Viscous Dissipation Effect on Flow through a Horizontal Porous Channel with Constant Wall Temperature and a Periodic Pressure Gradient

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Abstract

The objective of this paper is to investigate the effects of viscous dissipation, constant wall temperature and a periodic pressure field on unsteady flow through a horizontal channel filled with porous material. The coupled nonlinear differential equations governing the flow were solved analytically using the usual method of separation variables and simple perturbation techniques. Effects of various parameters such as the Darcy, Reynolds, prandtl and Eckert numbers were also studied and visualized.

Keywords: viscous Dissipation, periodic pressure gradient, porous media

1. Introduction.

The porous media heat transfer problems have numerous thermal engineering applications such as geothermal energy recovery, crude oil extraction, thermal insulation, ground water pollution, oil extraction, thermal energy storage, thermal insulations, and flow through filtering devices. Hamad and Bashir(2011).

The effect of a pressure field is very important in establishing the flows in certain engineering, industrial and geothermal design such as diffusers and nozzles, in boundary layer flow the shape of the velocity example, if the pressure gradient increases in the direction of flow, the boundary layer increases rapidly, Hughes and Brighton(1991).

The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Examples of flow situations where the dissipation is important includes high shear flows (such as thin lubricant films), in boundary layers in high speed supersonic flight and in large volume channel flows.

Several works has been published on the effect of viscous dissipation, these includes the work of Ahmed and Batin(2010) who studied the analytical model of MHD mixed convective radiating fluid with viscous dissipative heat, Rao and Babu(2010) who carried out finite element analysis of radiation and mass transfer flow past semi-infinite moving vertical plate with viscous dissipation. Most of the above studies have not considered the effect of viscous dissipation with a harmonic pressure gradient.

Hence the objective of this study is to consider the effect of viscous dissipation, periodic pressure gradient and constant wall temperature on the flow and heat transfer in a horizontal channel field with porous material.

2. Mathematical Formulations.

consider the unsteady flow of a viscous and incompressible fluid through horizontal channel filled with porous material in the presence of a periodic pressure gradient in the Cartesian plane, (x, y) with a flow velocity u, density ρ , temperature T, coefficient of kinematic viscosity γ , and specific heat c_p the wall temperature of the channel are assumed to be constant with the lower wall at T_0 and upper wall at T_w with $T_w \geq T_0$ under these assumptions and the boussinesq approximation the equation of motion and energy transfer together with the initial and boundary conditions are as given below.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dx} + \gamma\frac{\partial^2 u}{\partial y^2} - \frac{\gamma}{k}u \tag{1.1}$$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{1.2}$$

Subject to

$$\left.\begin{array}{l} u(y,0) = 0, T(y,0) = T_0, \ u(l,t) = 0 = u(0,t), \\ T(0,t) = T_0, T(l,t) = T_w \end{array}\right\} \tag{1.3}$$

The non-dimensional quantities are as follows

$$\left.\begin{array}{l} \mathrm{Re} = \frac{L^2}{t_0\gamma}, Da = \frac{kL}{\gamma t_0^2}, \mathrm{Pr} = \frac{\gamma}{\alpha}, \ and \ Ec = \frac{\gamma L}{c_p t_0^2 (T_w - T_0)}, \\ U = \frac{ut_0}{L}, \ \theta = \frac{T - T_0}{T_w - T_0}, G = -\frac{1}{\rho}\frac{dp}{dx}(\frac{t_0^2}{L}), \tau = \frac{t}{t_0}, Y = \frac{y}{L} \end{array}\right\} \tag{1.4}$$

Where,
Re=Reynolds Number, Da=Darcy Number, Pr=Prandtl, G=pressure term and Ec=Eckert Number. Using the above transformation in the governing equations and the corresponding boundary conditions we have the following dimensionless form of the governing equations.

