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Combined effects of internal heat generation and buoyancy force on boundary layer over a vertical plate with a convective surface boundary condition

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Abstract

This paper considers the effect of buoyancy force and internal heat generation on laminar thermal boundary layer over a vertical plate with a convective surface boundary condition. We assumed that left surface of the plate is in contact with a hot fluid while a stream of cold fluid flows steadily over the right surface with a heat source that decays exponentially. Using a similarity variable, the steady state governing non-linear partial differential equations have been transformed into a set of coupled non-linear ordinary differential equations, which are solved numerically by applying shooting iteration technique together with fourth order Runge-Kutta integration scheme. The effects of Prandtl number, local Biot number, the internal heat generation parameter and the local Grashof number on the velocity and temperature profiles are illustrated and interpreted in physical terms. A comparison with previously published results on special case of the problem shows excellent agreement

Keywords: Vertical plate; Convective boundary condition; Internal heat generation; Buoyancy forces

1. Introduction

Investigations of boundary layer flow and heat transfer of viscous fluids over a flat sheet are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Study of laminar boundary layer flow caused by a moving rigid surface was initiated by Sakiadis [1] and later the work was extended to the flow due to stretching of a sheet by Crane [2]. The flow of an incompressible fluid past a moving surface has several engineering applications. The aerodynamic extrusion of plastic sheets, the cooling of a large metallic plate in a cooling bath, the boundary layer along a liquid film in condensation process and a polymer sheet or filament extruded continuously from a die, or a long thread traveling between a feed roll and a wind-up roll are the examples of practical applications of a continuous flat surface. In certain dilute polymer solution (such as 5.4% of polyisobutylene in cetane and 0.83% solution of ammonium alginate in water [3,4]), the viscoelastic fluid flow occurs over a stretching sheet. Any fluid that does not behave in accordance with the Newtonian constitutive relation is called non-Newtonian [5–12]. Non-Newtonian fluids have gained considerable importance because the power required in stretching a sheet in a viscoelastic fluid is less than when it is placed in a Newtonian fluid; and the heat transfer rate for a viscoelastic fluid is found to be less than that of Newtonian fluid.

Recently, Aziz [13] examined a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Makinde and Olanrewaju [14] and Makinde et al. [15] extended Aziz [13] work by adding the buoyancy effects on thermal boundary layer and internal heat generation term to Aziz [13] work. Other papers that are relevant to the present work are those of Wang [16] and Anderson [17]. The papers by Abel and Mahesha [18, 19] are important contributions because they included effect of variable thermal conductivity, heat source, radiation, buoyancy, magneto-hydrodynamic effects, and viscoelastic behavior of the fluid.

This present work examined the combined effects of internal heat generation and buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition. Using a similarity approach, the governing equations are transformed into nonlinear ordinary differential equations and solved numerically using a shooting iteration technique together with fourth order Runge-Kutta integration scheme. The pertinent results are displayed graphically and discussed quantitatively

2. Mathematical analysis

We consider the steady flow of a stream of cold incompressible fluid at temperature T_∞ over the right surface of the vertical plate with a uniform velocity U_∞ while the left surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient

h_f . The cold fluid in contact with the right surface of the plate generates heat internally at the volumetric rate $\dot{q}$. The density variation due to buoyancy effects is taken into account in the momentum equation using the Boussinesq approximation. The continuity, momentum, and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=k\frac{\partial^2 T}{\partial y^2}+\dot{q} \tag{3}$$

where u and v are the x (along the plate) and the y (normal to the plate) components of the velocities, respectively, T is the local temperature, υ is the kinematics viscosity of the fluid, ρ is the fluid density, c_p is the specific heat at constant pressure and k is the thermal conductivity of the fluid. The velocity boundary conditions can be expressed as

$$u(x,0)=v(x,0)=0,\ \ u(x,\infty)=U_\infty. \tag{4}$$

The thermal boundary conditions at the plate left surface and far into the cold fluid at the plate right surface may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)], \tag{5}$$

$$T(x,\infty)=T_\infty, \tag{6}$$

Introducing a similarity variable η and a dimensionless stream function $F(\eta)$ and temperature $\theta(\eta)$ as

