Dynamics of a Small Hydro-Power Station (SHS) Turbine for Slow Moving Water Body

Arinola B. Ajayi, Frank N. Okafor
Department of Electrical and Electronics Engineering,
Faculty of Engineering, University of Lagos,
Lagos, Nigeria
abajayi@unilag.edu.ng
frankok@yahoo.comg

Abstract - **Climate change due to carbon oxides emissions from fossil fuel is a major environmental concern for the world today. The world is now moving towards "clean energy sources" such as solar, wind, hydro-power stations to mention but a few for electricity generation. The process (dynamics) of converting the energy of flowing water bodies to electricity and the quantum of the derivable power depends largely on the head, the speed and the impact angle of the incident force of the water body on the turbine blades. It therefore follows that the determination of the optimal impact angle of the incident force of the water body on the turbine blades for small hydro-power stations (SHS) is of major engineering interest in slow moving water bodies where the head and the speed are relatively '*low*'. This article presents analytical technique for theoretical determination of the optimal impact angle of the incident force of the slow moving water bodies on the turbine blade of a small hydro-power station to yield maximum electric power to ensure optimal turbine blade designs for impact angle enhanced efficiency. It also investigates the variation of impact angle with the power output so as to determine the optimal impact angle for maximum power output. This SHS can easily be deployed by small and cottage firms in slow moving waters without elaborate cost and technology, and the electricity generated can be sold to the neighboring consumers thereby reducing their dependency on fossil fuel generators and national grid for electricity thus reducing the carbon foot print of such benefiting communities.**

Index Terms— Climate Change; Electricity Generation; Optimal Impact Angle; Small Hydro-power Stations.

I. INTRODUCTION

Climate change due to carbon oxides emissions from fossil fuel is already a major environmental concern for the world today. There is a global concern about the fossil fuels environmental impact and degradation such as global warming; therefore, there is need for sustainable alternative energy sources that are affordable and environmentally friendly. The world is now moving towards "clean energy sources" for electricity generation such as solar, wind, small hydro-power stations (SHS) to mention a few.

In hydro power generation, the quantum of energy generation depends largely on the head, the speed and the impact angle of the incident water body on the turbine blades. But in slow moving water bodies such as streams and lagoons, the head and speed are relatively lower than in the water that can be used for large hydro stations. Therefore, for SHS the impact angle of the incident force from the moving water body on the turbine blade to a large extent determines the quantum of derivable power from the system. It therefore follows that the determination of the optimal impact angle of the force of water on the turbine blades for SHS is of major engineering interest. This research effort intends to study the dynamics of SHS deployed for electricity generation.

SHS can be used by individuals and small companies to generate electricity and the electricity generated sold to the people in their locality thereby reducing dependence on the National Grid and fossil fuel electricity. A miniature model for electricity generation from slow flowing waters such as lagoons and streams can be easily replicated anywhere. Several works has been carried out on small hydropower stations and turbines [1 – 4]. Mohamad, Mokhlis, Abu Bakar, and Ping [5] presented the background of islanding and reviewed the operations of small hydro generation and related their discussion with the controller designed for other type of turbines interfaced with synchronous generator. Tabrizi and Gifani [6] compared the cost of small hydro power plants substituting pressure reducer valve in water transmission pipelines and discovered that small hydro power to be better. Bracken and Lucas [7] noted that small-scale hydropower is developing rapidly in many countries in response to policies of encouraging renewable energy and reducing reliance on fossil fuels. This rapid increase in the construction of hydroelectric turbines, they also opined, provides a substantial risk to migrating biota, especially fish but gave exception to some turbines. They suggested that the cumulative potential impacts of multiple hydropower sites on downstream fish passage including lampreys should be considered by regulatory agencies when planning development within catchments but did not carry out any design of the hydropower turbine. Bhandari, Poudel, Lee and Ahn [8] summarized the mathematical modeling of various renewable energy system particularly PV, wind, hydro and storage devices, and also summarized mathematical models of various MPPT techniques for hybrid renewable energy systems. Boodoo, McClain, Upegui, and Lopez [9] investigated the potential impacts on hydropower generation by applying six Colombian and two international best practice environmental flow methodologies within a small Andean river catchment, the Chinchina River basin. They tried to determine post-impact flow and hydropower production levels under the eight methodologies and a no environmental flow scenario using Microsoft Excel flow diversion and hydropower production spread sheet. They observed that hydropower production decreased with increasing levels of environmental flow releases. It has been discovered that extensive work has not been done on impact angle and how it affects the maximum power output for maximal power conversion in small hydro turbines. The main objective of this research is to investigate the variation of impact angle with the power output so as to determine the optimal impact angle for maximum power output of a typical SHS energy conversion process and predict optimal impact for various turbine configurations.

