viscous fluid past a semi-infinite stationary plate. Hayat et al. [16] extended the analysis of reference [15] for a second grade fluid.

Convective heat transfer studies are very important in processes involving high temperatures such as gas turbines, nuclear plants, thermal energy storage, etc. Recently, Ishak [17] examined the similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Moreover, Aziz [18] studied a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition and also studied hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition (see [19]). Very recently, Makinde and Olanrewaju [20] investigated the buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition.

In this study, the recent work of Ishak [17], Aziz [18], and Makinde and Olanrewaju [20] is extended to include the effect of thermal radiation and internal heat generation. The numerical solutions of the resulting momentum and the thermal similarity equations are reported for representative values of the thermophysical parameters embedded in the fluid convection process. The objective of this paper is to explore the effects of the thermal radiation and the internal heat generation on the fluid under a convective surface boundary condition. The non-linear equations governing the flow are solved numerically using shooting technique with

Runge-Kutta of order six. Graphical results are reported first for emerging parameters and then discussed.

2. Mathematical formulation

We consider a two-dimensional steady incompressible fluid flow coupled with heat transfer by convection over a vertical plate. A stream of cold fluid at temperature T_∞ moving over the right surface of the plate with a uniform velocity U_∞ while the left surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f. The density variation due to buoyancy force effects is taken into account in the momentum equation and the thermal radiation and the internal heat generation effects are taking into account in the energy equation (Boussinesq approximation). The continuity, momentum, and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty)-\frac{\alpha}{k}\frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the x (along the plate) and the y (normal to the plate) components of the velocities, respectively, T is the temperature, υ is the kinematics viscosity of the fluid, and α is the thermal diffusivity of the fluid and β is the thermal expansion coefficient, Q is the heat release per unit mass, g is the gravitational acceleration and q_r is the radiative heat flux, respectively. The velocity boundary conditions can be expressed as

$$u(x,0)=v(x,0)=0, \tag{4}$$

$$u(x,\infty)=U_\infty. \tag{5}$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)], \quad (6)$$

$$T(x,\infty)=T_\infty. \quad (7)$$

The radiative heat flux q_r is described by Roseland approximation such that

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \quad (8)$$

where σ^* *and* K are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Chamkha [22], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \quad (9)$$

Using (8) and (9) in (3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3K}\frac{\partial T^4}{\partial y}. \quad (10)$$

Introducing a similarity variable η and a dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$ as

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}, \quad \frac{u}{U_\infty} = f', \; v = \frac{1}{2}\sqrt{\frac{U_\infty \upsilon}{x}}(\eta f' - f), \; \theta = \frac{T-T_\infty}{T_f - T_\infty}, \quad (11)$$

where prime symbol denotes differentiation with respect to η and $Re_x = U_\infty x/\upsilon$ is the local Reynolds number. Eqs. (1) – (7) reduce to

$$f''' + \frac{1}{2}ff'' + Gr_x\theta = 0, \quad (12)$$

$$\theta''[1+\frac{4}{3}\mathrm{Pr}\,Ra] + \frac{1}{2}\mathrm{Pr}\,f\theta' + \mathrm{Pr}\,\lambda_x\theta = 0, \quad (13)$$

$$f(0)=f'(0)=0,\ \theta'(0)=-Bi_x[1-\theta(0)], \tag{14}$$

$$f'(\infty)=1,\ \ \theta(\infty)=0, \tag{15}$$

where

$$Bi_x=\frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}},\mathrm{Pr}=\frac{\upsilon}{\alpha},Gr_x=\frac{\upsilon xg\beta(T_f-T_\infty)}{U_\infty^2},$$
$$Ra=\frac{4\alpha\sigma T_\infty^3}{kK},\qquad \lambda_x=\frac{xQ}{U_\infty}, \tag{16}$$

For the momentum and energy equations to have a similarity solution, the parameters Gr_x , λ_x, and Bi_x must be constants and not functions of x as in Eq. (16). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$, the thermal expansion coefficient β is proportional to x^{-1} and the heat release coefficient Q is proportional to x^{-1} . We therefore assume

$$h_f=cx^{-\frac{1}{2}},\beta=mx^{-1},\ Q=dx^{-1}, \tag{17}$$

where *c, d,* and *m* are constants. Substituting Eq. (17) into Eq. (16), we have

$$Bi=\frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}},Gr=\frac{\upsilon mg(T_f-T_\infty)}{U_\infty^2},\lambda=\frac{dQ}{U_\infty}, \tag{18}$$

With *Bi, λ, and* Gr defined by Eq. (18), the solutions of Eqs. (12)- (15) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever Bi_x , λ_x *and* Gr_x are defined as in Eq. (13).