$$\eta=\frac{y}{x}\sqrt{\mathrm{Re}_x},\ u=U_\infty F',\ v=\frac{\upsilon}{2x}\sqrt{\mathrm{Re}_x}\left(\eta F'-F\right),\theta=\frac{T-T_\infty}{T_f-T_\infty},$$

$$\lambda_x=\frac{\dot{q}\,x^2e^{\eta}}{k\,\mathrm{Re}_x\left(T_f-T_\infty\right)},\ Gr_x=\frac{\upsilon xg\beta\left(T_f-T_\infty\right)}{U_\infty^2},\tag{7}$$

where prime symbol denotes differentiation with respect to η and $Re_x=U_\infty x/\upsilon$ is the local Reynolds number. The local internal heat generation parameter λ_x is defined so that the internal heat generation $\dot{q}$ decays exponentially with the similarity variable η as stipulated in [6] and the local thermal Grashof number Gr_x. This type of model can be used in mixtures where a radioactive material is surrounded by inert alloys and in the electromagnetic heating of materials [18]. After substituting Eq.(8) into Eqs. (1) – (6), we obtain the following locally similar equations:

$$F'''+\frac{1}{2}FF'+Gr_x\theta=0\tag{8}$$

$$\theta''+\frac{1}{2}\mathrm{Pr}\,F\theta'+\lambda_x e^{-\eta}=0\tag{9}$$

$$F(0)=F'(0)=0,\theta'(0)=-Bi_x[1-\theta(0)]\tag{10}$$

$$F'(\infty)=1,\theta(\infty)=0\ ,\tag{11}$$

where

$$Bi_x=\frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}},\ \mathrm{Pr}=\frac{\rho c_p\upsilon}{\alpha}.\tag{12}$$

The solution generated whenever Bi_x, λ_x and Gr_x are defined as in Eqs. (8)-(12) are local similarity solutions. In order to have a true similarity solution the parameters Bi_x, λ_x and Gr_x must be constants and not depend on x. This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$, the internal heat generation $\dot{q}$ is proportional to x^{-1} and the thermal expansion coefficient β is proportional to x^{-1}. We therefore assume

$$h_f=cx^{-1/2},\quad \beta=mx^{-1},\quad \dot{q}=lx^{-1},\tag{13}$$

where c, m and l are constants but have the appropriate dimensions. Substituting Eq. (13) into Eqs. (7) and (12), we obtain

$$Bi=\frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}},\ Gr=\frac{\upsilon mg\left(T_f-T_\infty\right)}{U_\infty^2},\ \lambda=\frac{l\upsilon e^{\eta}}{\alpha U_\infty\left(T_f-T_\infty\right)}.\tag{14}$$

The Biot number lumps together the effects of convection resistance of the hot fluid and the conduction resistance of the flat plate. The parameter λ is a measure of the strength of the internal heat generation and the parameter Gr is the thermal Grashof number.

3. Numerical Solutions

The coupled nonlinear Eqs (8) and (9) with the boundary conditions in Eqs. (10) and (11) are solved numerically using the fourth-order Runge-Kutta method with a shooting technique and implemented on Maple [20]. The step size is used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence.