II. NOMENCLATURE

a area of the turbine blade

$\mathbf{F}, \mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z, \mathbf{F}_r$ force, force in the x, y, z, directions and resultant force respectively

h length of the turbine blade

i, j, k unit vectors along x, y, and z, axes of the rotating frame

I, J, K unit vectors along X, Y, and Z, axes of the inertia frame

$i, \dot{i}, \ddot{i}$; distance, first and second time derivatives of distance in i, direction

$j, \dot{j}, \ddot{j}$ distance, first and second time derivatives of distance in j direction

$k, \dot{k}, \ddot{k}$ distance, first and second time derivatives of distance in k direction

$\dot{m}$ mass flow rate of water

$\mathbf{M}_o$ Total moment about the centre of turbine blade shaft

M_0 moment about the centre of turbine blade shaft

P power developed by the turbine blade

$P0$(watts), $P10$(watts), $P20$(watts), $P30$(watts), $P40$(watts), $P45$(watts), $P50$(watts), $P60$(watts), $P70$(watts), $P80$(watts), $P90$(watts) power developed by the turbine when inclination angle to the vertical θ is 0^0, 20^0, 30^0, 40^0, 45^0, 50^0, 60^0, 70^0, 80^0, 90^0 respectively

$\mathbf{r}_p, \mathbf{v}_p, \mathbf{a}_p$ position vector, velocity and acceleration derivatives respectively of the water particle in the control volume about to hit the turbine blade

u velocity of water particle immediately after hitting the turbine blade

$\bar{v}$ velocity of the water body

$x, y, z,$ rotating frame axes

X, Y, Z inertia frame axes

$x, \dot{x}, \ddot{x}$; distance, first and second time derivatives of distance in x direction.

$y, \dot{y}, \ddot{y}$ distance, first and second time derivatives of distance in y direction.

$z, \dot{z}, \ddot{z}$ distance, first and second time derivatives of distance in z direction.

$\varphi, \dot{\varphi}, \ddot{\varphi}$; angle, angular velocity and angular acceleration of the turbine blade

$\theta, \dot{\theta}, \ddot{\theta}$ angle, angular velocity and angular acceleration of the turbine blade inclination to the vertical

ρ density of water at atmospheric temperature and pressure

α angular velocity of the turbine shaft and blade

$\mathbf{I}o$ mass moment of inertia of the blade and shaft

III. EQUATION FORMULATION

If the turbine blade is inclined to the vertical at angle θ and it is impinged by the flowing water body with velocity v, Fig. 1. The turbine blade will rotate counter clockwise with angular velocity $\dot{\varphi}$.

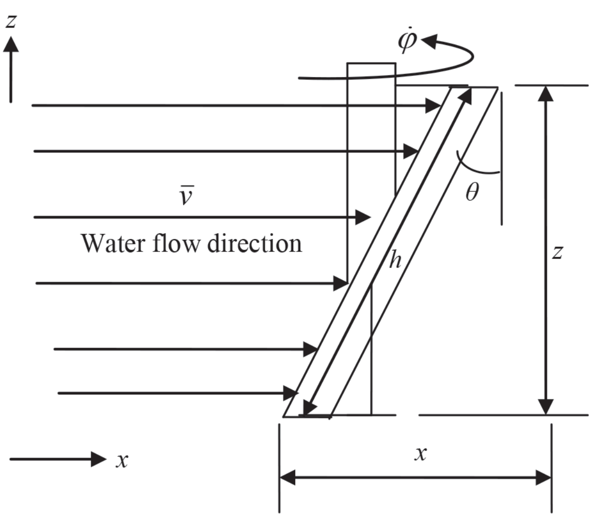

Fig. 1(a): Front view of the turbine impacted by water body.