Table 1: Computations showing comparison with Aziz [7] results for $Gr_x = 0$, $Ra = 0$, $\lambda_x=0$, and $Pr = 0.72$

Bi_x	$-\theta'(0)$ Aziz[7]	$\theta(0)$ Aziz[7]	$-\theta'(0)$ Present	$\theta(0)$ Present	$-\theta'(0)$ Ishak[17]	$-\theta'(0)$ Makinde and Olanrewaju [20]
0.05	0.0428	0.1447	0.042767	0.14466	0.042767	0.0428
0.10	0.0747	0.2528	0.074724	0.25275	0.074724	0.0747
0.20	0.1193	0.4035	0.119295	0.40352	0.119295	0.1193
0.40	0.1700	0.5750	0.169994	0.57501	0.169994	0.1700
0.60	0.1981	0.6699	0.198051	0.66991	0.198051	0.1981
0.80	0.2159	0.7302	0.215864	0.73016	0.215864	0.2159
1.00	0.2282	0.7718	0.228178	0.33205	0.228178	0.2282
5.00	0.2791	0.9441	0.279131	0.94417	0.279131	0.2791
10.00	0.2871	0.9713	0.287146	0.97128	0.287146	0.2871
20.00	0.2913	0.9854	0.291329	0.98543	0.291329	0.2913
30.00	-	-	0.292754	0.99024	-	0.2928

Table 2: Computations showing $f''(0)$, $\theta(0)$, *and* $\theta''(0)$ for different parameter values embedded in the flow model

Bi_x	Gr_x	Pr	λ_x	Ra	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0.1	0.72	0.1	0.1	0.386316	0.06681	0.33181
1.0	0.1	0.72	0.1	0.1	0.460825	0.17679	0.82320
10.0	0.1	0.72	0.1	0.1	0.483261	0.21388	0.97861
0.1	0.5	0.72	0.1	0.1	0.557241	0.069730	0.30269
0.1	1.0	0.72	0.1	0.1	0.723310	0.071736	0.28263
0.1	0.1	3.00	0.1	0.1	-0.07454	0.231312	-1.31312
0.1	0.1	7.10	0.1	0.1	-0.01586	0.261733	-1.61733
0.1	0.1	0.72	0.5	0.1	0.280070	0.110631	-0.10631
0.1	0.1	0.72	0.6	0.1	0.298365	0.102052	-0.02052
0.1	0.1	0.72	0.1	0.5	0.392337	0.065305	0.346940
0.1	0.1	0.72	0.1	1	0.398724	0.063698	0.363019
0.1	0.1	0.72	0.1	2	0.408879	0.061177	0.388227

3. Results and discussion

The ordinary differential equations (9)-(10) subject to the boundary conditions (11)-(12) are solved numerically using the symbolic algebra software Maple [21]. Table 1 presents the comparison for the values of $-\theta'(0)$ *and* $\theta(0)$ with those reported by Aziz [18], Ishak [17] and Makinde and Olanrewaju [20], which shows an excellent agreement for Pr = 0.72. Table 2 illustrates the values of the skin-friction coefficient and the local Nusselt number in terms of $f''(0)$ and $-\theta'(0)$, respectively for various values of embedded parameters. From table 2, it is understood that the skin-friction and the rate of heat transfer at the plate surface increases with an increase in local Grashof number, convective surface heat transfer parameter, internal heat generation parameter and the radiation absorption parameter. However, an increase in the fluid Prandtl number decreases the skin-friction but increases the rate of heat transfer at the plate surface. Figures (1)- (6) depict the fluid velocity profiles. Generally, the fluid velocity is zero at the plate surface and increases gradually away from the plate towards the free stream value satisfying the boundary conditions. It is clearly seen from figure 1 that Grashof number has profuse effects on the velocity boundary layer thickness. It is interesting to note that an increase in the intensity of convective surface heat transfer (Bi_x) produces a slight increase in the fluid velocity within the boundary layer (see figure 2). Figures (3)-(4) and (6) has no effect on the velocity profiles. When Ra = 0.1, then Prandtl number has effects on the velocity profile (see figure 5). Figures (7)-(12) illustrate the fluid temperature profiles within the boundary layer. The fluid temperature is maximum at the plate surface and decreases exponentially to zero value far away from the plate satisfying the boundary conditions. From these figures, it is noteworthy that the thermal boundary layer thickness increases with an increase in Bi_x, λ_x, Ra and decreases with increasing values of Gr_x and Pr. Hence, convective surface heat transfer, internal heat generation parameter and radiation parameter enhances thermal diffusion while an increase in the Prandtl number and the intensity of buoyancy force slows down the rate of thermal diffusion within the boundary layer.