4. Results and Discussion

Figures 2-9 illustrate the influence of the local Biot number Bi_x, local Grashof number Gr_x (the buoyancy effect), local internal heat generation parameter λ_x and the Prandtl number Pr on the velocity $F'(\eta)$, temperature $\theta(\eta)$, the local skin friction coefficient and the local Nusselt number, respectively. Comparison is made with previous results in tables 1 and 2 while table 3 shows the influence of embedded parameters on the overall flow structure. Attention is focused on positive values of the buoyancy parameters i.e. local Grashof number $Gr_x > 0$ (which corresponds to the cooling problem). It is clearly seen in tables 1 and 2 that the special cases of our results are in perfect agreement with those reported in [13-15]. From table 3, we observed that the local skin-friction and the rate of heat transfer at the plate right surface decreases as local Grashof number and local Biot number increases while the local skin-friction and the rate of heat transfer at the plate right surface increases as the internal heating parameter increases. Increase in Prandtl number brings an increase in the local skin-friction and the rate of heat transfer at the plate right surface. Figure 2 depicts the velocity profiles for various values of local Grashof number with other parameters remain constant. It was observed that increase in local Grashof number bring an increase in the velocity which thickens the velocity boundary layer. Moreover, the fluid velocity increases from the plate right surface, attains its peak value within the boundary layer and decreases to its free stream values satisfying the boundary condition. Similar trend is observed in figure 3 with respect to an increase in the internal heat generation parameter. The fluid within the boundary layer becomes lighter and flow faster due to internal heating. Figures 4 and 5 show the effect of increasing local Biot number and Prandtl number. It is interesting to note that the velocity boundary layer thickness decreases as local Biot number and Prandtl number increase. Figures 6-9 illustrate the fluid temperature profiles within the boundary layer. The fluid temperature is maximum at the plate right surfaces and decreases exponentially to zero value far away from the plate satisfying the boundary conditions. From these figures, it is noteworthy that the thermal boundary layer thickness increases with a decrease in local Biot number, Prandtl number and local Grashof number while it increases as the local internal heat generation parameter increases.

Table 1: Computations showing comparison with Aziz [13] results for $Gr_x=0$, $\lambda_x=0$ and $Pr=0.72$

Bi_x	Aziz[13] $\theta(0)$	Aziz [13] $-\theta'(0)$	Present $\theta(0)$	Present $-\theta'(0)$
0.05	0.1447	0.0428	0.1447	0.0428
0.10	0.2528	0.0747	0.2528	0.0747
0.20	0.4035	0.1193	0.4035	0.1193
0.40	0.5750	0.1700	0.5750	0.1700
0.60	0.6699	0.1981	0.6699	0.1981
0.80	0.7302	0.2159	0.7302	0.2159
1.00	0.7718	0.2282	0.7718	0.2282
5.00	0.9441	0.2791	0.9441	0.2791
10.00	0.9713	0.2871	0.9713	0.2871
20.00	0.9854	0.2913	0.9854	0.2913

Table 2: Computations showing comparison with Makinde & Olanrewaju [14] results

Bi_x	Pr	λ_x	Makinde& Olanrewaju[14] $\theta'(0)$	Makinde& Olanrewaju[14] $\theta(0)$	Present $\theta'(0)$	Present $\theta(0)$
0.1	0.72	1	0.1154879	2.15487958	0.1154879	2.15487958
1.0	0.72	1	0.3526541	1.35265410	0.3526541	1.35265410
10	0.72	1	0.4437910	1.04437910	0.4437910	1.04437910
0.1	3.0	1	0.0272290	1.27229008	0.0272290	1.27229008
0.1	7.10	1	-0.0101008	0.89899201	-0.0101008	0.89899201
0.1	0.72	5	0.8763365	9.76336572	0.8763365	9.76336572
0.1	0.72	10	1.8273973	19.273973	1.8273973	19.2739733

Table 3: Computation showing $f''(0), \theta'(0)$ *and* $\theta(0)$ for different parameter values

Bi_x	Pr	λ_x	Gr_x	$f''(0)$	$\theta'(0)$	$\theta(0)$
0.1	0.72	1	0.1	0.639970	0.089403	1.894034
1.0	0.72	1	0.1	0.570644	0.303644	1.303644
10	0.72	1	0.1	0.538641	0.396797	1.039679
0.1	3.0	1	0.1	0.479768	0.016571	1.165719
0.1	7.10	1	0.1	0.421585	-0.01547	0.845266
0.1	0.72	5	0.1	1.319375	0.611718	7.1171820
0.1	0.72	10	0.1	1.916164	1.157887	12.578876
0.1	0.72	1	1.0	2.023402	0.040122	1.4012239
0.1	0.72	1	10	7.698598	-0.01036	0.8963780
0.1	0.72	1	20	11.564248	-0.023012	0.769870