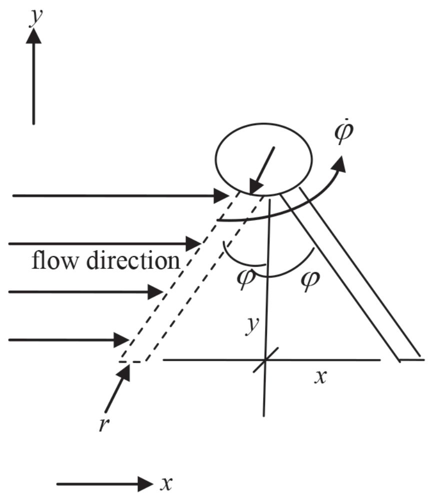

Fig. 1(b): The Plan of the turbine impacted by water body.

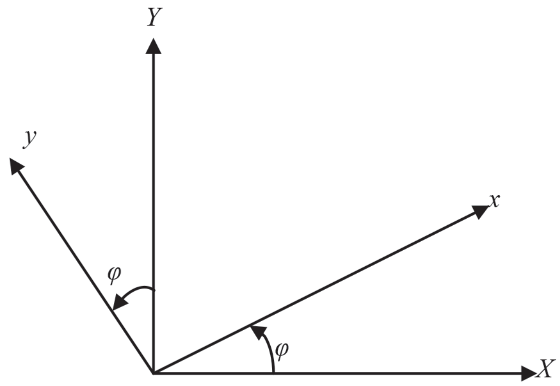

Fig. 2. The two coordinate system for the turbine/water dynamics

If two frames, inertia and rotating frames, Fig. 2, are assigned to the turbine blade. If the inertia frame coordinates are *XYZ* and the rotating frame coordinates are *xyz*, and the *Z* and *z* axes of the two frames coincide along the vertical, the angle between the rotating frame and inertia frame is gotten by rotating the rotating frame counter-clockwise through angle φ about *Z* (*z*) axes. Therefore, the unit vectors attached to the rotating frame are:

$$i = \cos\varphi \; I + \sin\varphi \; J \qquad \text{Eq. 1}$$

$$j = \cos\varphi \; J - \sin\varphi \; I \qquad \text{Eq. 2}$$

$$k = K \qquad \text{Eq. 3}$$

If the unit vectors above in Eq. 1, Eq. 2, and Eq. 3 are differentiated with respect to time, these will yield rate of change of these unit vectors, as expressed in Eq. 4, Eq. 5, and Eq. 6 respectively,

$$\dot{i} = -\dot{\varphi}\sin\varphi \; I + \dot{\varphi}\cos\varphi \; J = \dot{\varphi} \; j \qquad \text{Eq.4}$$

$$\dot{j} = -\dot{\varphi} \; \cos\varphi \; I - \dot{\varphi} \; \sin\varphi \; J = -\dot{\varphi} \; i \qquad \text{Eq.5}$$

$$\dot{k} = 0 \qquad \text{Eq.6}$$

The unit vectors are differentiated the second time to obtain Eq. 7, Eq. 8, and Eq. 9,

$$\ddot{i} = \ddot{\varphi} \; j + \dot{\varphi} \; \dot{j} = \ddot{\varphi} \; j - \dot{\varphi}^2 i \qquad \text{Eq.7}$$

$$\ddot{j} = -\ddot{\varphi} \; i - \dot{\varphi} \; \dot{i} = -\ddot{\varphi} \; i - \dot{\varphi}^2 j \qquad \text{Eq. 8}$$

$$\ddot{k} = 0 \qquad \text{Eq. 9}$$

The rotation of the moving frame attached to the shaft is $\dot{\varphi}$, this equals the angular velocity of the turbine shaft. The position vector associated with a water particle that is about to hit the turbine blade is

$$\mathbf{r}_p = \mathbf{r}_{P/B} = x \; i + y \; j + z \; k \qquad \text{Eq. 10}$$