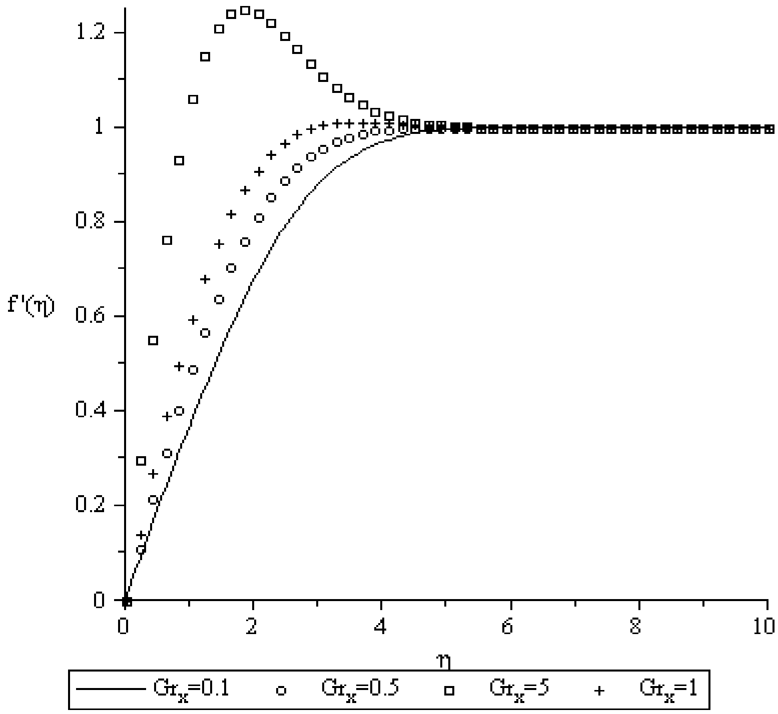

Figure 1: Velocity profiles for Pr = 0.72, λ_x = 0.1, Bi_x = 0.1, Ra = 0.1

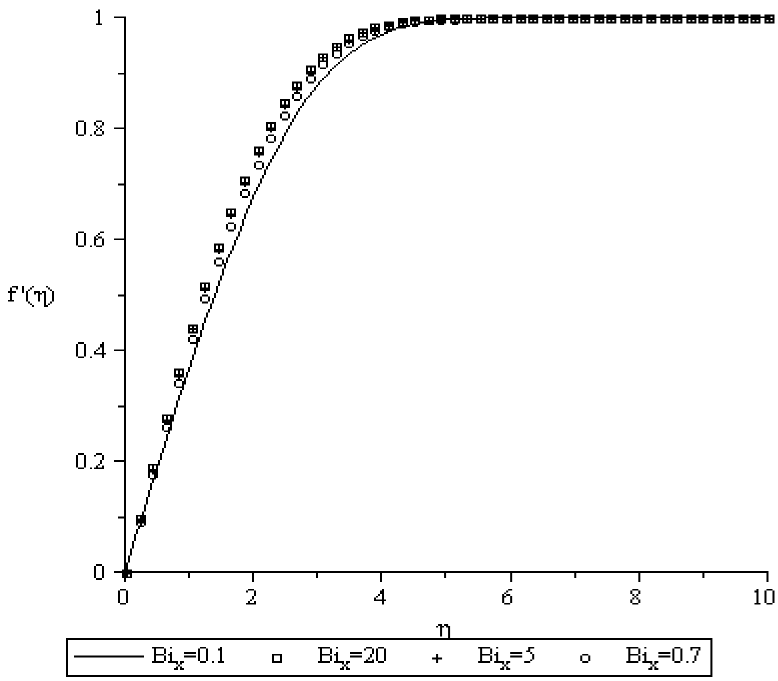

Figure 2: Velocity profiles for Pr = 0.72, Gr_x = 0.1, λ_x = 0.1, Ra = 0.1

