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A Note on Thermal Ignition in a Reactive Variable Viscosity Poiseuille Flow

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Abstract: In this study, we investigate a note on the thermal ignition in a strongly exothermic reaction of a variable viscosity combustible material flowing through a channel with isothermal walls under Arrhenius kinetics, neglecting the construction of the material under physical and reasonable conditions to give further insight into the theory of combustion. Numerical solutions are constructed for the governing non linear boundary-value problem using shooting technique together with Runge-Kutta method and important properties of the temperature field and thermal critically are discuss.

Key words: Thermal ignition, reactive variable, viscosity, exothemic reaction, boundary-value

INTRODUCTION

In petrochemical industries as well as petroleum refineries, the study of thermal ignition in a combustible reacting variable viscosity fluid is of great importance in order to ensure safety of life and properties (Bowes, 1984; Makinde and Osulusi, 2005).

Thermal ignition occurs when the reactions produce heat too rapidly for a stable balance between heat production and heat loss to be preserved. Hence, it is important to know the critical values of the basic physical quantities, such as the ambient temperature, surface characteristics, the chemistry of the reacting combustible material and the physical storage geometry at which ignition occur (Balakrishnan *et al.*, 1996; Bebernes and Eberly, 1989; Frank, 1969; Makinde, 2004, 2005).

Olarenwaju *et al.* (2006) examine the existence and uniqueness result for a two-step reactive-diffusive equation with variables pre-exponential factor. They established the criteria and conditions for existence and uniqueness of solution. They further discovered that there are certain values for n,m,r and β that the problem can accommodates for the solution to be stable.Similarly, Frank-kameretskii parameters δ_1, δ_2 must not exceed certain values for the solution to exist and the same time stable.

In this study, we compare the results of Gbolagade and Makinde (2005) with our new results and both agree together.

MATHEMATICAL FORMULATION

The classical formulation of this type of problem was first introduced by Frank-Kameretskii (1969) (Fig. 1). Neglecting the reactant consumption, the equation for the heat balance in the original variables together with the boundary conditions can be written as

$$\left.\begin{aligned} &\frac{d^2T}{dy^2} + Q\frac{C_0A}{K}e^{\frac{-E}{RT}} + \mu\!/\!_K\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2 = 0 \\ &\frac{d}{d\bar{y}}\left(\mu\frac{d\bar{u}}{d\bar{y}}\right) = -G \end{aligned}\right] \quad (1)$$

$$u = 0, T = T_0, \text{ on } \bar{y} = a \quad (2)$$

$$\frac{dT}{dy} = \frac{d\bar{u}}{d\bar{y}} = 0 \text{ on } \bar{y} = 0 \quad (3)$$

Where T is the absolute temperature
G : The constant for axial pressure gradient.
T_0 : The wall reference temperature.
K : The thermal conductivity of the material.
Q : The heat of reaction.
A : The rate constant.
E : The activation energy.
R : The universal gas constant.
C_0 : The initial concentration of the reactant species.
A : The chemical characteristics half width.

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$(\bar{x},\bar{y})$ is the distance measured in the axial and normal directions, respectively.

Following Gutlamann *et al.* (1989) we defined the dynamic viscosity of the combustible material as

$$\mu = \mu_0 e^{E/RT} \tag{4}$$

Where μ_0 is the combustible material references viscosity.

We use the following dimensionless variables in Eq. 1-3:

$$\left.\begin{array}{l} \theta = \frac{E}{RT_0^2}(T - T_0), \quad \varepsilon = \frac{RT_0}{E}, \quad y = \frac{\bar{y}}{a} \\ \lambda = \frac{QEAa^2c_o}{T_0^2Rk} e^{-E/RT_0}, \quad W = \frac{\mu_0\bar{u}}{Ga^2} e^{E/RT_0} \\ \beta = \frac{G^2a^2}{Qc_0A\mu_0} \end{array}\right] \tag{5}$$

and obtain the dimensionless governing equation together with the corresponding boundary conditions as

$$\begin{array}{l} \frac{dw}{dy} = -y\exp(\theta/1+\varepsilon\theta), \quad \frac{d^2\theta}{dy^2} + \\ \lambda(1+\beta y^2)\exp(\theta/1+\varepsilon\theta) = 0 \end{array} \tag{6}$$

$$\frac{d\theta(0)}{dy} = 0, \quad \theta(\pm 1) = 0, \quad w(1) = 0 \tag{7}$$

Where λ, ε, β represent the Frank-Kameretskii parameter, activation energy parameter and the viscous heating parameter, respectively.

In the next study, Eq. 6 and 7 are solved by using shooting techniques and Runge kutta instead of both perturbation and multivariate series summation techniques that take care of some terms of the problem (Makind, 2004, 2005; Olanrewaju *et al.*, 2006).