$$\frac{\partial U}{\partial \tau} = G + \frac{1}{\mathrm{Re}}\frac{\partial^2 U}{\partial Y^2} - \frac{1}{Da}U \tag{1.5}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial Y^2} + Ec\left(\frac{\partial U}{\partial Y}\right)^2 \tag{1.6}$$

Subject to

$$\left.\begin{array}{l} U(Y,0) = U(0,\tau) = U(L,\tau) = 0 \\ \theta(Y,0) = \theta(0,\tau) = 0, \theta(L,\tau) = 1 \end{array}\right\} \tag{1.7}$$

Method of Solution

In order to solve (1.5) to (1.7) assume that the flow experiences an applied harmonic pressure gradient of the form
$$G = G_0 \cos w\tau \tag{1.8}$$

Where G_0 is the amplitude of oscillation of the periodic pressure, so that (1.5) becomes

$$\frac{\partial U}{\partial \tau} = G_0 \cos w\tau + \frac{1}{\mathrm{Re}}\frac{\partial^2 U}{\partial Y^2} - \frac{1}{Da}U \tag{1.9}$$

(1.9) was solved analytically using separation of variables to obtain

$$U(Y,\tau) = \sum_{n=1}^{\infty} \frac{\lambda_n}{P_n + w_n} \left\{ R_n \cos(w\tau - \beta) - P_n e^{-P_n \tau} \right\} \sin \frac{n\pi Y}{L} \qquad (2.0)$$

Where

$$P_n = \frac{1}{Re}\left(\frac{n\pi}{L}\right)^2 + \frac{1}{Da} \quad and \quad \lambda_n = \frac{-2G_0}{n\pi}(\cos n\pi - 1)$$

Also

$$R_n^2 = P_n^2 + w^2 \quad and \quad \beta_n = \tan^{-1}\left(\frac{w}{P_n}\right)$$

Also (1.6) together with the initial and boundary conditions were solved analytically by assuming a perturbation of the form

$$\theta(Y,\tau) = \theta_0(Y,\tau) + \theta_1(Y) \qquad (2.1)$$

Using (2.1) we obtain

$$\left. \begin{array}{l} \theta_1(Y) = \dfrac{Y}{L} \\[2mm] and \\[2mm] \theta_0(Y,\tau) = \displaystyle\sum_{n=1}^{\infty} B_n(\tau)\sin\dfrac{n\pi Y}{L} \end{array} \right\} \qquad (2.2)$$

With

$$\theta(Y,\tau) = \frac{Y}{L} + \sum_{n=1}^{\infty} B_n(\tau)\sin\frac{n\pi Y}{L} \qquad (2.3)$$

Where

$$B_n(\tau) = \sum_{n=1}^{\infty} \lambda_m \left\{ \begin{array}{l} \dfrac{1}{\left(\dfrac{1}{Pr}\right)^2 + 4w^2}\left[2w\sin 2(w\tau-\beta) + \dfrac{1}{Pr}\cos 2(w\tau-\beta)\right] + \\[4mm] \dfrac{e^{-P_n\tau}}{\left(\dfrac{1}{Pr}-P_n\right)^2 + 4w^2}\left[2w\sin 2(w\tau-\beta) + \left(\dfrac{1}{Pr}-P_n\right)\cos 2(w\tau-\beta)\right] + \\[4mm] \dfrac{e^{-P_n\tau}}{\left(\dfrac{1}{Pr}-P_n\right)} + Pr \end{array} \right\} + Ce^{\frac{1}{Pr}\tau}$$

Where C is the integration constant which is determined from the initial condition, and

$$\lambda_m = \frac{Ec}{2L}\left(\frac{\pi}{4}\right)^2 (1 - \frac{L}{\pi}\cos n\pi)$$