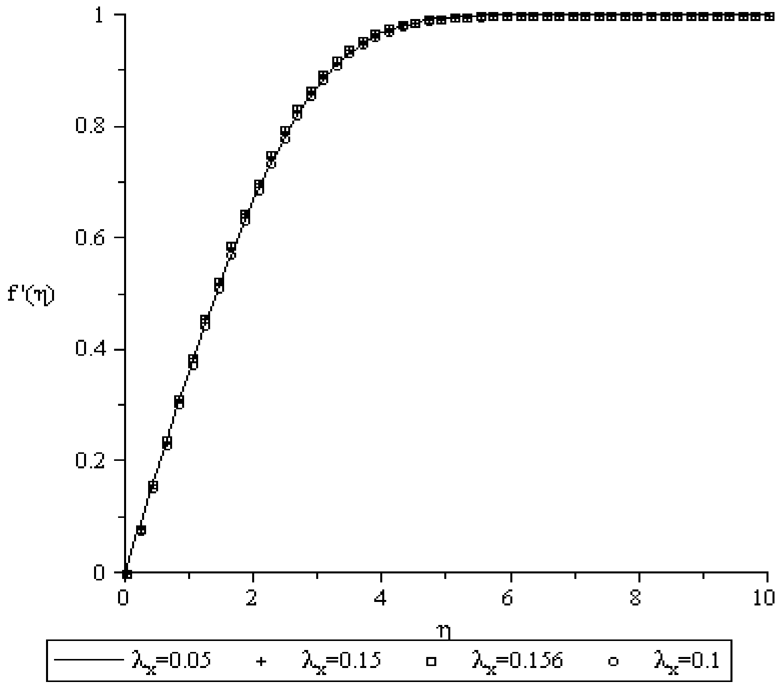

Figure 3: Velocity profiles for Pr = 0.72, Gr_x = 0.1, Bi_x = 0.1, Ra = 0.1

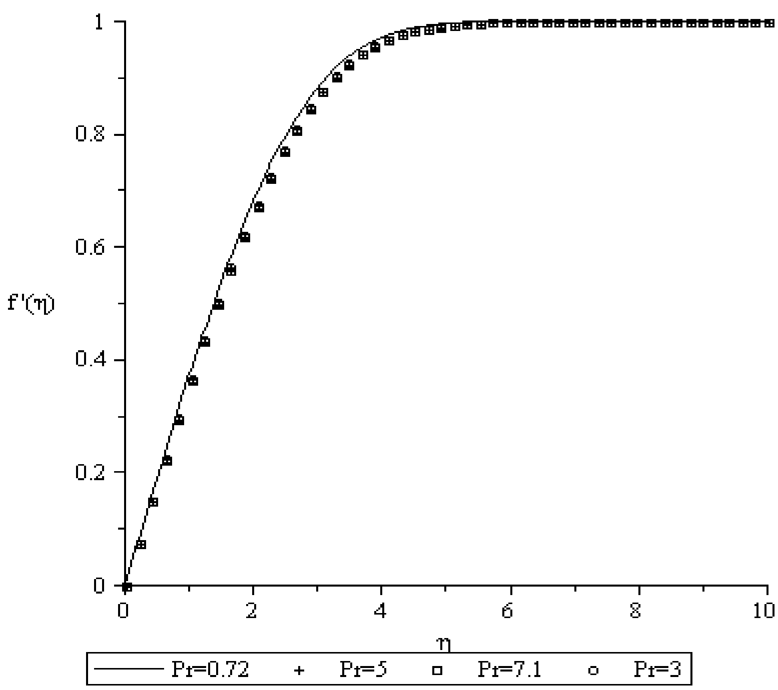

Figure 4: Velocity profiles for λ_x = 0.1, Gr_x = 0.1, Bi_x = 0.1, Ra = 0.5