The velocity and acceleration derivatives of the position vector are respectively:

$$v_p = \dot{x} \; i + \dot{y} \; j + \dot{z} \; k + x(\dot{\varphi} \; j) + y \; (-\dot{\varphi} \; i) + z(0)$$

Eq. 11

$$\mathbf{a}_p = \begin{Bmatrix} \ddot{x} \; i + \ddot{y} \; j + \ddot{z} \; k + x \; (\ddot{\varphi} \; j - \dot{\varphi}^2 i) + \\ y \; (-\ddot{\varphi} \; i - \dot{\varphi}^2 j) + 2(\dot{x} \; \dot{\varphi} \; j + \dot{y} \; (-\dot{\varphi} \; i)) \end{Bmatrix}$$

Eq. 12

From Fig. 1 (a) and (b) above *x, y,* and *z* can easily be determined thus:

$$x = h\sin\theta + r\sin\varphi \qquad \text{Eq. 13}$$

$$y = r\cos\varphi \qquad \text{Eq. 14}$$

$$z = h\cos\theta \qquad \text{Eq. 15}$$

The first time derivatives of Eq. 13, Eq. 14, and Eq. 15 are Eq. 16, Eq. 17, and Eq. 18 respectively:

$$\dot{x} = h\dot{\theta}\cos\theta + r\dot{\varphi}\cos\varphi \quad \text{Eq.16}$$

$$\dot{y} = -r\dot{\varphi}\sin\varphi \quad \text{Eq.17}$$

$$\dot{z} = -h\dot{\theta}\sin\theta \quad \text{Eq. 18}$$

The second time derivatives are Eq. 19, Eq. 20 and Eq. 21

$$\ddot{x} = h(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + r(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) \quad \text{Eq. 19}$$

$$\ddot{y} = -r(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) \quad \text{Eq. 20}$$

$$\ddot{z} = -h(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \quad \text{Eq. 21}$$

Expression for the forces generated by the moving water body is gotten by considering the momentum change of a control volume of water jet striking the turbine plate Figure 3. The control volume is attached to the turbine blade and it is fixed relative to the turbine blade, therefore, the control volume will be moving with the blade.

$$\dot{m} = \begin{Bmatrix}\text{Mass flow rate}\\ \text{outside the control}\\ \text{volume}\end{Bmatrix} - \begin{Bmatrix}\text{Mass flow rate required to fill}\\ \text{the control volume due to the}\\ \text{velocity of the turbine blade}\end{Bmatrix}$$

$$= \rho a\bar{v} - \rho a u = \rho a(\bar{v} - u) \quad \text{Eq.22}$$

Initial and final components of water velocity relative to turbine blade in *x* direction are respectively Eq 23 and Eq 24:

$$v_{xin} = (\bar{v} - u)(\cos\theta + \cos\varphi) \quad \text{Eq. 23}$$

$$v_{xout} = 0 \quad \text{Eq. 24}$$

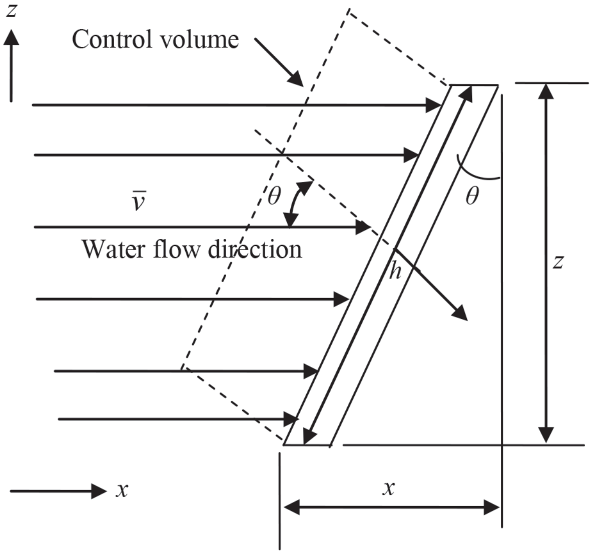

Fig. 3. Force exerted on the turbine blade.