Results and Discussion

In order to analyze the flow regime we consider the fundamental mode for which n=1, and depicts graphically the velocity and temperature profiles in Fig.1-4. In Fig.1 we have the velocity profile for Pr=0.71, Da=0.5, w=10, t=10, G_0=1, Ec=0.5, and L=10 for various values of the Reynolds number, it is observed that as the Reynolds number increases the velocity decreases and the velocity profile is parabolic and symmetric about the center of the channel. While in Fig.2 we show the temperature profile for various values of the Eckert number while the other parameters are kept constant. It is found that increase in Eckert number leads to a corresponding increase in the temperature profile. It is also seen that the temperature decreases at certain portion of the channel and then increases this could be due to the dissipation effect and the harmonic pressure term. And in Fig.3 a temperature profile for various values of the Prandtl number is shown it is observed that increase in the Prandtl number leads to temperature decrease at certain portion of the channel and then increases. Lastly in Fig.4 the velocity profile for various values of the Darcy number is shown, it is observed that increase in the Darcy number leads to increase in the velocity profile.
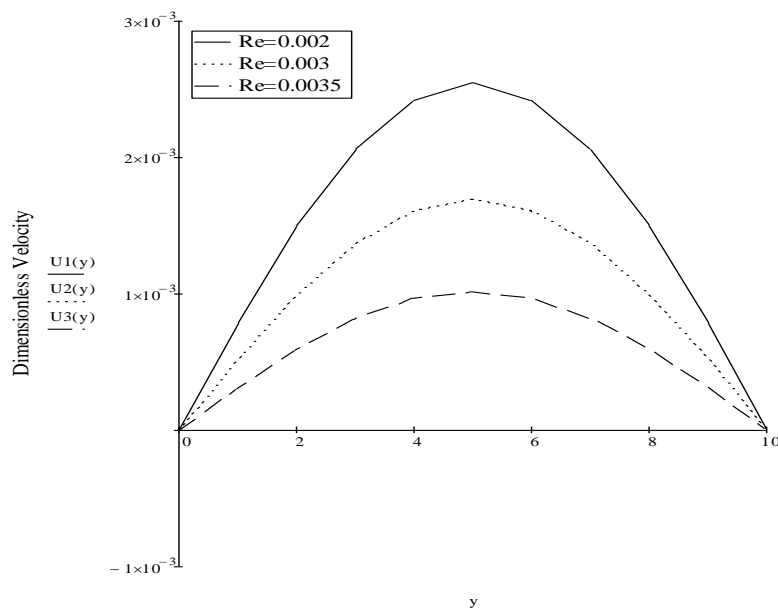

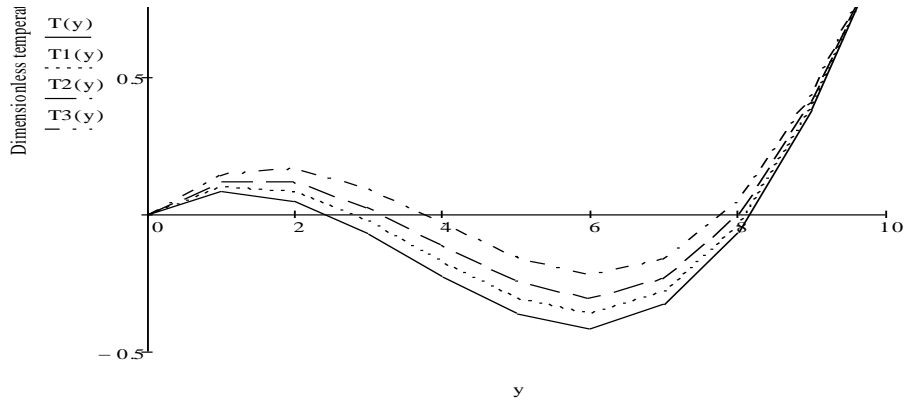Fig.1. Velocity profile for various Reynolds Number