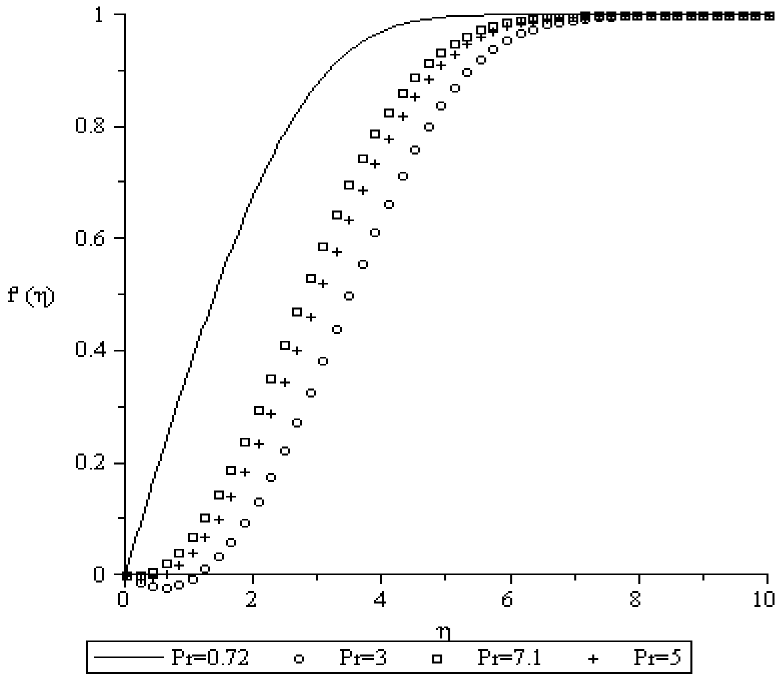

Figure 5: Velocity profiles for $\lambda_x = 0.1$, $Gr_x = 0.1$, $Bi_x = 0.1$, $Ra = 0.1$

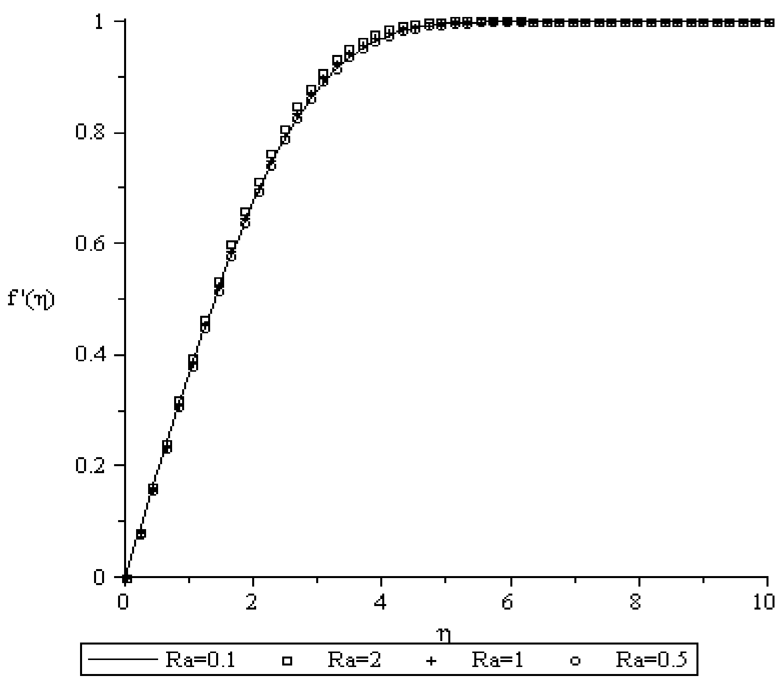

Figure 6: Velocity profiles for $\lambda_x = 0.1$, $Gr_x = 0.1$, $Bi_x = 0.1$, $Pr = 0.72$