METHOD OF SOLUTION

We let

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} y \\ w \\ \theta \\ \theta' \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 1 \\ -y_1 e^{y_3/1+\varepsilon y_3} \\ y_4 \\ -\lambda(1+\beta y_1^2) e^{y_3/1+\varepsilon y_3} \end{pmatrix} \tag{9}$$

Satisfying

$$\begin{pmatrix} y_1(-1) \\ y_2(-1) \\ y_3(-1) \\ y_4(-1) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ a \end{pmatrix}$$

RESULTS AND DISCUSSION

The results obtained are shown in the Table 1-4 .

Figure 1 shows the geometry of the problem $(\bar{x},\bar{y})$ where is the distance measured in the axial and normal directions, respectively.

Figure 2 shows the curve of temperature against position y for various values of â and fixed values of λ = 1 and ε = 1. It is shown that as β increases the temperature also increases. The fluid temperature increases with an increase in the viscous heating parameter.

Figure 3 shows the graph of temperature against position y for various values of λ and fixed values of β = 0 and ε = 1. The fluid temperature increases with increase in frank-Kamenetskii parameter which also agree with the literature.

Table 1: Computations showing the procedure rapid convergence for λ = 1, β = 0, ε = 1, α = 1.369106

s/n	y	w	θ	θ¹
0	-1	0.0000	0.0000	1.3691
1	-0.9000	-0.1008	0.1319	1.2627
2	-0.8000	-0.2004	0.2526	1.1451
3	-0.7000	-0.2951	0.3610	1.0186
4	-0.6000	-0.3819	0.4563	0.8848
5	-0.5000	-0.4586	0.5379	0.7452
6	-0.4000	-0.5233	0.6054	0.6012
7	-0.3000	-0.5748	0.6582	0.4537
8	-0.2000	-0.6122	0.6961	0.3038
9	-0.1000	-0.6349	0.7190	0.1523
10	0.0000	-0.6425	0.7266	-0.0000
11	0.1000	-0.6349	0.7190	-0.1524
12	0.2000	-0.6122	0.6961	-0.3039
13	0.3000	-0.5747	0.6582	-0.4538
14	0.4000	-0.5232	0.6054	-0.6013
15	0.5000	-0.4584	0.5380	-0.7454
16	0.6000	-0.3818	0.4563	-0.8850
17	0.7000	-0.2950	0.3610	-1.0188
18	0.8000	-0.2002	0.2526	-1.1454
19	0.9000	-0.1004	0.1320	-1.2630
20	1.0000	0.0004	0.0000	-1.3695

Table 2: Computations showing the procedure rapid convergence for $\lambda = 1$ $\alpha = 2.394968$ $\varepsilon = 1$ $\beta = 2$

s/n	y	w	θ	θ¹
0	-1	0.0000	0.0000	2.3950
1	-0.9000	-0.1046	0.2245	2.0860
2	-0.8000	-0.2128	0.4174	1.7745
3	-0.7000	-0.3173	0.5795	1.4777
4	-0.6000	-0.4135	0.7130	1.2038
5	-0.5000	-0.4984	0.8203	0.9552
6	-0.4000	-0.5699	0.9041	0.7311
7	-0.3000	-0.6266	0.9666	0.5282
8	-0.2000	-0.6677	1.0097	0.3423
9	-0.1000	-0.6926	1.0351	0.1681
10	0.0000	-0.7009	1.0434	-0.0002
11	0.1000	-0.6925	1.0350	-0.1685
12	0.2000	-0.6677	1.0097	-0.3428
13	0.3000	-0.6266	0.9665	-0.5287
14	0.4000	-0.5698	0.9040	-0.7315
15	0.5000	-0.4983	0.8202	-0.9557
16	0.6000	-0.4134	0.7129	-1.2043
17	0.7000	-0.3171	0.5794	-1.4783
18	0.8000	-0.2125	0.4173	-1.7753
19	0.9000	-0.1042	0.2245	-2.0873
20	1.0000	0.0006	0.0000	-2.3970

Table 3: Computations showing the procedure rapid convergence for $\alpha = 1.162605$ $\lambda = 0.878$ $\varepsilon = 1$ $\beta = 0$

s/n	y	w	θ	θ¹
0	-1	0.0000	0.0000	1.1626
1	-0.9000	-0.0999	0.1119	1.0700
2	-0.8000	-0.1976	0.2140	0.9689
3	-0.7000	-0.2896	0.3057	0.8609
4	-0.6000	-0.3736	0.3862	0.7472
5	-0.5000	-0.4475	0.4551	0.6290
6	-0.4000	-0.5099	0.5120	0.5073
7	-0.3000	-0.5594	0.5566	0.3828
8	-0.2000	-0.5954	0.5886	0.2562
9	-0.1000	-0.6172	0.6079	0.1284
10	0.0000	-0.6245	0.6143	-0.0000
11	0.1000	-0.6172	0.6079	-0.1285
12	0.2000	-0.5954	0.5886	-0.2563
13	0.3000	-0.5594	0.5566	-0.3828
14	0.4000	-0.5098	0.5121	-0.5074
15	0.5000	-0.4474	0.4552	-0.6292
16	0.6000	-0.3735	0.3863	-0.7474
17	0.7000	-0.2895	0.3057	-0.8610
18	0.8000	-0.1973	0.2141	-0.9691
19	0.9000	-0.0997	0.1119	-1.0702
20	1.0000	0.0003	0.0000	-1.1629