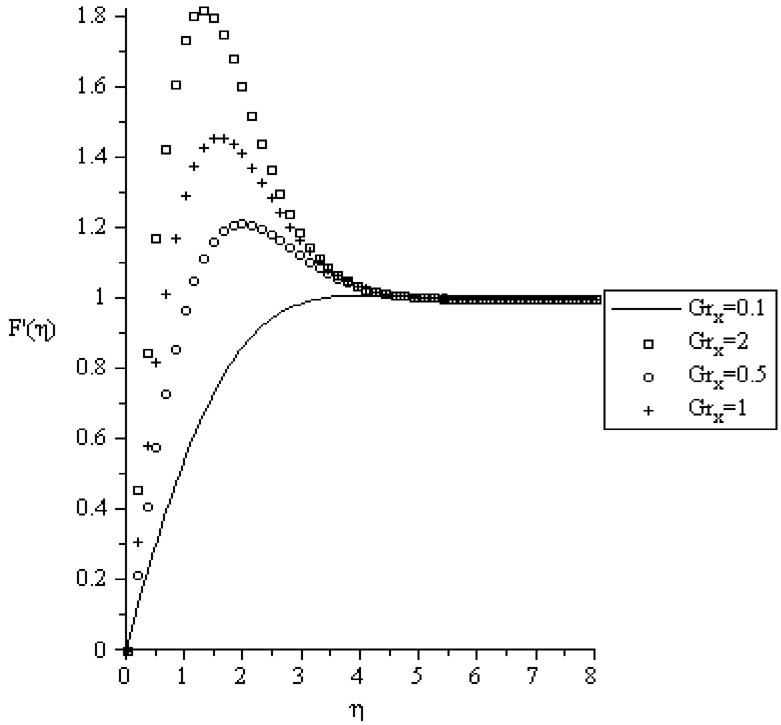

Figure 2: Velocity profiles for Bi=0.1, λ_x=1, Pr=0.72

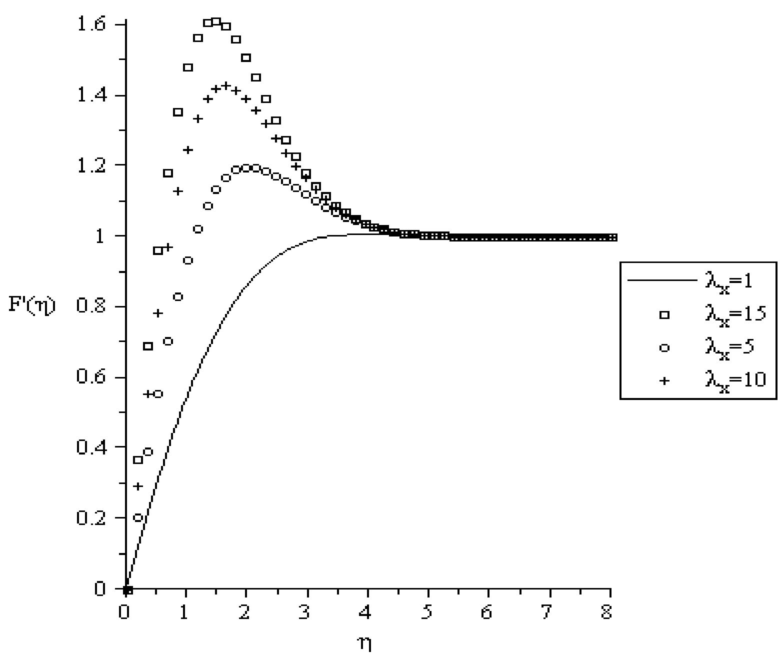

Figure 3: Velocity profiles for Gr_x=0.1, Bi=0.1, Pr=0.72

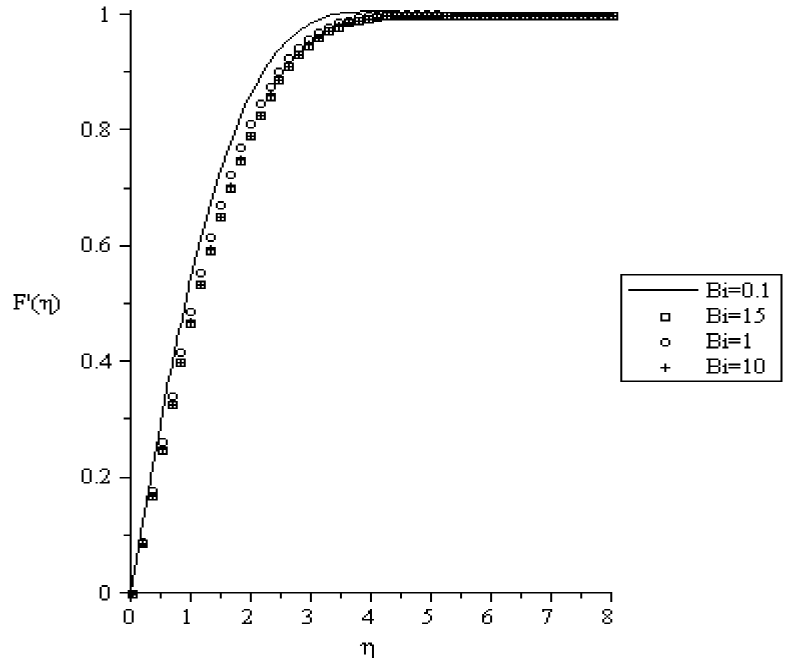

Figure 4: Velocity profiles for Gr_x=0.1, λ_x=1, Pr=0.72

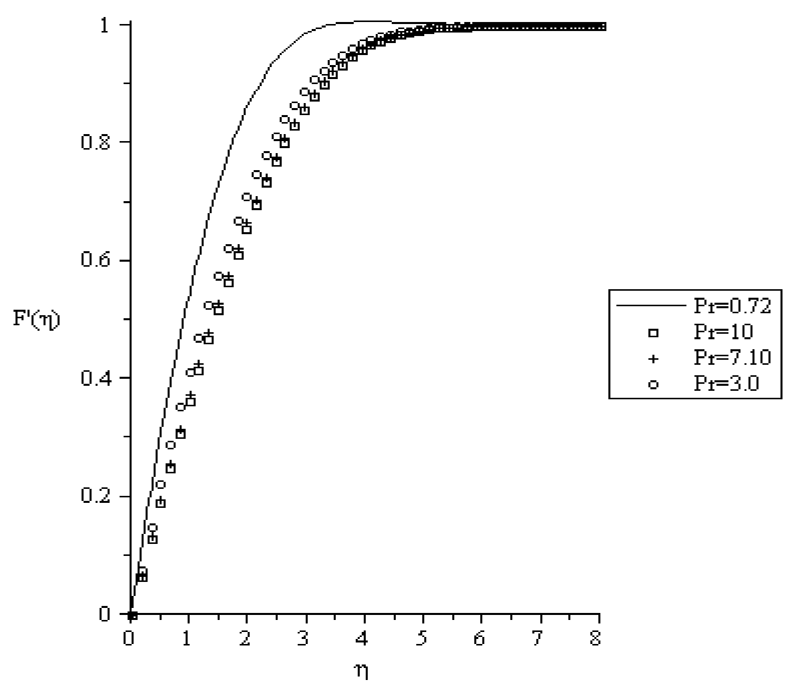

Figure 5: Velocity profiles for Gr_x=0.1, Bi=0.1, λ_x=1

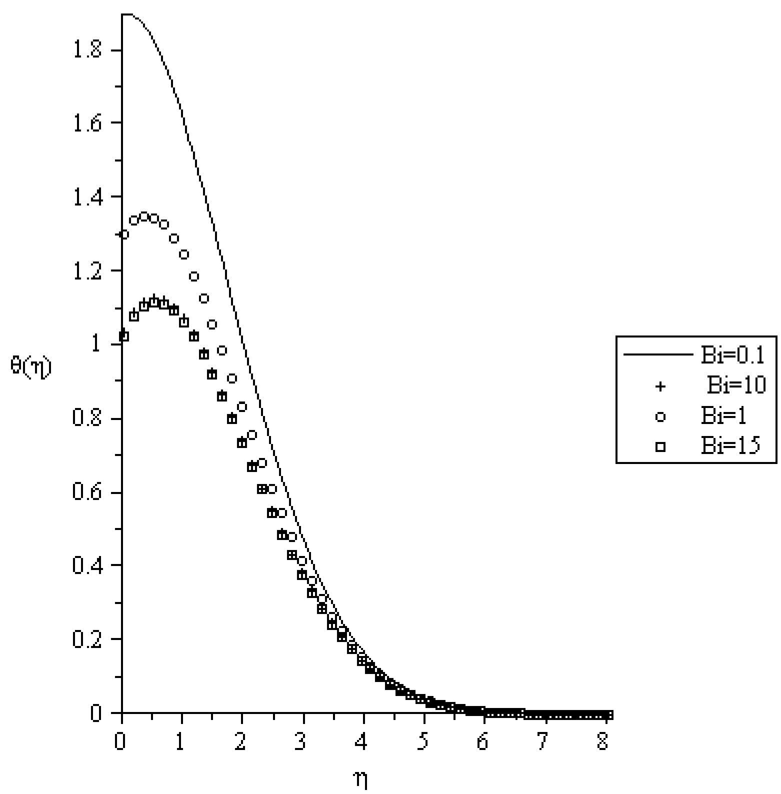

Figure 6: Temperature profiles for Gr_x=0.1, λ_x=1, Pr=0.72