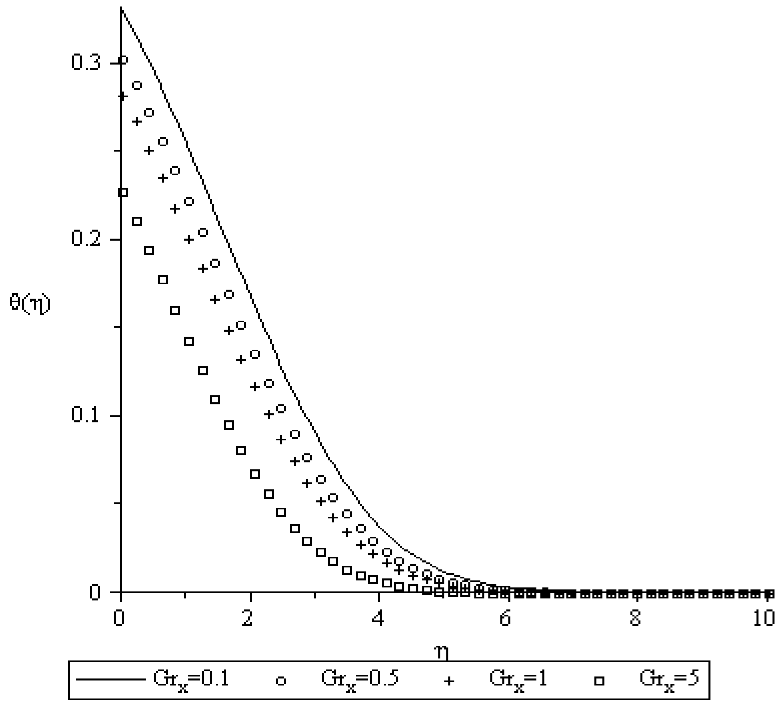

Figure 7: Temperature profile for Pr = 0.72, λ_x = 0.1, Bi_x = 0.1, Ra = 0.1

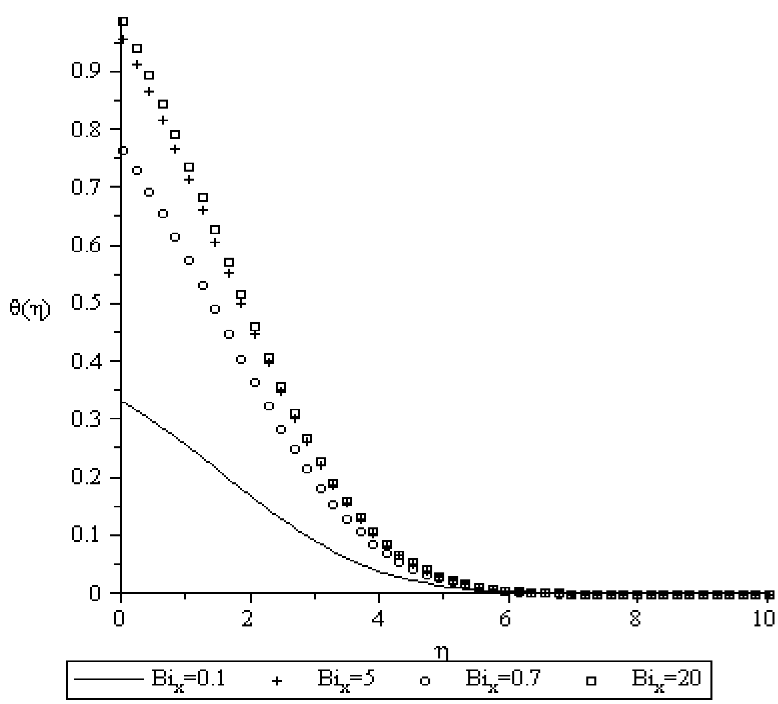

Figure 8: Temperature profiles for Pr = 0.72, λ_x = 0.1, Bi_x = 0.1, Ra = 0.1

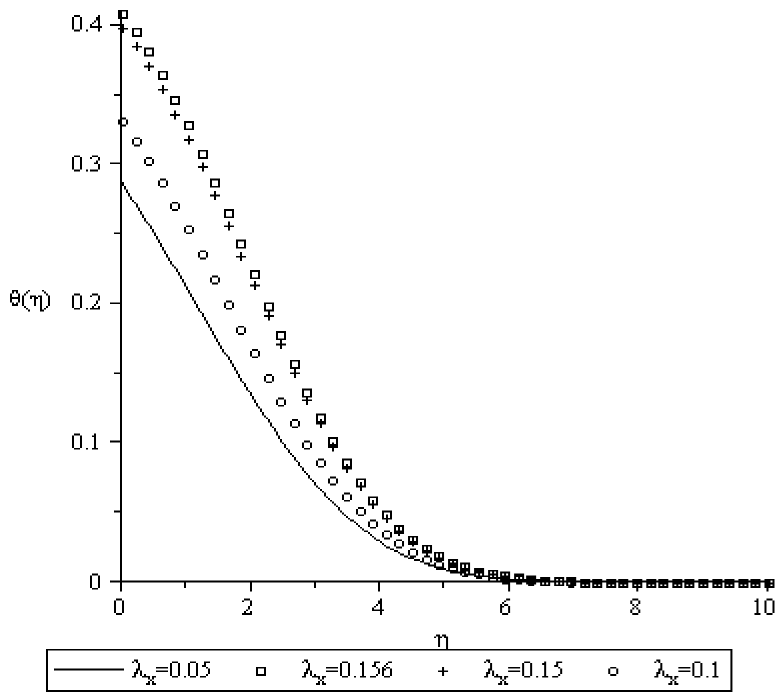