Initial and final components of water velocity relative to turbine blade in *y* direction are Eq 25 and Eq. 26 respectively:

$$v_{yin} = (\bar{v} - u)\sin\varphi \quad \text{Eq. 25}$$

$$v_{yout} = 0 \quad \text{Eq. 26}$$

Initial and final components of water velocity relative to turbine blade in *z* direction are Eq. 27 and Eq. 28 respectively:

$$v_{zin} = (\bar{v} - u)\sin\theta \quad \text{Eq. 27}$$

$$v_{zout} = 0 \quad \text{Eq. 28}$$

The force exerted on the turbine blades by the fluid in *x, y,* and *z* directions are given in equations Eq. 29, Eq. 30 and Eq. 31 as:

$$\mathbf{F} = -\dot{m}(v_{out} - v_{in}) = \dot{m}(v_{in} - v_{out}) \quad \text{Eq. 29}$$

$$\dot{m} = \rho a(\bar{v} - u) \quad \text{Eq. 30}$$

$$\mathbf{F}_x = \dot{m}(v_{xin} - v_{xout}) \quad \text{Eq. 31 a}$$

$$\mathbf{F}_y = \dot{m}(v_{yin} - v_{yout}) \quad \text{Eq. 31 b}$$

$$\mathbf{F}_z = \dot{m}(v_{zin} - v_{zout}) \quad \text{Eq. 31c}$$

$$\mathbf{F}_r = \sqrt{\mathbf{F}_x^2 + \mathbf{F}_y^2 + \mathbf{F}_z^2} \quad \text{Eq. 34}$$

The free body diagram of the turbine blade is shown in Fig. 4, and it is being acted upon by gravitational and water moments. The moment developed is therefore,

$$\mathbf{M}_o = \sum M_o = I_o\alpha = -\frac{r}{2}(mg) + \mathbf{F}_r \bullet y \qquad \text{Eq. 35}$$

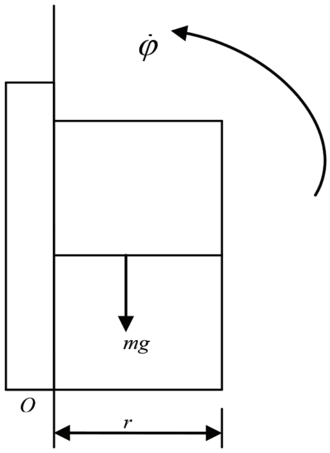

Fig. 4. The turbine blade

Power developed by the turbine shaft is

$$P = \mathbf{F}_r \bullet \mathbf{V}_p + \mathbf{M}_o \bullet \dot{\varphi} \qquad \text{Eq. 36}$$

IV. SIMULATIONS AND PARAMETRIC RESULTS

The basic values used for the simulations were tabulated in Table 1 below.

TABLE I. PARAMETRIC VALUES FOR SIMULATIONS

S/N	DESCRIPTION	SYMBOLS	VALUES
1	Turbine blade width	r	2m
2	Turbine blade length	h	1.2m
3	Stream velocity	$\overline{v}$	1, 3, 5 m/s
4	Blade inclination to the vertical	θ	0^0 - 90^0
5	Water density	ρ	1000kg/m^3
6	Estimated mass of one turbine blade	m	2kg
7	Acceleration due to gravity	g	9.81 m/s^2
8	Turbine blade swept angle	φ	$-90^0 - +90^0$

Fig. 5 -7 shows the result of the parametric simulations at different velocities. Fig. 5 is for 1m/s, Fig 6 is for 3m/s and Fig. 7 is for 5m/s, for a particular configuration of the SHS. It was discovered that the peak power was produced at angle between 15^0 and 20^0 inclination of the blade to perpendicular axis of the flow. It is also observed that the inclination of the blade to the vertical axis gives maximum power at angle 90^0 followed by 50^0.

