

EXISTENCE AND UNIQUENESS OF SOLUTION FOR THE KURZWEIL EQUATION ASSOCIATED WITH QUANTUM STOCHASTIC DIFFERENTIAL EQUATION

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ABSTRACT

We study the existence and uniqueness of solution of the Kurzweil equations associated with the Non-Lipschitz quantum stochastic differential equation. This is accomplished within the framework of the Hudson and Parthasarathy formulations of quantum stochastic calculus and the Schwabic generalized ordinary differential equations.

INTRODUCTION

A non-commutative generalization of the classical Stochastic differential equation is the following quantum Stochastic differential equation (QSDE) introduced by Hudson and Parthasarathy [1]:

$$\begin{aligned} dX(t) &= E(t, X(t))d\Lambda_\pi(t) + F(t, X(t))dA_f^\dagger(t) + G(t, X(t))dA_g(t) + H(t, X(t))dt \\ X(t_0) &= X_0, \quad t \in [t_0, T], \end{aligned} \quad (1)$$

In Equation (1), the coefficients E, F, G, H lie in a certain class of Stochastic processes for which quantum stochastic integrals against the gauge, creation, annihilation process $\Lambda_\pi, A_f^\dagger, A_g$ and the Lebesgue measure are defined.

QSDE (1) often arises as mathematical models, which describe, among other things, quantum dynamical systems and several physical problems in quantum Stochastic control theory, quantum Stochastic evolutions [2, 3, 4 and 5].

In the work of Ekhaguere [2], an equivalent form of QSDE (1) was established as

$$\frac{d}{dt}(\eta, X(t)\xi) = P(X(t), t)(\eta, \xi) \quad (2)$$

$X(t_0) = X_0$, $t \in [t_0, T]$ and η, ξ is lying in some dense subspaces of some Hilbert spaces. The map $F : \mathcal{A} \times [t_0, T] \rightarrow \text{sesq}(D \otimes E)$ is a sesquilinear valued-map, well defined in [4].

In [4], the Kurzweil (generalized) equation associated with QSDE (2) was established. The existence and uniqueness of solution for the Kurzweil equation associated with the QSDE (2) was established as