Figure 9: Temperature profiles for Pr = 0.72, Gr_x = 0.1, Bi_x = 0.1, Ra = 0.1

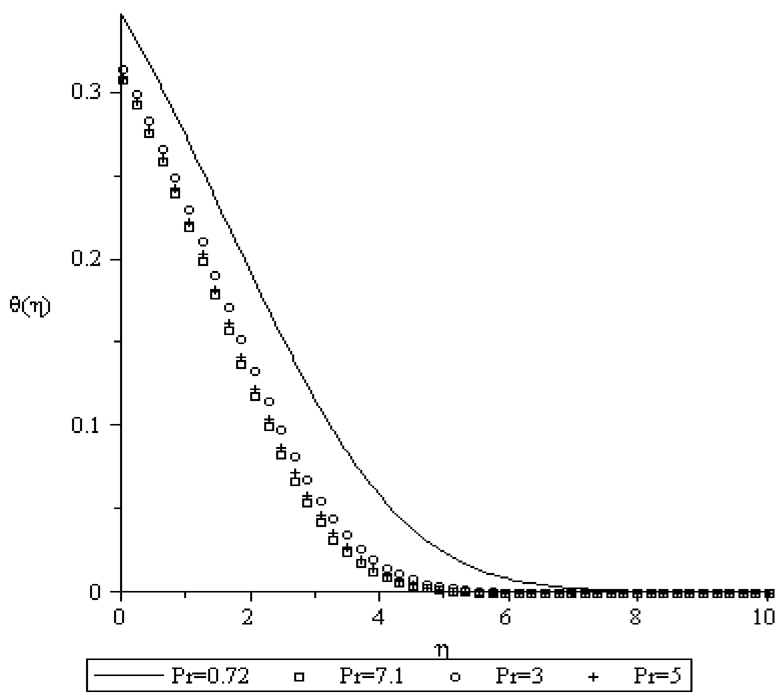

Figure 10: Temperature profiles for λ_x = 0.1, Gr_x = 0.1, Bi_x = 0.1, Ra = 0.5

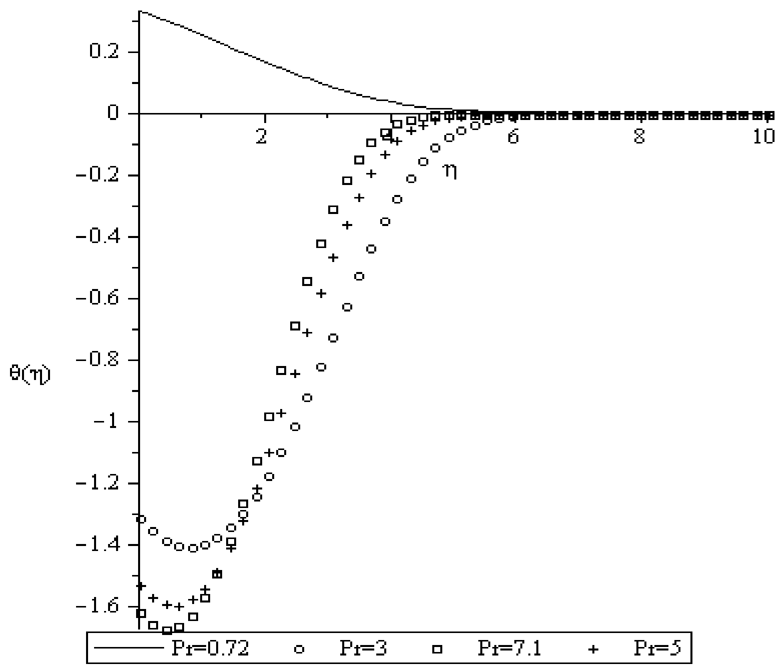

Figure 11: Temperature profiles for $\lambda_x = 0.1$, $Gr_x = 0.1$, $Bi_x = 0.1$, $Ra = 0.1$