Table 4: Computations showing the procedure rapid convergence for $\lambda = 1$. $\varepsilon = 1$, $\beta = 1$, $\alpha = 1.866365$

s/n	y	w	θ	θ¹
0	-1	0.0000	0.0000	1.8664
1	-0.9000	-0.1027	0.1766	1.6605
2	-0.8000	-0.2066	0.3322	1.4495
3	-0.7000	-0.3063	0.4666	1.2413
4	-0.6000	-0.3979	0.5805	1.0403
5	-0.5000	-0.4787	0.6747	0.8483
6	-0.4000	-0.5469	0.7502	0.6655
7	-0.3000	-0.6011	0.8078	0.4910
8	-0.2000	-0.6404	0.8484	0.3233
9	-0.1000	-0.6642	0.8725	0.1604
10	0.0000	-0.6722	0.8805	-0.0001
11	0.1000	-0.6642	0.8725	-0.1606
12	0.2000	-0.6404	0.8484	-0.3236
13	0.3000	-0.6011	0.8078	-0.4913

Table 4: Continued

s/n	y	w	θ	θ¹
14	0.4000	-0.5468	0.7502	-0.6658
15	0.5000	-0.4786	0.6747	-0.8486
16	0.6000	-0.3977	0.5805	-1.0407
17	0.7000	-0.3061	0.4666	-1.2417
18	0.8000	-0.2064	0.3322	-1.4500
19	0.9000	-0.1023	0.1766	-1.6611
20	1.0000	0.0005	0.0000	-1.8673

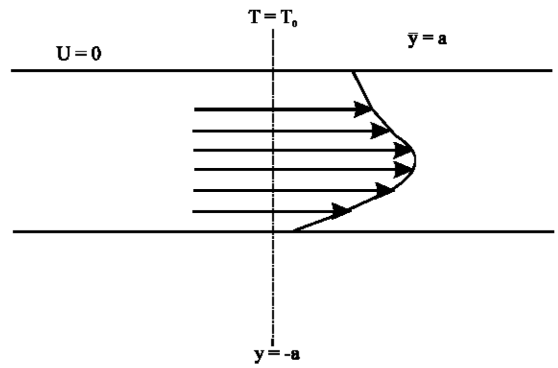

Fig. 1: Geometry of the problem

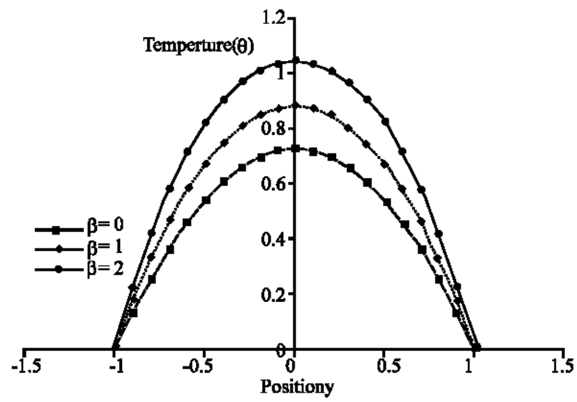

Fig. 2: Graph of temperture against positiony y for various values of β and fixed values of λ = 1 and ε = 1

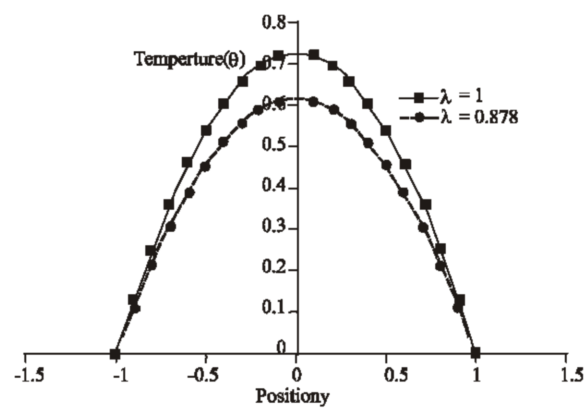

Fig. 3: Graph of temperture against positiony y for various values of λ and fixed values of β = 1 and ε = 1

CONCLUSION

The steady flow of reactive viscosity fluid in a channel with isothermal walls is investigated using shooting techniques.The procedure reveals accurately the state thermal ignition criticality conditions for as well as their dependent on viscous heating parameter.Finally,the shooting method can be used as an effective tool to investigate several other parameter dependent nonlinear boundary-value problem.

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