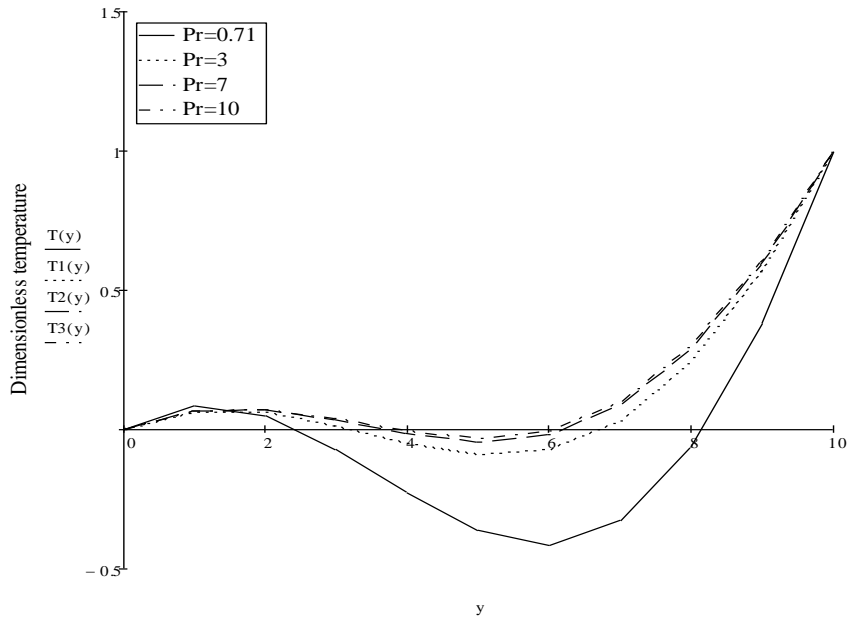Fig.2: Temperature profile for different values of Eckert numbers

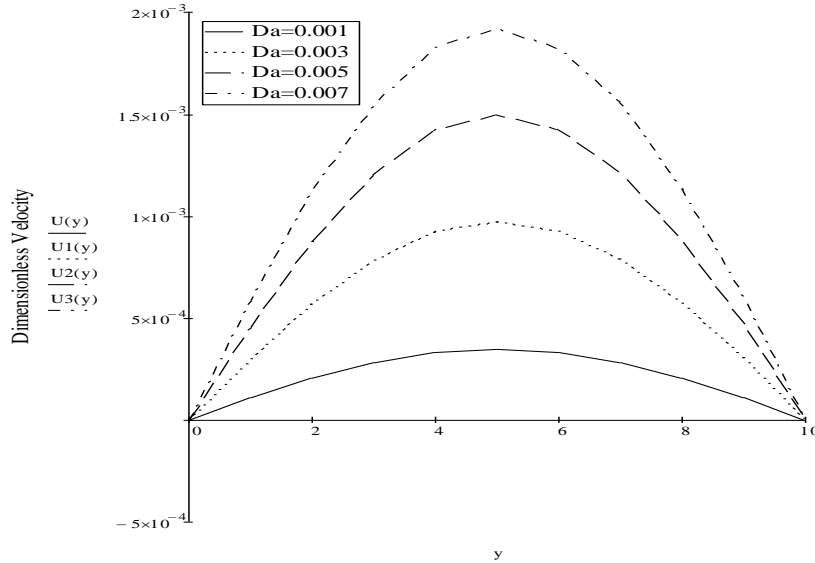Fig.3: Temperature profile for different values of Prandtl Numbers

Fig.1. Velocity profile for various Darcy Number

Conclusion

This study presents a theoretical study of the flow of an incompressible fluid through a horizontal channel filled with porous materials with viscous dissipation and periodic pressure term, the governing equations were solved analytically using perturbation and separation of variables. The results obtained were analyzed for various thermo-physical parameters entering the dimensionless equations. The following were observed:

1. Increase in the Reynolds number leads to a decrease in the velocity Velocity profile.
2. Increase in the Darcy leads to increase in the velocity profile
3. Increase in the Eckert number leads to increase in the temperature Temperature profile.
4. Increase in the Prandtl number leads to decease in temperature at certain at portion of the channel and decrease at other portion of the channel.
5. Flow velocity is parabolic and symmetric about the channel width.

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