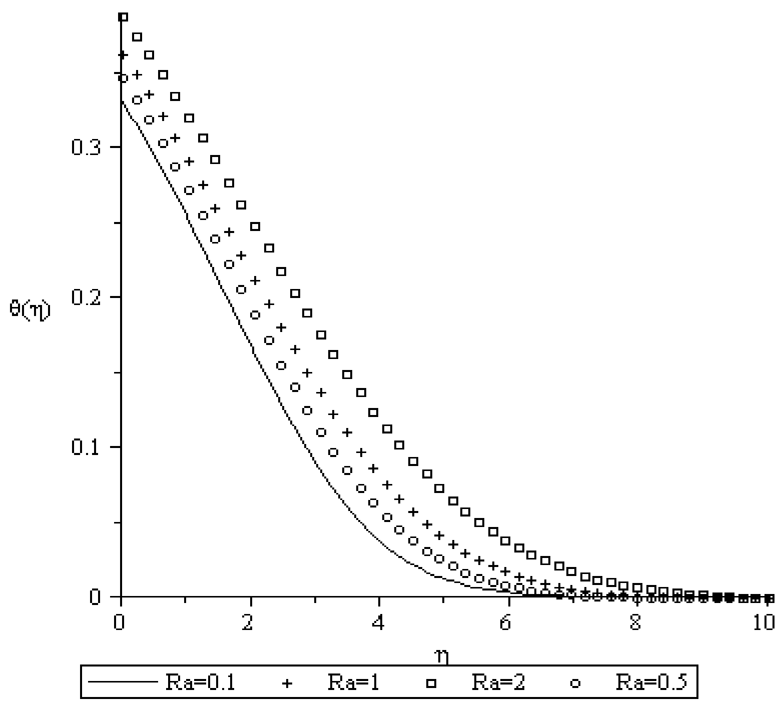

Figure 12: Temperature profiles for $\lambda_x = 0.1$, $Gr_x = 0.1$, $Bi_x = 0.1$, $Pr = 0.72$

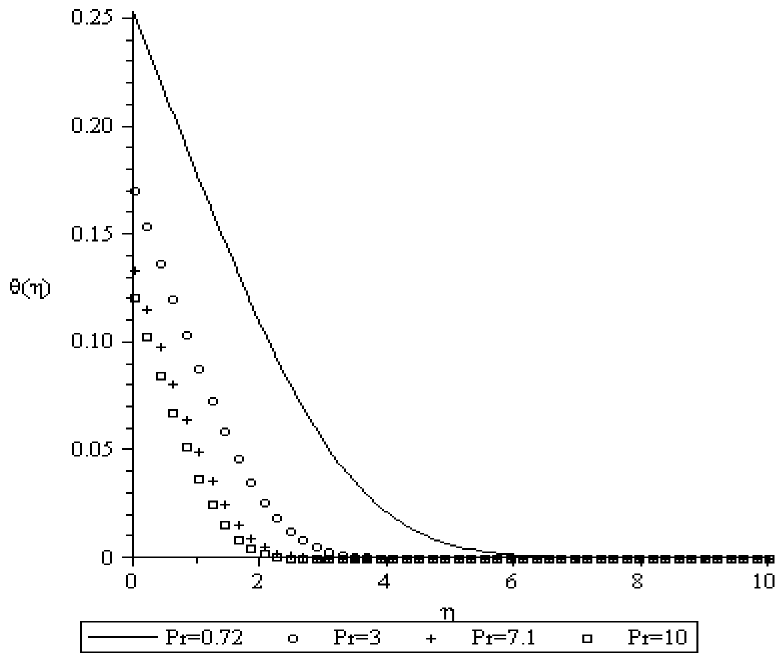

Figure 13: Temperature profiles for Aziz [7] result for $\lambda_x = 0$, $Gr_x = 0$, $Bi_x = 0.1$, Ra = 0

4. Conclusions

Analysis has been carried out to study the effects of internal heat generation, thermal radiation, and buoyancy force on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary. A similarity solution for the momentum and the thermal boundary layer equations is possible if the convective heat transfer of the fluid heating the plate on its left surface is proportional to $x^{-1/2}$, the thermal expansion coefficient β and Q is proportional to x^{-1}. Numerical solutions of the similarity equations were reported for the various parameters embedded in the problem. The combined effects of increasing the Prandtl number and the Grashof number tends to reduce the thermal boundary layer thickness along the plate while the effects of increasing the Biot number, internal heat generation parameter and the radiation absorption parameter enhances thermal diffusion.

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