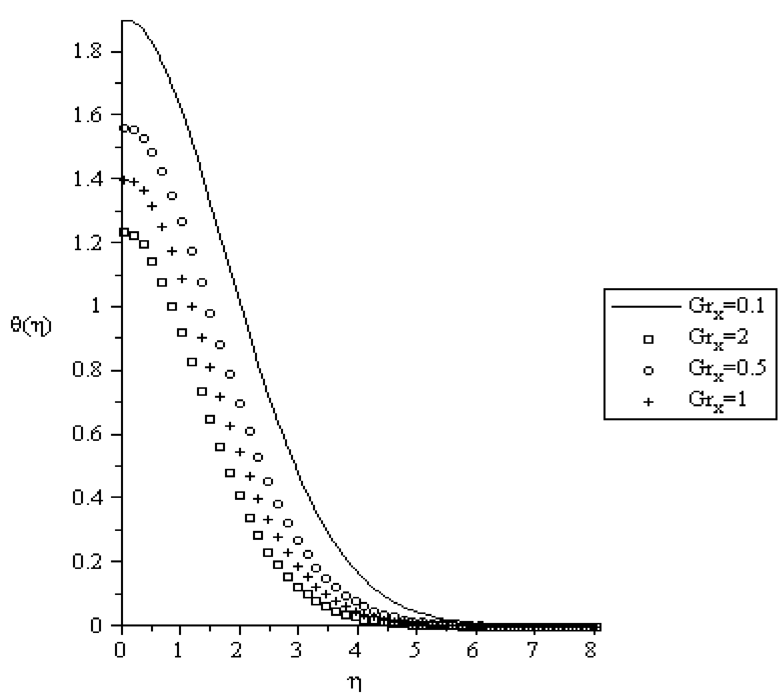

Figure 7: Temperature profiles for Bi=0.1, λ_x=1, Pr=0.72

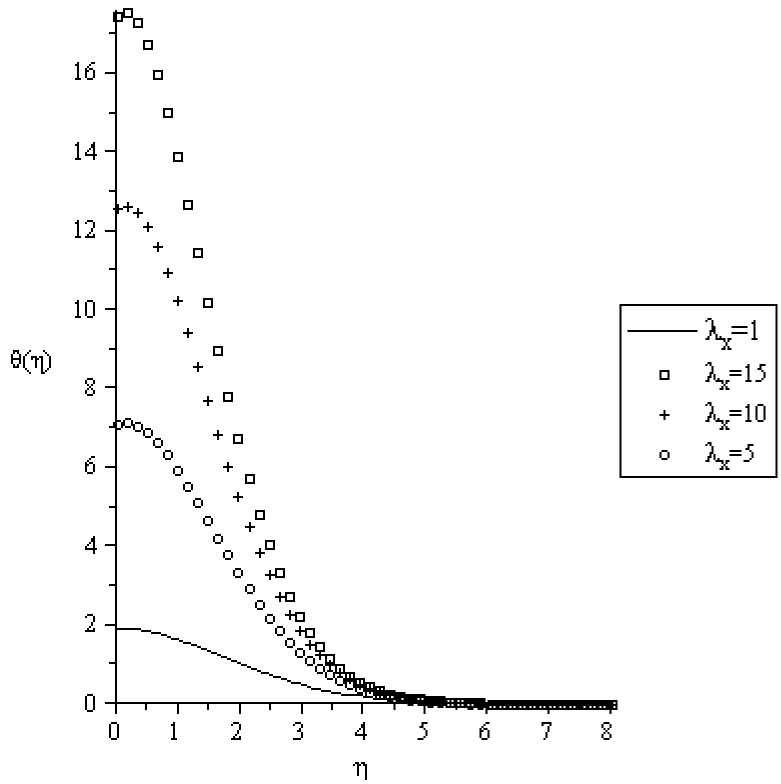

Figure 8: Temperature profiles for Gr_x=0.1, Bi=0.1, Pr=0.72

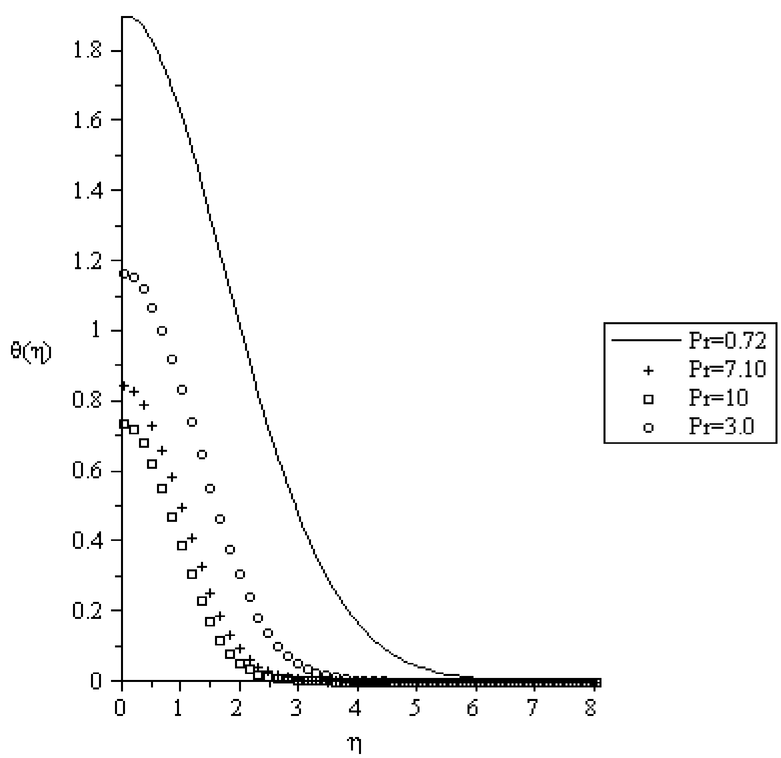

Figure 9: Temperature profiles for Gr_x=0.1, Bi=0.1, λ_x=1

5. Conclusions

The problem on boundary layer flow past a vertical plate due to gravity and fluid density variation due to temperature with internal heat generation and buoyancy effects has been considered. Using similarity variable and the fourth-order Runge-Kutta method coupled with shooting technique, the governing equations are tackled numerically and the influence of various embedded parameters have been discussed quantitatively. Our results reveal among others that;

- the velocity boundary layer thickness increases with an increase in local Grashof number due to buoyancy effects and local internal heat generation.

- the thermal boundary layer thickness increases with a decrease in local Biot number, Prandtl number and local Grashof number while it increases as the local internal heat generation parameter increases.

- the local skin-friction and the rate of heat transfer at the plate right surface decreases as local Grashof number and local Biot number increases while the local skin-friction and the rate of heat transfer at the plate right surface increases as the internal heating parameter increases.

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