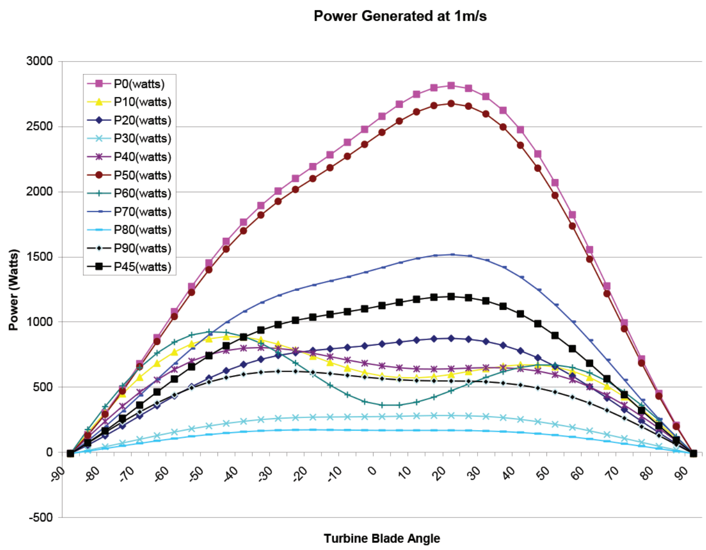

Fig. 5. Power generated at different turbine angle combinations at 1 m/s

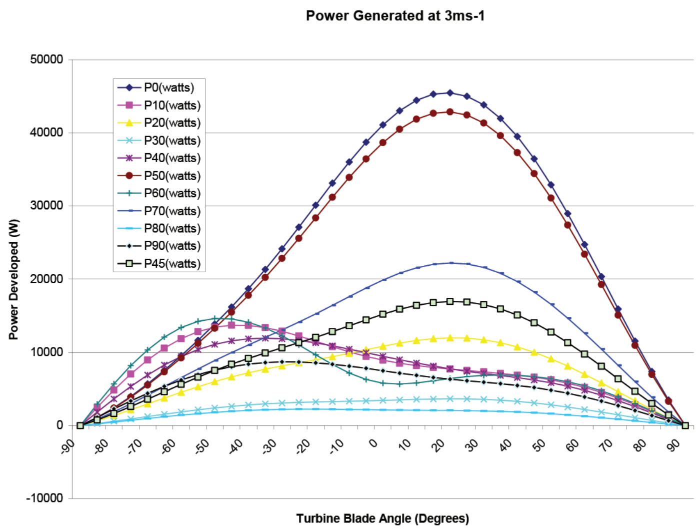

Fig. 6. Power generated at different turbine angle combinations at 3 m/s

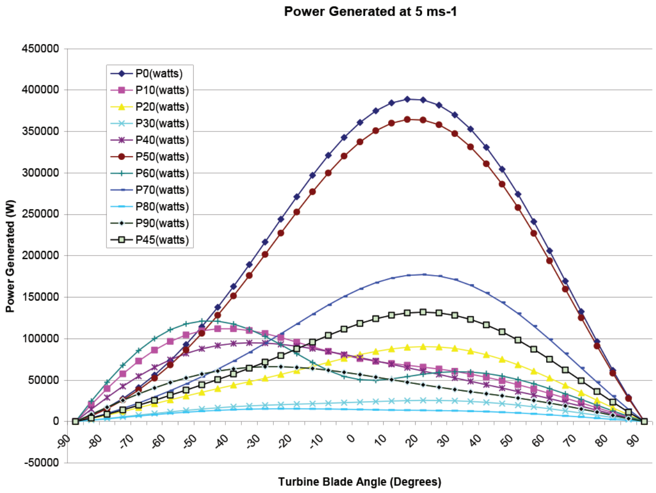

Fig. 7. Power generated at different turbine angle combinations at 5 m/s

V. CONCLUSION

The analysis of the optimal impact angle of the water body on the turbine blade has been presented. The necessary equations were first derived and used for simulation in other to determine the optimal impact angle. It was discovered that the optimal impact angle will be 50^0 inclination of the blade to the vertical in the *x-z* plane, when the blade is at 15^0 – 20^0 inclination in the *x-y* plane. The maximum power is determined from the graph as the peak of the graph. It is also discovered that for this configuration, three blades will give the maximum power output.

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